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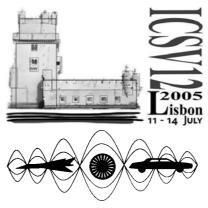
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Twelfth International Congress on Sound and Vibration

# A CRITERION FOR DETECTING NONSTATIONARY EVENTS

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#### **Abstract**

This paper tackles the problem of detecting nonstationary events in a signal. An ergodic process is stationary if its law is invariant whatever the time translation is. So the control of the invariance of the law parameters can lead to a stationarity measure. We propose such a test stated from a time-frequency representation so as to localize both in time and frequency nonstationary events. At each frequency we define two observation subsets. The first one is the subset of time frequency points, whose process is random and stationary eventually added with a deterministic process. The second one gathers all time-frequency points, whose process is non stationary. The problem is thus simplified into two competing sets or hypotheses between which we have to choice. A binary hypothesis test is stated from the Neyman-Pearson criterion. We construct a decision rule to have maximum probability of detection while not allowing the probability of false alarm to exceed a given value. Given that no a priori information is known, a recursive algorithm is performed in order to estimate the unknown parameters of the decision rule. The test converges when the subsets become steady. To be independent of the choice of the false alarm probability, the decision rule is applied for different values of this probability. To initialize the algorithm, we only assume that a time stationary part exists at each frequency. Under all these assumptions, all types of nonstationarities can be detected. The proposed detector is a post-processing to a timefrequency estimator. In this paper, we use a spectrogram or a gliding correlogram, which sets the size of the nonstationary events to detect. The detector could be adapted to any other time-frequency estimator if its statistical law is known. Applications on real signals are carried out and show that the proposed method performs well.

#### INTRODUCTION

To detect nonstationary occurrences in a signal is a current problem in a great deal of domains. Natural sources are indeed nonstationary rather than stationary. Acoustic signals such as speech, breath sounds, bioacoustic sounds emitted by whales and dolphins, seismic signals such as those emitted by avalanches or earthquakes contain occurrences that can be of interest to detect. Engineering systems also produce nonstationary signals such as cavitation signals, transient signals got from shocks on a structure, vibratory signals in rotating machine or sensor signals. The processing of these signals is not trouble-free. Each possible method takes accounts of some particular characteristics of the signal.

In this paper, we do not made any specific assumption about the nonstationarities of the signal. We propose a method that is suitable to all signals, whatever the variations might be, and that is of interest whenever a simple Cusum test or an energy detector does not perform well. This method only depends on a time-frequency representation (TFR), which must afford an adequate resolution. Under this assumption, the method can be widely applied to the detection of transients, frequency modulated signals, frequency jumps, births or deaths of deterministic or random components. Only amplitude modulations will be badly detected. As is to be expected if any *a priori* information is used, the method is not optimal but robust and automatic.

### PRINCIPLE OF THE METHOD

An ergodic random process is completely stationary if its probability density function (pdf) remains unaltered under a time shift. This definition is a severe requirement and is often relax by considering a stationarity up to a given order n. Under this weaker condition, the moments of process should be identical up to order n only. These definitions imply the ergodicity of the process, that is to say the equality between time and ensemble averages. We *a priori* assume this property, so that the moments estimated with observations from a single realization converges in mean square to the corresponding ensemble average. Furthermore, if the process is Gaussian, the stationarity up to order 2 necessarily implies complete stationarity [PRI 81].

In this paper, we are dealing with nonstationary random signals, whose probability density function changes over time. Consequently, the mean and the variance change over time and are thus functions of time. The detection scheme implies the knowledge of the probability density function of the process. The detector is here designed for a Gaussian noise, but it could be extended to other types of random process.

In this framework, a measure of the stationarity results in the time invariance control of the statistical properties of the process. Such an approach requires a time observation scale and a way of measuring the moment invariance. In addition, we aim at estimating both the time and the frequency locations of each nonstationary occurrence. As we already said in the introduction, we design the detector in the time-

frequency plane to cope with this problem. The TFR is estimated with a spectrogram or a gliding correlogram, whose time and frequency resolutions are given *a priori*. The detector is then a post-processing, whose properties depend on the TFR properties.

# STATISTICAL TEST IN THE TIME-FREQUENCY PLANE

One of important statistical tools in signal theory detection is hypothesis testing. The hypotheses are statements of the possible decisions that are being considered. Among possible criterion for making decisions, the Bayes criterion is optimal because the average cost of all possible decision or risk is minimized [VTR 68], [WHA 71], [PRO 89]. Applied to signal processing, the hypotheses were first defined from time data [ARQ 82]. When data are nonstationary, it can be of interest to define the decision rule in a more adapted space. In 1980, Altès defined a detector from a spectrogram [ALT 80]. Kumar and Carroll extended it to a Wigner Ville distribution [KUM 84]. At that time, these authors agreed about the fact that the performance in a time-frequency space is identical to that obtained in a time space only. See also [KAY 85] and [FLA 86]. Marinovich decomposed the TFR in singular values in order to separate the signal components from the noise [MAR 91]. Later on, Lemoine showed the limits of an optimal detector [LEM 95].

Matz and Hlawatsch proposed a composite hypothesis test defined in a time-frequency region [MAT 99]. But, in this case, a training data set is necessary. Richard and Lengellé defined a performing linear detector, which suppresses the redundancy in a Wigner-Ville distribution [RIC 98]. Friedlander and Porat proposed a detector based on the maximum likelihood function applied on parts of the RTF only [FRI 89], [POR 92]. This algorithm is suitable to transient detection but, indeed, is not optimal.

In this paper, we propose a detector, which takes advantage of the locality of the TFR as well. But we do not want using any *a priori* information on the signal because many applications require this context. We cannot thus determine the Bayes risk and we will consider the Neyman-Pearson criterion, which involves neither *a priori* probabilities nor cost estimates. In comparison with previous works, theis detector assigns a label to each time frequency point without defining regions or contours of these regions [HOR 02], [MAR 03].

Let x[n] be a discrete time signal. The observation set  $\mathcal{L}_x[k]$  is a subset of  $\mathbb{R}^2$  so that

$$\mathcal{L}_{x}[k] = \{(n,k) \in \mathbb{R}^{2} / \exists S_{x}[n,k] \text{ for a given } x[n]\},$$
 (1)

with k the frequency variable and  $S_x[n,k]$  the RTF of x[n]. As such,  $\mathcal{L}_x[k]$  is a cross section of  $S_x[n,k]$  at a constant frequency k. Let us define two hypotheses:

- $H_0$  where x[n] is a stationary random process b[n] with zero mean and constant variance, added with a stationary deterministic signal d[n] eventually;
- $H_1$  where x[n] is nonstationary.

Under  $H_0$ , the observation set  $\mathcal{L}_{H_0}[k]$  is defined as

$$\mathcal{L}_{H_0}[k] = \left\{ (n,k) \in \mathcal{L}[k] / p_{S,|H_0} = p_0 \text{ et } E(S_x[n,k]) \text{ min} \right\}, \tag{2}$$

with  $p_{S_x|H_0}$  the pdf of the observation  $S_x[n,k]$  under  $H_0$ . Under  $H_1$ , the observation set  $\mathcal{L}_{H_1}[k]$  is the complement of  $\mathcal{L}_{H_0}[k]$ . Due to the definition of  $H_0$ ,  $S_x[n,k]$  is distributed as gamma  $\Gamma(r/2,\alpha,0)$ , which defines  $p_0$  [DUR 99]. The parameter r is an equivalent degree of freedom, which is derived from the normalized variance  $\mathrm{Varn}(S_x[n,k])$  of the observation

$$\operatorname{Varn}\left(S_{x}[n,k]\right) = \frac{\operatorname{Var}\left(S_{x}[n,k]\right)}{\left(E\left(S_{x}[n,k]\right)\right)^{2}} = \frac{2}{r}.$$
 (3)

The parameter  $\alpha$  is defined as

$$\alpha = E(S_b[k, \nu])/r, \tag{4}$$

where  $S_b[n,k]$  is the RTF of b[n]. If a deterministic signal d[n] is present,  $S_x[n,k]$  is proportional to a noncentral chi-square variable,  $\chi'_r^2(\delta)$ , with the same proportionality parameter  $\alpha$ , a degree of freedom r', and with noncentrality parameter  $\delta = r S_d[n,k]/S_b[n,k]$ , where  $S_d[n,k]$  is the RTF of d[n]. In this case, the normalized variance is

$$\operatorname{Varn}\left(S_{x}\left[n,k\right]\right) = \frac{\operatorname{Var}\left(S_{x}\left[n,k\right]\right)}{\left(E\left(S_{x}\left[n,k\right]\right)\right)^{2}} = \frac{\left(2r'+4\delta\right)}{\left(r'+\delta\right)^{2}}.$$
 (5)

Whether the RTF estimator is either a spectrogram or a correlogram, the normalized variance depends on the method parameters and mostly on the average rate. So the degree r' is nearly equal to the degree r.

Knowing  $p_0$ , we can write at each frequency k the decision rule,

$$S_{x}[n,k] \stackrel{H_{1}}{\stackrel{>}{\underset{H_{0}}{\leq}}} \eta_{Pfa}[k], \tag{6}$$

which allows making a decision between the two hypothesis  $H_0$  and  $H_1$ . The probability of false alarm Pfa is given by the user and, therefore, permits the calculation of the detection threshold  $\eta_{Pfa}[k]$  after inverting the integral,

$$Pfa = \int_{\eta_{Pfa}[k]}^{+\infty} p_0(s) \ ds \ . \tag{7}$$

We note that the normalized variance in (5) is always lower than the normalized variance in (3). So, if we use the gamma distribution instead of the non-central chi-square distribution, the effective *Pfa* will be lower.

### THRESHOLD OF THE DECISION RULE

A gamma variable is proportional to a chi-square variable  $\chi_r^2$ . So, Eq. (7) can be written as

$$Pfa = \int_{\eta_{pfa}[k]}^{+\infty} \frac{1}{\alpha} g_{\chi_r^2} \left( \frac{s}{\alpha} \right) ds = \int_{\lambda_{pfa}[k] = \eta_{pfa}[k]/\alpha}^{+\infty} g_{\chi_r^2} \left( u \right) du$$
 (8)

where  $g_{\chi^2_r}(u)$  is the pdf of a  $\chi^2_r$ . Therefore, inverted Eq. (8) gives the threshold, which depends on the given Pfa and on the parameter  $\alpha$ . We have to estimate this last parameter defined in Eq. (4). The moment about zero of  $S_b[n,k]$  can be estimated from the average of the points belonging to  $\mathcal{L}_{H_0}[k]$ . Thus, we have

$$\hat{\alpha}[k] = \frac{1}{card\left(\mathcal{L}_{H_0}[k]\right)} \sum_{n/S_b[n,k] \in \mathcal{L}_{H_0}[k]} S_b[n,k] / r, \qquad (9)$$

where  $\hat{\alpha}[k]$  is an estimation of  $\alpha$  at frequency k.

Finally, taking (8) into account, the test (6) becomes

$$S_{x}[n,k] \in \mathcal{L}_{H_{1}}[k]$$

$$S_{x}[n,k] \geqslant \lambda_{pfa}[k] \times \hat{\alpha}[k]$$

$$\downarrow^{H_{0} \text{ is true}} S_{r}[n,k] \in \mathcal{L}_{H_{0}}[k]$$

$$(10)$$

The threshold  $\lambda_{Pfa}[k]$  is equal to 1 minus the  $Pfa^{th}$ -quantile of a  $\chi_r^2$  and depends on the given Pfa only.

# RECURSIVE TIME-FREQUENCY TEST

The partition of  $\mathcal{L}_x[k]$  is *a priori* not known. We propose an iterative algorithm to be able to apply (10). At each iteration,  $\mathcal{L}_{H_0}[k]$  and the test threshold are updated. Let us define the partition set under  $H_0$  at iteration i+1,

$$\mathcal{L}_{H_0}^{i+1}(v) = \left\{ S_x[n,k] / S_x[n,k] \le \lambda_{Pfa}[k] \hat{\alpha}^i[k] \right\}, \tag{11}$$

with  $\hat{\alpha}^i[k]$  the value of  $\hat{\alpha}[k]$  at iteration *i*. The knowledge of the threshold at the previous iteration allows the determination of the partition sets. The test will converge when  $\mathcal{L}_{H_0}[k]$  will be stable,

$$\mathcal{L}_{H_0}(v) = \mathcal{L}_{H_0}^{I}(v) \quad \text{when} \quad \mathcal{L}_{H_0}^{I}(v) = \mathcal{L}_{H_0}^{I-1}(v). \tag{12}$$

We then deduce the complement

$$\mathcal{L}_{H_1}[k] = \mathcal{L}_{H_1}^{I}[k], \tag{13}$$

which is the desired set.

The remaining problem is the initialization. We propose to initialize  $\mathcal{L}_{H_0}^0[k]$  with the p % lowest samples  $S_x[n,k]$  of  $\mathcal{L}_x[k]$ . The value p is given a priori, which may induce a bias on the estimation of  $\hat{a}^0[k]$ . If the signal is only a stationary white noise,  $r \times \hat{a}^0[k]$  might subestimate the value of  $E(S_r[n,k])$  and, subsequently, the Pfa will be higher than the desired value. On the other hand, if more than p % samples are nonstationary,  $r \times \hat{a}^0[k]$  might overestimate the value of  $E(S_r[n,k])$  and the Pfa would be lower than the desired value. But, in both cases, the estimation will be adjusted at the next iteration, the number of false alarms will decrease whereas the detection probability will increase. In addition, since we want the detector to be not sensitive to the choice of the Pfa, the decision rule is applied with several values of Pfa, which yields a classification of the detected points. The user has to choice this set of Pfa values.

Lastly, the result of this criterion can be represented as a time-frequency map. Each element of  $\mathcal{L}_{H_1}[k]$  is coded by the normalized value  $S_r[n,k]/\lambda_{Pfa}[k]\alpha^I[k]$ , whose maximum value is I. All elements of  $\mathcal{L}_{H_0}[k]$  are coded by 0. We can also compute an alternative map, where each element of  $\mathcal{L}_{H_1}[k]$  is classified according to the value of Pfa that detects this element at the last iteration. Each class is defined as

$$\operatorname{Class}\left(Pfa^{c} \leq Pfa < Pfa^{c+1}\right) = \left\{S_{x}\left[n,k\right] \in \bigcup_{k} \mathcal{L}_{H_{0}}\left[k\right] / \lambda_{Pfa^{c}}\left[k\right] \hat{\alpha}^{I}\left[k\right] \leq S_{x}\left[n,k\right] < \lambda_{Pfa^{c+1}} \hat{\alpha}^{I}\left[k\right]\right\}$$

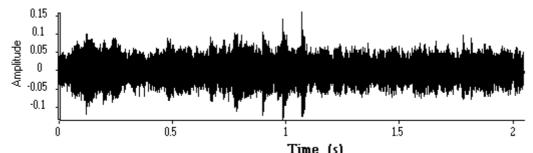
$$\tag{14}$$

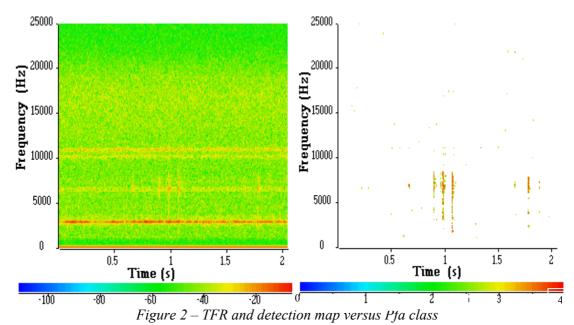
with  $Pfa^c$  and  $Pfa^{c+1}$  successive values of  $Pfa^c$  in the Pfa set. We usually choice  $\{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$ , which defines 5 classes.

#### TRANSIENT DETECTION IN AN HYDRAULIC NOISE

We have tested the proposed detector for many vibratory and acoustic signals (ASpect data base). In this paper, we present the detection results of a vibratory signal provided by Electricité de France (EDF). The signal is a measure of an hydraulic noise in a primary pump of a nuclear reactor. The measure has the particularity to have recorded the shocks of a bolt against the walls of hydraulic circuits. Figure 1 shows the signal of 102 400 points sampled at 50 000 Hz. The RTF of this signal and the detection map are presented in Figure 2.

The criterion detects 4 main patterns corresponding to the shocks that we can identify by attentively listening the signal. The first three shocks are visible on the time representation between 0.9 and 1.1s. But, the last one is very difficult to recognize whereas the test detects it without any difficulty. We also observe that the hydraulic noise, which is stationary through all the observation time, is well not detected by the proposed test.





Spectrogram parameters: Blackman window, segments de 512 points), Detection parameters: p=50.

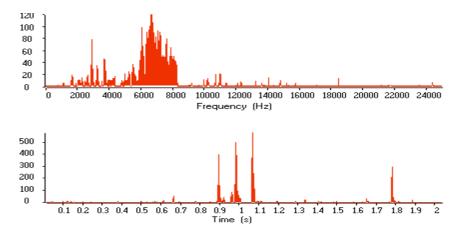


Figure 3 – Frequency and time average of  $\mathcal{L}_{H_0} [k]$  elements.

Figure 3 shows projections of the detection map over the time axis and over the frequency axis. In addition to the detection map, this projections could be useful for defining simple tests for diagnostic or surveillance.

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