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Vincent Rossetto, A. C. Maggs

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## Comment on “Elasticity Model of a Supercoiled DNA Molecule”

Bouchiat and Mézard [1] have proposed a “wormlike rod chain model” to describe recent experiments on DNA mechanics [2]. Their model can be solved using an analogy with quantum mechanics, thanks to the *local* nature of the effective Hamiltonian. The Letter concluded that the continuum limit of DNA elasticity is singular and linear response theory cannot be used in the absence of a cut-off: For example, the curves of extension versus torsion flatten in the continuum limit. We show here that this singular behavior is an artifact of an inexact formulation of the writhing geometry. An exact formulation of the experimental geometry needs no extra regularization.

The writhe,  $Wr = \chi_W^C/2\pi$ , of a closed chain is given by the Călugăreanu-White formula [3]

$$\chi_W^C = \frac{1}{2} \iint_0^L \frac{\mathbf{r}(s) - \mathbf{r}(s')}{|\mathbf{r}(s) - \mathbf{r}(s')|^3} \cdot \frac{d\mathbf{r}(s)}{ds} \times \frac{d\mathbf{r}(s')}{ds} ds ds', \quad (1)$$

where  $\mathbf{r}(s) = \int_0^s \mathbf{t}(s') ds'$  is the position in space and  $L$  is the length of the chain. Fuller showed that if we represent the tangent vector  $\mathbf{t}$  by a point on the unit sphere the area,  $\chi_W^F$ , enclosed by  $\mathbf{t}(s)$  on the sphere [4], is related to  $\chi_W^C$ . The relation between Fuller’s and Călugăreanu’s formulas is

$$\chi_W^F \equiv \chi_W^C \pmod{4\pi}. \quad (2)$$

Fuller’s result, expressed as a local integral, was used in [1] to calculate the writhe. The authors find a Cauchy probability distribution,  $p(\chi_W^F)$ , varying as  $1/(\chi_W^F)^2$  for large  $\chi_W^F$  due to winding of trajectories about the south pole of the sphere. It is this broad distribution of probability which leads to the breakdown of linear response. We proceed by examining numerically the distribution function for writhe in the two formulations.

We numerically generate a large equilibrated ensemble of open semiflexible chains of 8 times the persistence length, made of  $8N$  straight links with  $\mathbf{t}(0)$  and  $\mathbf{t}(L)$  parallel. This assures that the path of  $\mathbf{t}(s)$  on the sphere is a closed loop. In order to use the Călugăreanu-White formula, the chain must also be closed in real space. To do this we extend the chain at each end with a long straight segment [5] in the directions  $-\mathbf{t}(0)$  and  $\mathbf{t}(L)$  and then join the two extremities with an arc of a circle, effectively closing the chain at infinity; it is this augmented version of Eq. (1) which determines the rotation angle measured in experiments on open chains. We calculate  $\chi_W^F$  and  $\chi_W^C$  for each chain from our ensemble and plot the integrated probability  $P(\chi_W) = \int_{-\infty}^{\chi_W} p(\chi) d\chi$ . For the Cauchy distribution, one expects  $\chi_W^F \sim \frac{1}{1-P}$  for large values of  $\chi_W^F$ .

Results are displayed in Fig. 1. The curves of  $P$  as a function of  $\chi_W^F$  depend on discretization and converge to a

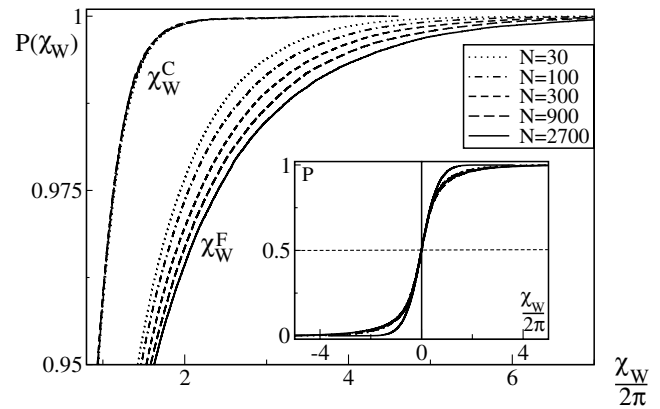


FIG. 1. Inset: The integrated writhe distribution function  $P(\chi_W)$  computed with two different formulations of the writhe.  $\chi_W^C$  is not computed for  $N = 2700$  due to computer limitations. The main figure is a zoom of the top right corner of the inset. One can compare the continuous evolution of  $\chi_W^F$  as  $N$  grows with the stability of  $\chi_W^C$  in the same limit. An asymptote in  $\chi_W^F \sim \frac{1}{1-P}$  observed in this figure corresponds to the Cauchy distribution. The singularity of  $\chi_W^C$  is clearly weaker than  $\frac{1}{1-P}$ . Statistical fluctuations of the curves are smaller than their width.

Cauchy tail. We have additionally verified that the curves of  $\chi_W^F$  scaled by the factor  $\sqrt{\ln N}$  converge very accurately to a master curve as predicted in [1]. On the other hand,  $\chi_W^C$  converges to a limiting curve as  $N$  grows. We find no evidence of divergence of  $\chi_W^C$  as  $\frac{1}{1-P}$  nor evidence of scaling with  $\sqrt{\ln N}$ . From Fig. 1 we see that even for  $N$  small, the Fuller formulation strongly overestimates the statistical weight of large writhe fluctuations.

To conclude, we believe that fluctuations of semiflexible chains are not divergent when going to the continuum limit. A DNA chain has a persistence length of order 50 times its radius; it is in a regime where Eq. (1) has reached the continuum limit to a very good approximation. The singular limit emphasized in [1] is due to the ambiguity introduced by the modulo relation Eq. (2).

V. Rossetto and A.C. Maggs

PCT (UMR CNRS 7083), ESPCI, 10 rue Vauquelin  
75231 Paris CEDEX 05, France

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