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Risk-averse asymptotics for reservation prices

Laurence Carassus^{*} Miklós Rásonyi[†]

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Abstract

An investor's risk aversion is assumed to tend to infinity. In a fairly general setting, we present conditions ensuring that the respective utility indifference prices of a given contingent claim converge to its superreplication price.

Keywords: Utility indifference price, Superreplication price, Convergence, Utility maximization, Risk aversion.

MSC 2000 Subject Classification: Primary: 91B16, 91B28 ; Secondary: 93E20, 49L20

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JEL classification: C61, C62, G11, G12

1 Introduction

In this article we investigate the effect of increasing risk aversion on utility-based prices. We are dealing with the utility indifference price (or reservation price), defined in [12] for the first time. This is the minimal amount added to an option seller's initial capital which allows her to attain the same utility that she would have attained from her initial capital without selling the option, see Definition 4.2 below. Intuitively, when risk aversion tends to infinity, reservation price should tend to the superreplication price (i.e. the price of hedging the option without any risk).

This result was shown in [19] for Brownian models and in [9] in a semimartingale setting when the agent has constant absolute risk aversion (i.e. for exponential utility functions). Certain other classes of utility functions were treated in [4], models with transaction costs were considered in [5].

However, an extension of this result to general utility functions was lacking. In [6] and [7] the case of discrete-time markets was treated for utilities on the positive axis as well as on the real line. Now we prove this result in a continuous-time semimartingale framework, under suitable hypotheses.

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In section 2 we model the agent's preferences and introduce a growth condition (related to the elasticity of utility functions), in section 3 the market model and a compactness assumption are discussed. In section 4 the concept of utility indifference price is formally defined and the two main theorems are proved.

2 Risk averse agents

We consider agents trading in the market with initial endowment $z \in \mathbb{R}$. We assume that

Assumption 2.1 U_n , $n \in \mathbb{N}$ are twice continuously differentiable, strictly concave and increasing functions on \mathbb{R} such that for each $x \in \mathbb{R}$,

$$r_n(x) := \frac{-U_n''(x)}{U_n'(x)} \xrightarrow{n \rightarrow +\infty} \infty. \quad (1)$$

The function r_n is called the (absolute) risk aversion of an agent with utility function U_n . This concept was introduced in [1] and [17]. Here we are interested in the case where this measure of risk aversion tends to infinity.

We take the Fenchel conjugates of U_n :

$$V_n(y) := \sup_{x \in \mathbb{R}} \{U_n(x) - xy\}, \quad y \in (0, \infty). \quad (2)$$

As easily checked, V_n is a finite convex function.

We stipulate a growth condition on the conjugates of the utility functions we consider. Such assumptions are often referred to as "elasticity conditions".

Assumption 2.2 For each $[\lambda_0, \lambda_1] \subset (0, \infty)$ there exist positive constants C_1, C_2, C_3 such that for all n and for all $y > 0$,

$$V_n(\lambda y) \leq C_1 V_n(y) + C_2 y + C_3. \quad (3)$$

holds for each $\lambda \in [\lambda_0, \lambda_1]$.

Remark 2.3 For n fixed, condition (3) is equivalent to

$$\limsup_{x \rightarrow \infty} \frac{xU_n'(x)}{U_n(x)} < 1, \quad \liminf_{x \rightarrow -\infty} \frac{xU_n'(x)}{U_n(x)} > 1, \quad (4)$$

as shown in [11].

The first of the two conditions in (4) was introduced in [15], the second one in [20]. A utility function U_n satisfying (4) is said to have *reasonable asymptotic elasticity* (terminology of [20]). Thus condition (3) is a (dually formulated) uniform reasonable asymptotic elasticity condition. Another uniform asymptotic elasticity condition appears as Assumption 2.3 of [8]. About the derivation of equivalences like that of (4) and (3) consult section 6 of [15], section 4 of [20] and [11].

3 Market Model

Our market is modelled by an adapted d -dimensional stochastic process S on a given continuous-time stochastic basis $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P)$. We think that S represents the evolution of the (discounted) prices of d assets. For simplicity we assume S locally bounded (to avoid technical complications related to σ -martingales). Absence of arbitrage is stipulated by

$$\mathcal{M} \neq \emptyset,$$

where \mathcal{M} denotes the set of measures $Q \sim P$ such that S is a local martingale under Q .

We make the following compactness assumption.

Assumption 3.1 There exists $Q_0 \in \mathcal{M}$ such that the sequence $V_n(dQ_0/dP)$ is uniformly integrable (with respect to P). We denote by \mathcal{M}_v the set of such Q_0 s.

Remark 3.2 Similar conditions have already appeared in investigations on the stability of optimal strategies with respect to perturbations of utility functions, see [16] and [14]. Both papers consider a sequence U_n of utility functions converging to a limiting utility U and show convergence of the corresponding optimal strategies/utility prices.

In [16] a sequence U_n is assumed to be dominated by some \bar{U} with conjugate function \bar{V} such that $\bar{V}(Z)$ is integrable, where Z is (the candidate for) the density of the minimal martingale measure. In Assumption (UI) of [14] we find essentially the uniform integrability of $V_n(ydQ_n/dP)$ for each $y > 0$, where Q_n is a suitable sequence in \mathcal{M} . Note that [14] also considers perturbations of the underlying probability measure and that in both mentioned papers the U_n are defined on the positive axis.

This elasticity hypothesis 2.2 allows us to prove that the set \mathcal{M}_v is in fact large.

Lemma 3.3 Assume that there is $x_0 \in \mathbb{R}$ such that $U_n(x_0)$ is bounded from below. Under Assumptions 2.2 and 3.1, the set \mathcal{M}_v is dense in \mathcal{M} with respect to the total variation norm topology.

Proof. This result was essentially reported in Proposition 6 of [3], without proof. Fix $Q_0 \in \mathcal{M}_v$. We know from [13] that the set

$$\mathcal{M}_b := \{Q \in \mathcal{M} : dQ/dQ_0 \text{ is bounded} \}$$

is dense in \mathcal{M} . It follows by concavity of \mathcal{M} that also

$$\mathcal{M}_{bb} := \{\alpha Q_0 + (1 - \alpha)Q : 0 < \alpha < 1, Q \in \mathcal{M}_b\}$$

is a dense subset of \mathcal{M} . We shall show that $\mathcal{M}_{bb} \subset \mathcal{M}_v$. Take an arbitrary $Q \in \mathcal{M}_b$ such that $dQ/dQ_0 \leq K$ and any fixed $0 < \alpha < 1$. We have

$$V_n(\alpha dQ_0/dP + (1 - \alpha)dQ/dP) \geq U_n(x_0) - x_0(\alpha dQ_0/dP + (1 - \alpha)dQ/dP).$$

Moreover,

$$V_n(\alpha dQ_0/dP + (1 - \alpha)dQ/dP) \leq C_1 V_n(dQ_0/dP) + C_2(dQ_0/dP) + C_3,$$

by Assumption 2.2 (choose C_1, C_2, C_3 corresponding to the interval $[\alpha, \alpha + (1 - \alpha)K]$ and note that $\alpha + (1 - \alpha)dQ_0/dQ$ falls into this interval).

Uniform integrability of $V_n(\alpha dQ_0/dP + (1 - \alpha)dQ/dP), n \in \mathbb{N}$ now follows from Assumption 3.1. \square

4 Utility indifference prices

It follows from $\mathcal{M} \neq \emptyset$ that S is a semimartingale and we may define the set of admissible trading strategies as predictable S -integrable processes with a finite credit line (to avoid doubling strategies).

Definition 4.1

$$\mathcal{A} := \{\phi : \phi \text{ predictable, } S\text{-integrable and for some } w > 0, V^{0,\phi} \geq -w\},$$

where we write $V_t^{x,\phi} := x + (\phi \cdot S)_t$ for the value process of strategy ϕ starting from initial endowment x .

Fix $T > 0$ and a bounded random variable G , interpreted as a contingent claim to be delivered at the end of the period $[0, T]$. By optimal trading an agent with initial capital $x \in \mathbb{R}$ and utility U_n delivering the claim G may attain

$$u_n(x, G) := \sup_{\phi \in \mathcal{A}} EU_n(V_T^{x,\phi} - G), \quad (5)$$

this is well-defined as admissible strategies have bounded from below value processes.

Definition 4.2 The utility indifference price for U_n and initial endowment z is

$$p_n(z, G) := \inf\{p \in \mathbb{R} : u_n(z + p, G) \geq u_n(z, 0)\}, \quad (6)$$

the minimal extra capital that allows for delivering G while attaining the same utility as without claim delivery.

The utility-free concept of the superhedging price of a bounded contingent claim (random variable) G is defined as

$$\pi(G) := \inf\{x \in \mathbb{R} : \text{there is } \phi \in \mathcal{A} \text{ such that } V_T^{x,\phi} \geq G \text{ a.s.}\}.$$

The following dual characterization is a fundamental result of mathematical finance, see e.g. [10] and the references therein.

Theorem 4.3 Under $\mathcal{M} \neq \emptyset$, we have

$$\pi(G) = \sup_{Q \in \mathcal{M}} E_Q G.$$

We are now ready to state our main result i.e. the convergence of utility indifference prices to the superreplication price. We will present this result under two types of assumptions. The first one (see Assumption 4.4) refers to the existence of a initial wealth for which all the investors have (asymptotically) a common preference for x_0 and also a common non zero growth rate for their preferences near x_0 .

The second kind of assumptions impose the elasticity Assumption 2.2 and the compactness Assumption 3.1 on a normalized family of utility functions (see Theorem 4.13).

Assumption 4.4 There exists some $x_0, \alpha, \beta \in \mathbb{R}$ with $\alpha \in (0, \infty)$ and $|\beta| < \infty$ such that :

$$\begin{aligned} U'_n(x_0) &\xrightarrow{n \rightarrow +\infty} \alpha \\ U_n(x_0) &\xrightarrow{n \rightarrow +\infty} \beta. \end{aligned}$$

Remark 4.5 We will see in the examples below that this assumption is satisfied for power and exponential utility functions. In those examples, we have that $U_n(0) = 0$ and $U'_n(0) = 1$. This means that all investors consider that utility of nothing is zero and that for very small wealth their utility functions are approximatively linear with slope one.

The following proposition is the first step to Theorem 4.7 :

Proposition 4.6 Under Assumptions 2.1 and 4.4, for each $y > 0$,

$$V_n(y) \xrightarrow{n \rightarrow +\infty} \beta - x_0 y.$$

Proof. First, the argument of Lemma 4 in [6] shows that as $n \rightarrow \infty$,

$$U'_n(x) \rightarrow \infty, \quad x < x_0, \quad U'_n(x) \rightarrow 0, \quad x > x_0. \quad (7)$$

Let I_n be the inverse of U'_n (which exists by strict concavity of U_n). We claim

$$I_n(y) \rightarrow x_0, \quad \text{for } y > 0.$$

Indeed, let $y > 0$. If we had $I_{n_k}(y) \geq x_0 + \varepsilon$ for some $\varepsilon > 0$ and a subsequence n_k then $U'_{n_k}(I_{n_k}(y)) = y \leq U'_{n_k}(x_0 + \varepsilon)$, but this latter tends to 0 by (7), a contradiction. The proof of the reverse inequality is similar and $I_n(y) \rightarrow x_0$.

First by definition $V_n(y) \geq U_n(x_0) - x_0 y$. Since V_n can be calculated as $V_n(y) = U_n(I_n(y)) - I_n(y)y$; concavity of U_n implies that $V_n(y) \leq U_n(x_0) + (I_n(y) - x_0)U'_n(x_0) - I_n(y)y$. Thus

$$0 \leq V_n(y) - (U_n(x_0) - x_0 y) \leq (I_n(y) - x_0)(U'_n(x_0) - y) \rightarrow 0,$$

$n \rightarrow \infty$, showing the claim. \square

Theorem 4.7 If Assumptions 2.1, 2.2, 3.1 and 4.4 hold then for each bounded random variable G the quantities $u_n(x_0, G)$, $n \in \mathbb{N}$ are finite and the corresponding utility indifference prices $p_n(x_0, G)$ tend to $\pi(G)$ as $n \rightarrow \infty$.

Remark 4.8 The convergence in Theorem 4.7 holds only for x_0 . We will see in Theorem 4.13 how the convergence for all initial capital z can be checked under a second type of assumptions.

Proof. It is a standard fact that that $p_n(x_0, G) \leq \pi(G)$, see page 152 of [6]. For the reverse inequality, we argue by contradiction. Suppose that for some $\varepsilon > 0$ and a subsequence n_k one has $p_k := p_{n_k}(x_0, G) \leq \pi(G) - \varepsilon$ for each k . We may suppose that $n_k = k$ and $p_k \rightarrow \pi(G) - \varepsilon$, $k \rightarrow \infty$.

Take arbitrary $\phi \in \mathcal{A}$ and $Q \in \mathcal{M}_v$. Then $E_Q(V_T^{x_0+p_k, \phi} - G) \leq x_0 + p_k - E_Q G$ ($V^{0, \phi}$ is a supermartingale by results of [2]). Proposition 4.6 and $Q \in \mathcal{M}_v$ imply

$$\begin{aligned} \limsup_{k \rightarrow \infty} E(V_k(dQ/dP) + (dQ/dP)(V_T^{x_0+p_k, \phi} - G)) & \quad (8) \\ & \leq \beta - x_0 + (x_0 + \limsup_{k \rightarrow \infty} p_k - E_Q G). \end{aligned}$$

Introduce

$$v_k^\phi := \inf_{Q \in \mathcal{M}_v} E(V_k(dQ/dP) + (dQ/dP)(V_T^{x_0+p_k, \phi} - G)).$$

It follows from Lemma 3.3 that $\sup_{Q \in \mathcal{M}_v} E_Q G = \sup_{Q \in \mathcal{M}} E_Q G$, thus (8) and Theorem 4.3 imply

$$\limsup_{k \rightarrow \infty} v_k^\phi \leq \beta + \lim_{k \rightarrow \infty} p_k - \pi(G) = \beta - \varepsilon.$$

By the definition of conjugate functions we have,

$$\limsup_{k \rightarrow \infty} EU_k(V_T^{x_0+p_k, \phi} - G) \leq \limsup_{k \rightarrow \infty} v_k^\phi \leq \beta - \varepsilon.$$

It follows that u_k are finite and $\limsup_{k \rightarrow \infty} u_k(x_0 + p_k, G) \leq \beta - \varepsilon$. But $u_k(x_0 + p_k, G) \geq u_k(x_0, 0) \geq U_k(x_0)$, thus $\limsup_{k \rightarrow \infty} u_k(x_0 + p_k, G) \geq \beta$ a contradiction. \square

Remark 4.9 Let us consider the condition that there exists $Q_0 \in \mathcal{M}$ such that dQ_0/dP and dP/dQ_0 are both bounded (by some $K > 0$).

Note that a convex function attains its maximum on an interval at one of the endpoints. Hence

$$U_n(x_0) - |x_0|K \leq V_n(dQ_0/dP) \leq |V_n(K)| + |V_n(1/K)|$$

which is bounded by Proposition 4.6, showing that this condition is stronger than Assumption 3.1.

We claim that, replacing Assumption 3.1 by this condition, one may drop Assumption 2.2 from the hypotheses of Theorem 4.7. Indeed, the result of [13] directly implies that measures Q_0 with the above property are dense in \mathcal{M} , without appeal to Assumption 2.2. The rest of the proof is identical.

Unfortunately, in continuous-time models one rarely finds such a Q_0 . In discrete-time models, however, such measures often exist, see [18] for an extensive discussion.

We now turn to our second Theorem. To state its hypothesis, we first need to introduce some normalization of the functions U_n . Fix some initial wealth $z \in \mathbb{R}$ and set :

$$\tilde{U}_n(x) := \frac{U_n(x) - U_n(z)}{U'_n(z)}, \quad n \in \mathbb{N}, x \in \mathbb{R}. \quad (9)$$

We now restate Assumptions 2.2 and 3.1 for the family $(\tilde{U}_n)_n$.

Assumption 4.10 For each $[\lambda_0, \lambda_1] \subset (0, \infty)$ there exist constants C_1, C_2, C_3 such that for all n and for all $y > 0$,

$$\tilde{V}_n(\lambda y) \leq C_1 \tilde{V}_n(y) + C_2 y + C_3. \quad (10)$$

holds for each $\lambda \in [\lambda_0, \lambda_1]$.

Assumption 4.11 There exists $Q_0 \in \mathcal{M}$ such that the sequence $\tilde{V}_n(dQ_0/dP)$ is uniformly integrable (with respect to P).

Remark 4.12 We think that Assumptions 4.10 and 4.11 should be considered as technical assumptions to be checked on \tilde{U}_n . We will see below that they hold true in our examples.

Theorem 4.13 Assume that Assumptions 2.1, 4.10 and 4.11 hold. Then for each bounded random variable G the indifference prices $p_n(z, G)$ tend to $\pi(G)$ as $n \rightarrow \infty$.

Proof. Obviously, $\tilde{U}_n(z) = 0$ and $\tilde{U}'_n(z) = 1$, thus Assumption 4.4 is satisfied for the sequence \tilde{U}_n at $x_0 = z$. Let \tilde{u}_n be defined analogously as in (5), with \tilde{U}_n replacing U_n . Then since the \tilde{U}_n are affine transforms of the U_n , we may alternatively write

$$p_n(z, G) = \inf\{p \in \mathbb{R} : \tilde{u}_n(z + p, G) \geq \tilde{u}_n(z, 0)\}.$$

It is obvious that if Assumption 2.1 hold for U_n then it holds also for \tilde{U}_n , so Theorem 4.7 applied for \tilde{U}_n and $x_0 = z$ allow us to conclude. \square

Example 4.14 (The exponential case) Let

$$U_n(x) = \frac{1 - \exp\{-\alpha_n x\}}{\alpha_n},$$

with some $\alpha_n > 0$ tending to ∞ as $n \rightarrow \infty$. It is straightforward that $U_n(0) = 0$, $U'_n(0) = 1$, $r_n(x) = \alpha_n$ and thus U_n satisfy Assumption 2.1 and 4.4 for $x_0 = 0$. Moreover, calculation gives $V_n(y) = (1/\alpha_n)[y \ln y + 1 - y]$ showing that Assumption 3.1 holds provided that there is $Q_0 \sim P$ with $E_{Q_0} \ln(dQ_0/dP) < \infty$, i.e. whenever a finite-entropy martingale measure exists. It is enough to show (3) for one function, namely $V(y) = y \ln y$. This is trivial since $\lambda y \ln(\lambda y) \leq \lambda y \ln y + \lambda |\ln \lambda| y$ for $\lambda, y > 0$.

So Theorem 4.7 applies and the respective reservation prices $p_n(0, G)$ converge to $\pi(G)$.

In fact, one can check that Theorem 4.13 applies for each $z \in \mathbb{R}$. We have thus retrieved the result in [9]. (They make a weaker moment assumption on G ; this extension follows by our method, too.)

Example 4.15 (The power case)

$$U_n(x) = -\frac{1}{\alpha_n}[(x+1)^{-\alpha_n} - 1]1_{\{x>0\}} - \frac{1}{\beta_n}[(1-x)^{\beta_n} - 1]1_{\{x \leq 0\}}$$

with $\alpha_n > 0, \beta_n > 1$ tending to ∞ . These functions are continuously differentiable with strictly monotone derivatives, hence they are strictly concave. They are also twice continuously differentiable on $\mathbb{R} \setminus \{0\}$ and satisfy (1) on $\mathbb{R} \setminus \{0\}$. Take now $\beta_n := \alpha_n + 2$ then U''_n exists and (1) holds in 0, too.

$$\begin{aligned} U'_n(x) &= (x+1)^{-(\alpha_n+1)}1_{\{x>0\}} - (1-x)^{\alpha_n+1}1_{\{x<0\}} \\ U''_n(x) &= -(\alpha_n+1)(x+1)^{-(\alpha_n+2)}1_{\{x>0\}} - (\alpha_n+1)(1-x)^{\alpha_n}1_{\{x<0\}} \end{aligned}$$

Thus $U_n(0) = 0$, $U'_n(0) = 1$, $r_n(x) = (\alpha_n + 1)(\frac{1_{\{x>0\}}}{x+1} + \frac{1_{\{x<0\}}}{1-x})$ and U_n satisfy Assumptions 2.1 and 4.4 for $x_0 = 0$.

It remains to check (3) and Assumption 3.1. We have, for $y \in (0, \infty)$,

$$V_n(y) = \left[\frac{1}{\alpha_n + 2} - y + \left(1 - \frac{1}{\alpha_n + 2} \right) y^{\frac{\alpha_n + 2}{\alpha_n + 1}} \right] 1_{\{y>1\}} + \left[\frac{1}{\alpha_n} + y - \left(1 + \frac{1}{\alpha_n} \right) y^{\frac{\alpha_n}{\alpha_n + 1}} \right] 1_{\{y \leq 1\}}.$$

Thus if there exists $Q_0 \in \mathcal{M}$ such that $dQ_0/dP \in L^{1+\varepsilon}$ for some $\varepsilon > 0$ then Assumption 3.1 will be satisfied for n large enough since $(\alpha_n + 2)/(\alpha_n + 1)$ converges to 1. Finally, (3) is trivially satisfied by the scaling properties of the power functions. We may conclude by Theorem 4.7 that the reservation prices $p_n(0, G)$ tend to $\pi(G)$ as $n \rightarrow \infty$.

Tedious calculations show that we get the same conclusion for any initial capital $z \in \mathbb{R}$ and we may apply Theorem 4.13.

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