



HAL
open science

Stabilization of walls for nano-wires of finite length

Gilles Carbou, Stéphane Labbé

► **To cite this version:**

Gilles Carbou, Stéphane Labbé. Stabilization of walls for nano-wires of finite length. 2009. hal-00369556v1

HAL Id: hal-00369556

<https://hal.science/hal-00369556v1>

Preprint submitted on 20 Mar 2009 (v1), last revised 24 Mar 2010 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

STABILIZATION OF WALLS FOR NANO-WIRES OF FINITE LENGTH

GILLES CARBOU

MAB, UMR 5466, CNRS, Université Bordeaux 1,
351 cours de la Libération, 33405 Talence cedex, France.

STÉPHANE LABBÉ

Université Joseph Fourier, Laboratoire Jean Kuntzmann, CNRS, UMR 5224,
51 rue des Mathématiques, B.P. 53, 38041 Grenoble Cedex 9, France.

ABSTRACT. We address the problem of control of the magnetic moment in a ferromagnetic nanowire of finite length by means of a magnetic field. Based on theoretical results for the 1D Landau-Lifschitz equation, we establish a stabilization result for the static solutions.

1. INTRODUCTION

This paper is concerned with stabilization of wall configurations in a mono-dimensional model of finite length nanowire. This kind of object can be found in nano electronic devices. The three dimensional model is the following (see [2], [10] and [17].) We denote by $m : \mathbb{R}_t^+ \times \Omega_x \longrightarrow \mathbb{R}^3$ the magnetic moment, defined on the ferromagnetic domain Ω . This unitary vector field links the magnetic induction B with the magnetic field H by the relation $B = H + \bar{m}$, where \bar{m} is the extension of m by zero outside Ω . The behaviour of m is governed by the Landau-Lifschitz equation:

$$(1) \quad \left\{ \begin{array}{l} \frac{\partial m}{\partial t} = -m \wedge H_e - m \wedge (m \wedge H_e), \\ H_e = \varepsilon^2 \Delta m + h_d(m) + H_a, \\ \frac{\partial m}{\partial \mathbf{n}} = 0 \text{ on } \partial\Omega, \end{array} \right.$$

where the demagnetizing field $h_d(m)$ is characterized by

$$(2) \quad \text{curl } h_d(m) = 0 \text{ and } \text{div } (h_d(m) + \bar{m}) = 0 \text{ in } \mathbb{R}^3,$$

where H_a is an applied magnetic field and where \mathbf{n} is the outward unit normal on $\partial\Omega$.

Existence of weak solutions for (1) is established in [4], [12] and [16]. Existence of strong solutions is proved in [5] and [6]. Numerical simulations are performed

1991 *Mathematics Subject Classification.* Primary: xxx; Secondary: xxx.

Key words and phrases. Landau-Lifschitz equation, control, stabilization.

The authors were partially supported by the ANR project SICOMAF ("SIMulation et COntôle des MATériaux Ferromagnétiques").

in [13]. For thin domains, equivalent 2-d models are justified in [3], [1], [14]. For nanowires, 1-d models are discussed in [15], [7] and [8].

In this paper we deal with the following one-dimensional model of finite nanowires. After renormalization, the wire is assimilated to the segment $] -L/\varepsilon, L/\varepsilon[e_1$, where (e_1, e_2, e_3) is the canonical basis of \mathbb{R}^3 , where ε is the exchange length, and L is the length of the wire. The magnetic moment m is then defined on $\mathbb{R}_t^+ \times] -L/\varepsilon, L/\varepsilon[$. The equivalent demagnetizing field is given by

$$h_d(m) = -m_2 e_2 - m_3 e_3,$$

i.e. $-h_d$ is the orthogonal projection onto the plane orthogonal to the wire. In addition we assume that we apply a magnetic field in the direction of the wire axis. Therefore we consider the following system:

$$(3) \quad \begin{cases} \frac{\partial m}{\partial t} = -m \wedge H_e - m \wedge (m \wedge H_e), \\ H_e = \varepsilon^2 \frac{\partial^2 m}{\partial x^2} - m_2 e_2 - m_3 e_3 + h e_1, \\ \frac{\partial m}{\partial x}(t, -L/\varepsilon) = \frac{\partial m}{\partial x}(t, L/\varepsilon) = 0. \end{cases}$$

This model will be justified by Γ -convergence arguments in Section 2.

Let us focus on the wall configurations, that is static solutions separating domains of almost constant magnetization. In [7] and [8], we studied wall configurations for an infinite nanowire. They were described by the "canonical" profile $(\text{th } x, 1/\text{ch } x, 0)$, and all its translations in x and rotations around the wire axis. We proved the asymptotic stability and the controllability for these configurations.

Here, for a finite wire, the situation is quite different. We call wall a configuration obtained by rotation around the wire axis of a profile $(\sin \theta_0, \cos \theta_0, 0)$, where $\theta_0 :] -L/\varepsilon, L/\varepsilon[\rightarrow] -\pi/2, \pi/2[$ satisfies $\theta_0(-\pi/2) < 0 < \theta_0(\pi/2)$ (in the infinite wire case, the canonical profile is obtained taking $\theta_0(x) = \text{Arcsin th}(x/2)$.) The walls exist if and only if the wire is longer enough compared to the exchange length:

Theorem 1.1. *If $L/\varepsilon > \pi$, there exists wall configurations. They are centered in the middle of the wire, that is $\theta_0(0) = 0$.*

Concerning the stability, we can prove that the wall configurations given by Theorem 1.1 are unstable.

Theorem 1.2. *Assume that $L/\varepsilon > \pi$. Let θ_0 given by Theorem 1.1. The static solution $M_0 = (\sin \theta_0, \cos \theta_0, 0)$ is linearly unstable for the Landau-Lifschitz equation (3) with $h = 0$.*

This phenomenon was expected. Let us consider a small translation of the centered wall. Without energetic cost, the Landau-Lifschitz equation induces then a displacement of the wall and pushes it outside the wire. Then the magnetic moment tends to $+\pi/2$ or $-\pi/2$ (i.e the minimizers of the ferromagnetism energy.) In the case of an infinite wire, obviously, this translation cannot make the wall disappear.

We prove now a stabilization result. We control the system by an applied field directed along the wire axis.

Theorem 1.3. *Let L and ε as in Theorem 1.1, and let $M_0 = (\sin \theta_0, \cos \theta_0, 0)$ be the canonical profile given by this theorem. We consider the following control:*

$$h(m(t, \cdot)) = \left[-\frac{\varepsilon}{2L} \int_{-L/\varepsilon}^{L/\varepsilon} m_1(t, s) ds \right].$$

Then M_0 is stable for the Landau Lifschitz equation controled with the applied field $H_a = h(m)e_1$.

Remark 1. The control given here is quite natural: when the wall is translated to the right hand side, the average of the profile first component in then negative and our applied field he_1 (with $h > 0$) pushes the wall to the left hand side.

The paper is organized as follows. In Section 2 we justify the one dimensional model by Gamma convergence arguments. In Section 3 we prove the existence of wall profiles. We address the unstability of these profiles in Section 4 by linearization of the Landau-Lifschitz equation. The last section is devoted to the stabilization of the walls by a convenient applied magnetic field.

2. MODELIZATION

In this article, we develop a model of nano-wires of finite length. The behavior of the magnetization in such wires is modeled by the following pde: for u_0 in $C^\infty(\cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot]$, find u in $C^\infty([0, T] \times \cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot]$

$$(4) \quad \begin{cases} \frac{\partial u}{\partial t} = -u \wedge H_e - u \wedge (u \wedge H_e), & x \in \cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot], \\ H_e = \frac{\partial^2 u}{\partial x^2} - u_2 e_2 - u_3 e_3 + h e_1, \\ \frac{\partial u}{\partial x} \left(-\frac{L}{\varepsilon} \right) = \frac{\partial u}{\partial x} \left(\frac{L}{\varepsilon} \right) = 0, \\ u(x, 0) = u_0(x), \quad \forall x \in \cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot], \end{cases}$$

this model is obtain as limit of a three dimensional model in the case of a wire of circular section.

The dynamical equation is build from the static equation in order to define the various contributions of the effective field H_e . In the finite wire, the static problem is the following:

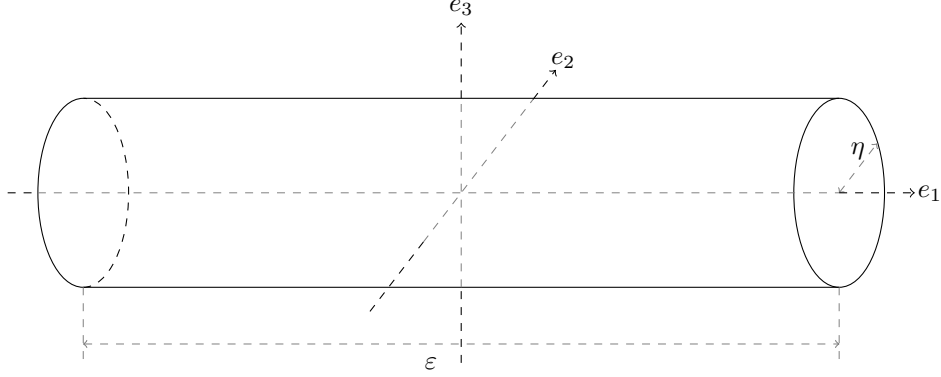
$$(5) \quad \begin{cases} \text{find } u \text{ in } H^1(\Omega_{\varepsilon, \eta}, S^2) \text{ such that} \\ \mathcal{E}_{\varepsilon, \eta}(u) = \min_{v \in H^1(\Omega_{\varepsilon, \eta}, S^2)} \mathcal{E}_{\varepsilon, \eta}(v), \end{cases}$$

where, for $B_d(x, r)$ the sphere of radius r and center x in \mathbb{R}^d

$$\Omega_{\varepsilon, \eta} = \cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot] \times B_2(0, \eta),$$

and for all v in $H^1(\Omega_{\varepsilon, \eta}, \mathbb{R}^3)$ one has

$$\mathcal{E}_{\varepsilon, \eta}(v) = \frac{1}{2} \int_{\Omega_{\varepsilon, \eta}} |\nabla v|^2 dx + \frac{1}{2} \int_{\Omega_{\varepsilon, \eta}} |h_d(v)|^2 dx - h \int_{\Omega_{\varepsilon, \eta}} v \cdot e_1 \partial_x.$$

FIGURE 1. Cylinder $\Omega_{\varepsilon, \eta}$.

To apply a gamma convergence result, we introduce the following rescaling: for all v in $H^1(\Omega_{\varepsilon, \eta})$, we set $\underline{v} \circ l(\eta) = v$ where $l(\eta)$ is the stretching of ratio η in the direction of (e_2, e_3) :

$$\underline{v}(x, y, z) = v(x, \eta y, \eta z).$$

Clearly, \underline{v} is an element of $H^1(\Omega_{\varepsilon, 1}, \mathbb{R}^3)$. Then, one can introduce the following energy:

$$\underline{\mathcal{E}}_{\varepsilon, \eta}(\underline{v}) = \frac{1}{\eta^2} \mathcal{E}_{\varepsilon, \eta}(v \circ l(\eta))$$

Proposition 1. *For all $(\underline{v})_{\eta \in \mathbb{R}_*^+}$ in $H^1(\Omega_{\varepsilon, 1}, S^2)$, sequence of minimizers of (5), $(\underline{\mathcal{E}}_{\varepsilon, \eta}(\underline{v}_\eta))_{\eta \in \mathbb{R}_*^+}$ is a bounded sequence of \mathbb{R} .*

Proof. In order to exhibit an upper bound for $\underline{\mathcal{E}}_{\varepsilon, \eta}(\underline{v}_\eta)$, we write:

$$\forall \eta \in \mathbb{R}_*^+, \mathcal{E}_{\varepsilon, \eta}(\underline{v}_\eta \circ l(\eta)) \leq \mathcal{E}_{\varepsilon, \eta}(e_1),$$

then

$$\begin{aligned} \mathcal{E}_{\varepsilon, \eta}(\underline{v}_\eta \circ l(\eta)) &\leq -h\eta^2\pi\frac{L}{\varepsilon} - \frac{1}{2} \int_{\Omega_{\varepsilon, \eta}} h_d(e_1) \cdot e_1 \, dx \\ &\leq \eta^2\frac{L\pi}{\varepsilon}(2-h). \end{aligned}$$

Then, the lower bound is obtain by canceling the positive contributions of the energy and maximizing the external contribution:

$$-\eta^2h\frac{\pi L}{\varepsilon} \leq \mathcal{E}_{\varepsilon, \eta}(\underline{v}_\eta \circ l(\eta)).$$

So, we can conclude

$$-h\frac{\pi L}{\varepsilon} \leq \underline{\mathcal{E}}_{\varepsilon, \eta}(\underline{v}_\eta) \leq \frac{L\pi}{\varepsilon}(2-h). \quad \square$$

Thanks to this proposition, one can write the following limit problem:

$$(6) \quad \begin{cases} \text{find } u \text{ in } \underline{H}^1(\Omega_{\varepsilon, 1}, S^2) \text{ such that} \\ \underline{\mathcal{E}}_{\varepsilon}(u) = \min_{v \in \underline{H}^1(\Omega_{\varepsilon, 1}, S^2)} \underline{\mathcal{E}}_{\varepsilon}(v), \end{cases}$$

where

$$\underline{H}^1(\Omega_{\varepsilon, 1}, \mathbb{R}^3) = \{u \in H^1(\Omega_{\varepsilon, 1}, \mathbb{R}^3) \text{ such that } \frac{\partial u}{\partial y} = \frac{\partial u}{\partial z} = 0\},$$

and for all v in $H^1(\Omega_{\varepsilon,1}, \mathbb{R}^3)$:

$$\underline{\mathcal{E}}_{\varepsilon}(v) = \frac{1}{2\pi} \int_{\Omega_{\varepsilon,1}} \left| \frac{\partial v}{\partial x} \right|^2 dx + \frac{1}{2\pi} \int_{\Omega_{\varepsilon}} |v \cdot e_1|^2 dx - \frac{h}{\pi} \int_{\Omega_{\varepsilon}} v \cdot e_1 dx,$$

and the problem (5) is rescaled

$$(7) \quad \begin{cases} \text{find } u \text{ in } H^1(\Omega_{\varepsilon,1}, S^2) \text{ such that} \\ \underline{\mathcal{E}}_{\varepsilon,\eta}(u) = \min_{v \in H^1(\Omega_{\varepsilon,1}, S^2)} \underline{\mathcal{E}}_{\varepsilon,\eta}(v), \end{cases}$$

Then, we state the following theorem

Theorem 2.1. $\underline{\mathcal{E}}_{\varepsilon,\eta}$ gamma-converges to $\underline{\mathcal{E}}_{\varepsilon}$ in sense of $H^1(\Omega_{\varepsilon,1}, \mathbb{R}^3)$ in $H^1(\Omega_{\varepsilon,1}, S^2)$, it is to say:

(i) (lower semi continuity) for all sequence $(v_{\eta})_{\eta \in \mathbb{R}_*^+}$ of $H^1(\Omega_{\varepsilon,1}, S^2)$ such that

$$\lim_{\eta \rightarrow 0} v_{\eta} = v_0 \text{ and } (\underline{\mathcal{E}}_{\varepsilon,\eta}(v_{\eta}))_{\eta \in \mathbb{R}_*^+} \text{ bounded,}$$

the limit v_0 is an element of $\underline{H}^1(\Omega_{\varepsilon,1}, B^2)$ such that:

$$\liminf_{\eta \rightarrow 0} \underline{\mathcal{E}}_{\varepsilon,\eta}(v_{\eta}) \geq \underline{\mathcal{E}}_{\varepsilon}(v_0),$$

(ii) (construction) for all u_0 , solution of (6), there exists $(v_{\eta})_{\eta \in \mathbb{R}_*^+}$, sequence of $H^1(\Omega_{\varepsilon,1}, S^2)$, such that:

$$\lim_{\eta \rightarrow 0} v_{\eta} = u_0,$$

and

$$\limsup_{\eta \rightarrow 0} \underline{\mathcal{E}}_{\varepsilon,\eta}(v_{\eta}) \leq \underline{\mathcal{E}}_{\varepsilon}(u_0).$$

Proof. (i) **Lower semi continuity:** Let $(u_{\eta})_{\eta \in \mathbb{R}_*^+}$, a sequence of $H^1(\Omega_{\varepsilon,1}, \mathbb{R}^3)$ of limit u_0 and such that the $(\underline{\mathcal{E}}_{\varepsilon,\eta}(v_{\eta}))_{\eta \in \mathbb{R}_*^+}$ is bounded. Then, one has :

$$\begin{aligned} \left| \frac{1}{\eta^2} \underline{\mathcal{E}}_{\varepsilon,\eta}(u_{\eta} \circ l(\eta)) \right| &= \left| \frac{1}{2} \int_{\Omega_{\varepsilon,1}} |\partial_x u|^2 dx + \frac{1}{2} \int_{\mathbb{R}^3} |h_d(u_{\eta} \circ l(\eta)) \circ l\left(\frac{1}{\eta}\right)|^2 dx \right. \\ &\quad \left. - \lambda \int_{\Omega_{\varepsilon,1}} e_1 \cdot u_{\eta} dx + \frac{1}{2\eta} \int_{\Omega_{\varepsilon,1}} |\nabla_X u_{\eta}|^2 dx \right|, \end{aligned}$$

where ∇_X is the gradient operator through the directions e_2 and e_3 . Then, we state that

$$\lim_{\eta \rightarrow 0} \|\nabla_X u_{\eta}\|_{0, \Omega_{\varepsilon,1}} = 0,$$

so, the limit u_0 only variate in the e_1 direction. Then, using the fact that h_d is a L^p multiplier of order 0, then,

$$\forall \eta \in \mathbb{R}_*^+, \frac{1}{\eta^2} \int_{\mathbb{R}^3} |h_d(u_{\eta} \circ l(\eta))|^2 dx < \infty$$

and, from the system (2) we get, in the sense of distributions

$$\eta(\partial_x h_{d,1} - \partial_x u_{\eta,1}) = -\nabla_X \cdot \tilde{h}_d(u_{\eta}) + \nabla_X \cdot \tilde{u}_{\eta},$$

where, for all Z in \mathbb{R}^3 , we set $\tilde{Z} = (Z \cdot e_2)e_2 + (Z \cdot e_3)e_3$. Then we have

$$\lim_{\eta \rightarrow 0} \|\nabla_X \cdot \tilde{h}_d(u_{\eta})\|_{0, \Omega_{\varepsilon,1}} = 0,$$

by the fact proved above, that

$$\lim_{\eta \rightarrow 0} \|\nabla_X \cdot \tilde{u}_\eta\|_{0, \mathbb{R}^3} = 0.$$

Then, we use that

$$\operatorname{curl}_X \tilde{h}_d(u_\eta) = 0,$$

and conclude that

$$\lim_{\eta \rightarrow 0} \|\tilde{h}_d(u_\eta)\|_{0, \mathbb{R}^3} = 0.$$

Then, u_0 is an element of $\underline{H}^1(\Omega_{\varepsilon,1}, S^2)$ and we can conclude, using the positivity of the difference between $\underline{\mathcal{E}}_{\varepsilon,\eta}$ and $\underline{\mathcal{E}}_\varepsilon$ that

$$\liminf_{\eta \rightarrow 0} \underline{\mathcal{E}}_{\varepsilon,\eta}(u_\eta) \geq \underline{\mathcal{E}}_\varepsilon(u_0)$$

(ii) Reconstruction In fact, this part is straightforward choosing, for a given solution u_0 of (6), the sequence $(u_\eta)_{\eta \in \mathbb{R}_*^+}$ constant equal to u_0 . In this case, trivially, one has

$$\limsup_{\eta \rightarrow 0} \underline{\mathcal{E}}_{\varepsilon,\eta}(v_\eta) \leq \underline{\mathcal{E}}_\varepsilon(u_0).$$

□

This theorem gives the behavior of minimizers for the nanowire. Then one can state that the limit energy is the following, for all u in $H^1(] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[, \mathbb{R}^3)$:

$$\mathcal{E}_\varepsilon(u) = 2\pi \underline{\mathcal{E}}_\varepsilon(u \cdot \chi_{B^2(0,1)}) = \int_{\Omega_{\varepsilon,1}} \left| \frac{\partial v}{\partial x} \right|^2 dx + \int_{\Omega_\varepsilon} |v \cdot e_1|^2 dx - h \int_{\Omega_\varepsilon} v \cdot e_1 dx,$$

then, in order to find out the effective field, we write (see [10]):

$$H_e = \frac{d\mathcal{E}_\varepsilon}{du},$$

it is to say

$$H_e = \frac{\partial^2 u}{\partial x^2} - u_2 e_2 - u_3 e_3 + h e_1.$$

Then, the limit dynamic system is obtain using the Landau Lifchitz combined with the new effective field H_e . This system is then given by (4) as we expected. The boundary conditions comes naturally from the fact that the solutions are in $H^1(] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[, \mathbb{R}^3)$.

3. EXISTENCE OF PARTICULAR EQUILIBRIUM STATES: THE WALLS

In this section we are interested in characterizing equilibrium states of the magnetization in a finite nano-wire when $h = 0$, it is to say when there is no external magnetic field. In this case, we look for solutions which can be written as follows:

$$M_0(x) = \begin{pmatrix} \sin \theta_0 \\ \cos \theta_0 \\ 0 \end{pmatrix}, \quad \forall x \in] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[,$$

where θ_0 is a map from $] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[, \mathbb{R}$ into \mathbb{R} such that M_0 is a stationary solution to (4). In fact, we want M_0 to verify:

$$-M_0 \wedge H_e - M_0 \wedge (M_0 \wedge H_e) = 0, \quad \forall x \in] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[,$$

with

$$H_e = \frac{\partial^2 M_0}{\partial x^2} - \cos \theta_0 e_2,$$

then, one has the following relation

$$(8) \quad -\theta_0'' - \sin \theta_0 \cos \theta_0 = 0, \quad \forall x \in]-\frac{L}{\varepsilon}, \frac{L}{\varepsilon}[,$$

with, on the boundaries

$$(9) \quad \theta_0'(-\frac{L}{\varepsilon}) = \theta_0'(\frac{L}{\varepsilon}) = 0.$$

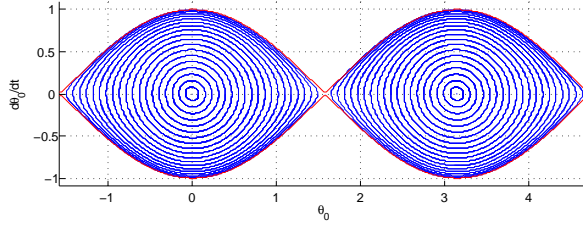


FIGURE 2. Phase portrait.

Setting $-\gamma_0 = \theta_0(0)$ ($\gamma_0 > 0$), we have, integrating the equation (8) and using (9):

$$(10) \quad (\theta_0')^2 + \sin^2 \theta_0 = \sin^2 \gamma_0.$$

The length of the nano-wire has to be such that the function θ_0 goes from $-\gamma_0$ to γ_0 . From formula (9), we deduce the length:

$$\ell(\gamma_0) = \int_{-\gamma_0}^{\gamma_0} \frac{d\theta}{\sqrt{\sin^2 \theta_0 - \sin^2 \theta}},$$

Then, using the length expression computed above, we deduce the following theorem

Theorem 3.1 (Existence of equilibrium states). *For every (ε, L) in \mathbb{R}_*^+ such that $\frac{L}{\varepsilon} > \pi$, there exists an equilibrium state for (4).*

Proof. In order to ensure that an equilibrium state as defined above, we must verify that the length given by the solution is at least greater to the effective length $2\frac{L}{\varepsilon}$. First of all, one has

$$\ell(\gamma_0) = \int_{-1}^1 \frac{\gamma_0}{\sin \gamma_0} \frac{d\theta}{\sqrt{1 - \frac{\sin^2 \theta}{\sin^2 \gamma_0}}},$$

we then see that

$$\lim_{\gamma_0 \rightarrow \frac{\pi}{2}} \ell(\gamma_0) = +\infty,$$

The first remark is that γ_0 is in the interval $]0, \frac{\pi}{2}[$. Now, computing the derivative of $\ell(\gamma_0)$ in γ_0 , we conclude that ℓ is strictly increasing on $]0, \frac{\pi}{2}[$. Then, the comparison

could be done at the limit $\gamma_0 = 0$:

$$\begin{aligned} \lim_{\gamma_0 \rightarrow 0} \ell(\gamma_0) &= \lim_{\gamma_0 \rightarrow 0} \int_{-1}^1 \frac{\gamma_0}{\sin \gamma_0} \frac{du}{\sqrt{1 - \frac{\sin^2(u \gamma_0)}{\sin^2 \gamma_0}}} \\ &= \int_{-1}^1 \frac{du}{\sqrt{1 - u^2}} \\ &= 2\pi, \end{aligned}$$

then, we require that $2\frac{L}{\varepsilon} > 2\pi$. □

4. UNSTABILITY OF WALLS WITHOUT APPLIED FIELD

In this section, we consider that L and ε given such that $\frac{L}{\varepsilon} > \pi$ and also a given wall θ_0 . We consider, as in [7] and [8], the mobile frame $(M_0(x), M_1(x), M_2)$ given by:

$$\forall x \in]-\frac{L}{\varepsilon}, \frac{L}{\varepsilon}[, M_0(x) = \begin{pmatrix} \sin \theta_0(x) \\ \cos \theta_0(x) \\ 0 \end{pmatrix}, M_1(x) = \begin{pmatrix} -\cos \theta_0(x) \\ \sin \theta_0(x) \\ 0 \end{pmatrix}, M_2(x) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Then, we describe the small perturbations of the static wall M_0 as follows:

$$\forall t \in \mathbb{R}^+, \forall x \in [-\frac{L}{\varepsilon}, \frac{L}{\varepsilon}], u(t, x) = r_1 M_1 + r_2 M_2 + \sqrt{1 - r_1^2 - r_2^2} M_0.$$

We denote $r = (r_1, r_2)$. In these coordinates, one can write:

$$\begin{aligned} H_e &= M_0 \left(2\partial_x r_1 \partial_x \theta_0 + r_1 \partial_{xx} \theta_0 - (\partial_x \theta_0)^2 - r_1 \sin \theta_0 \cos \theta_0 + \sin^2 \theta_0 \right) \\ &\quad + M_1 \left(\partial_{xx} r_1 - r_1 ((\partial_x \theta_0)^2 - \cos^2 \theta_0) - \partial_{xx} \theta_0 - \sin \theta_0 \cos \theta_0 \right) \\ &\quad + M_2 (\partial_{xx} r_2) + Q(r), \end{aligned}$$

where $Q(r)$ is the non linear part in r of H_e . Then, we can state that

$$\partial_t r = J \begin{pmatrix} (\mathcal{L} - \cos^2 \theta_0) r_1 \\ \mathcal{L} r_2 \end{pmatrix} + F(r, \partial_x r, \partial_{xx} r),$$

with $J = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$, and $\mathcal{L}r = -\partial_{xx} r_2 + g_0 r_2$, where $g_0(x) = \sin^2 \theta_0 - (\theta_0')^2$.

The linear unstability of the wall structure computed in the previous section is given by the study of the operator \mathcal{L} .

Proposition 2. \mathcal{L} is a linear, positive operator. Its first eigenvalue, 0, is associated to the eigenfunction $\cos \theta_0$ and its second eigenvalue, 1, is associated to the eigenfunction $\sin \theta_0$.

Proof. We set: $f = \theta_0' \tan \theta_0$, then

$$\mathcal{L} = \ell^* \ell, \text{ where } \ell = \partial_x + f,$$

then, we can conclude that \mathcal{L} is a positive operator and that $\cos \theta_0$ is in the kernel of \mathcal{L} . Thus 0 is the first eigenvalue of \mathcal{L}_2 .

Furthermore, we have:

$$\mathcal{L}(\sin \theta_0) = \sin \theta_0,$$

it is to say that 1 is eigenvalue of \mathcal{L} associated to the eigenfunction $\sin \theta_0$. In addition we remark that $\sin \theta_0$ vanishes once in the domain, so by Sturm-Louville theorem, 1 is the second eigenvalue of \mathcal{L}_2 . \square

We can now prove Theorem 1.2.

Proof. From the previous proposition, since $-\cos^2 \gamma_0 < 0$, we conclude that $\mathcal{L} - \cos^2 \gamma_0$ has got one eigenvalue strictly negative, then, zero is unstable for the linearized of (3) around M_0 . \square

5. STABILIZATION OF WALLS

Now, we discuss the stabilization of M_0 by the command H_a (the applied field). We recall that we introduced the following command:

$$h(m) = \frac{-\varepsilon}{2L} \int_{-\frac{L}{\varepsilon}}^{\frac{L}{\varepsilon}} m_1(t, s) ds.$$

We want to prove that the profile M_0 is a stable stationary solution for the following system:

$$(11) \quad \begin{cases} \frac{\partial m}{\partial t} = -m \wedge H_e - m \wedge (m \wedge H_e), \\ \frac{\partial m}{\partial x}(-L/\varepsilon) = \frac{\partial m}{\partial x}(L/\varepsilon) = 0, \\ H_e = \frac{\partial^2 m}{\partial x^2} - m_2 e_2 - m_3 e_3 + h(m) e_1. \end{cases}$$

Proof. To start with, let us introduce $M_0 = (\sin \theta_0, \cos \theta_0, 0)$ given in Theorem 1.1. We recall that

$$\theta_0'' + \sin \theta_0 \cos \theta_0 = 0,$$

$$\theta'(-L/\varepsilon) = \theta'(L/\varepsilon) = 0,$$

Furthermore, on $[-L/\varepsilon, L/\varepsilon]$,

$$\cos^2 \theta_0 - (\theta_0')^2 = \cos^2 \gamma_0,$$

where $\gamma_0 = \theta_0(L/\varepsilon)$.

Since $h(M_0) = 0$, we remark that M_0 is a stationary solution of (11).

First step: moving frame.

As in the previous section, in the spirit of [7], we will describe the problem in the moving frame

$$(M_0(x), M_1(x), M_2(x)).$$

We write the solutions to (11) as:

$$m(t, x) = M_0(x) + r_1(t, x)M_1(x) + r_2(t, x)M_2(x) + \nu(r(t, x))M_0(x)$$

where $\nu(r) = \sqrt{1 - r_1^2 - r_2^2} - 1$. In this moving frame, we get

$$H_e = (g_0 + a_0)M_0 + (a_1 + \tilde{a}_1)M_1 + a_2M_2$$

with

$$\begin{aligned} g_0(x) &= \sin^2 \theta_0 - (\theta'_0)^2 \\ a_0 &= 2\theta'_0 \partial_x r_1 + r_1 \theta''_0 - \cos \theta_0 \sin \theta_0 r_1 + \partial_{xx} \nu - \nu (\theta'_0)^2 + \nu \sin^2 \theta_0 + h(r) \sin \theta_0, \\ a_1 &= \partial_{xx} r_1 + \cos^2 \gamma_0 r_1 - S(r_1) \cos \theta_0, \\ \tilde{a}_1 &= -2\partial_x \nu \theta'_0 - \varphi(r) \cos \theta_0, \\ a_2 &= \partial_{xx} r_2. \end{aligned}$$

where

$$S(r_1) = \frac{\varepsilon}{2L} \int_{-\frac{L}{\varepsilon}}^{\frac{L}{\varepsilon}} r_1 \cos \theta_0 ds, \quad \varphi(r) = -\frac{\varepsilon}{2L} \int_{-\frac{L}{\varepsilon}}^{\frac{L}{\varepsilon}} \nu(r) \sin \theta_0 ds, \quad \text{and } h(r) = S(r_1) + \varphi(r).$$

Using these coordinates in the Landau-Lifchitz equation (9) and projecting on M_1 and M_2 yield:

$$(12) \quad \partial_t r = \Lambda r + F(x, r, \partial_x r, \partial_{xx} r),$$

where

$$\Lambda r = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \tilde{\mathcal{L}}_1(r_1) \\ \mathcal{L}(r_2) \end{pmatrix}$$

with

$$\begin{aligned} \mathcal{L} &= -\partial_{xx} + g_0, \\ \tilde{\mathcal{L}}_1 &= \mathcal{L} - \cos^2 \gamma_0 + \cos \theta_0 S, \end{aligned}$$

and where the non linear part F is given by

$$F(x, r, \partial_x r, \partial_{xx} r) = F_1(r) \partial_{xx} r + F_2(r) (\partial_x r, \partial_x r) + F_3(x, r) \partial_x r + F_4(r) + F_5(r),$$

with

- $F_1 \in \mathcal{C}^\infty(\mathbb{R}^2; \mathcal{M}_2(\mathbb{R}))$:

$$\begin{aligned} F_1(r) &= \begin{pmatrix} (r_2)^2 + (\nu(r))^2 + 2\nu(r) & \nu(r) - r_1 r_2 \\ r_1 r_2 - \nu(r) & (r_1)^2 + (\nu(r))^2 + 2\nu(r) \end{pmatrix} \\ &\quad + \begin{pmatrix} -r_1 - r_2 - \nu(r) r_1 \\ r_1 - r_2 - \nu(r) r_2 \end{pmatrix} \nu'(r), \end{aligned}$$

- $F_2 \in \mathcal{C}^\infty(\mathbb{R}^2; \mathcal{L}_2(\mathbb{R}^2; \mathbb{R}^2))$:

$$F_2(r) (\partial_x r, \partial_x r) = \begin{pmatrix} -r_1 - r_2 - \nu(r) r_1 \\ r_1 - r_2 - \nu(r) r_2 \end{pmatrix} \nu''(r) (\partial_x r, \partial_x r),$$

- $F_3 \in \mathcal{C}^\infty([-L/\varepsilon, L/\varepsilon] \times \mathbb{R}^2; \mathcal{L}(\mathbb{R}^2; \mathbb{R}^2))$:

$$F_3(x, r) \partial_x r = 2\theta'_0(x) \begin{pmatrix} -r_1 - r_2 - \nu(r)r_1 \\ r_1 - r_2 - \nu(r)r_2 \end{pmatrix} \partial_x r_1 \\ - 2\theta'_0(x) \begin{pmatrix} (r_2)^2 + (\nu(r))^2 + 2\nu(r) + 1 \\ r_1 r_2 - \nu(r) - 1 \end{pmatrix} \nu'(r) (\partial_x r),$$

- $F_4(r) \in \mathcal{C}^\infty([-L/\varepsilon, L/\varepsilon] \times \mathbb{R}^2; \mathbb{R}^2)$:

$$F_4(r) = (\cos^2 \gamma_0 r_1) \begin{pmatrix} (r_2)^2 + (\nu(r))^2 + 2\nu(r) \\ r_1 r_2 - \nu(r) \end{pmatrix} - g_0 \nu(r) \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \\ + ((\theta''_0 - \cos \theta_0 \sin \theta_0) r_1 + g_0 \nu(r)) \begin{pmatrix} -r_1 - r_2 - \nu(r)r_1 \\ r_1 - r_2 - \nu(r)r_2 \end{pmatrix}.$$

- F_5 is given by

$$F_5(r) = -h(r) \cos \theta_0 \begin{pmatrix} (r_2)^2 + (\nu(r))^2 + 2\nu(r) \\ r_1 r_2 - \nu(r) \end{pmatrix} - \varphi(r) \cos \theta_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ + h(r) \sin \theta_0 \begin{pmatrix} -r_1 - r_2 - \nu(r)r_1 \\ r_1 - r_2 - \nu(r)r_2 \end{pmatrix}.$$

Remark 2. The command h makes the linear part of (12) positive. Indeed, on one hand, we know that $\mathcal{L} \geq 0$ with $\text{Ker } \mathcal{L} = \mathbb{R} \cos \theta_0$. On the other hand, $\tilde{\mathcal{L}}_1 = \mathcal{L} + \cos \theta_0 S - \cos^2 \gamma_0$. On $\mathbb{R} \cos \theta_0$, $\tilde{\mathcal{L}}_1 \cos \theta_0 = \alpha_0 \cos \theta_0$ with

$$(13) \quad \alpha_0 = \frac{\varepsilon}{2L} \int_{-L/\varepsilon}^{L/\varepsilon} \cos^2 \theta_0(x) dx - \cos^2 \gamma_0 > 0$$

since for $x \in]-L/\varepsilon, L/\varepsilon[$, $\cos^2 \theta_0(x) > \cos^2 \gamma_0$, and on $(\cos \theta_0)^\perp$,

$$\tilde{\mathcal{L}}_1|_{(\cos \theta_0)^\perp} = \mathcal{L} - \cos^2 \theta_0 \geq 1 - \cos^2 \theta_0$$

since $\mathcal{L} \geq 1$ on $(\cos \theta_0)^\perp$ (see Proposition 2).

Second step: new coordinates.

The Landau Lifschitz equation (11) is invariant by rotation around the wire axis, so we can build a family of static solutions.

For $\tau \in \mathbb{R}$ let us introduce the rotation around the x -axis given by

$$\rho_\tau = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \tau & -\sin \tau \\ 0 & \sin \tau & \cos \tau \end{pmatrix}.$$

We denote $M_\tau(x) = \rho_\tau M_0(x)$, and R_τ its projection on the moving frame:

$$R_\tau(x) = \begin{pmatrix} M_\tau(x) \cdot M_1(x) \\ M_\tau(x) \cdot M_2(x) \end{pmatrix} = \begin{pmatrix} \cos \theta_0(x) \sin \theta_0(x) (\cos \tau - 1) \\ \cos \theta_0(x) \sin \tau \end{pmatrix}.$$

Since M_τ is solution to (11), R_τ is a static solution of (12), that is

$$(14) \quad \Lambda R_\theta + F(x, R_\theta, \partial_x R_\theta, \partial_{xx} R_\theta) = 0.$$

Now in order to avoid the problems due to the zero eigenvalue of Λ , we describe r in the new coordinates (τ, σ, W) defined by

$$(15) \quad r(t, x) = R_{\tau(t)}(x) + \sigma(t) \begin{pmatrix} \cos \theta_0(x) \\ 0 \end{pmatrix} + W(t, x),$$

where $(\tau, \sigma) \in \mathcal{C}^1(\mathbb{R}^+; \mathbb{R}^2)$ and $W \in \mathcal{C}^1(\mathbb{R}^+; H^2)$ such that both coordinates of W are in $(\cos \theta_0)^\perp$.

Indeed, as in [11], we can prove that for a given $r \in H^2(-L/\varepsilon, L/\varepsilon)$ in a neighbourhood of 0, there exists a unique $(\tau, \sigma, W) \in \mathbb{R} \times \mathbb{R} \times \mathcal{W}$ such that

$$\rho = R_\theta + \sigma \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} + W,$$

where

$$\mathcal{W} = \left\{ W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} \in H^2, \int_{-L/\varepsilon}^{L/\varepsilon} W_1(x) \cos \theta_0(x) dx = \int_{-L/\varepsilon}^{L/\varepsilon} W_2(x) \cos \theta_0(x) dx = 0 \right\}.$$

Remark 3. On $(\cos \theta_0)^\perp$, the operators \mathcal{L} and $\mathcal{L} - \cos^2 \theta_0$ are non negative, so we introduce the following norms on \mathcal{W} , respectively equivalent to the H^2 and the H^3 norms:

$$\|W\|_{\mathcal{H}^2} = \left(\|\mathcal{L}^{\frac{1}{2}} (\mathcal{L} - \cos^2 \theta_0)^{\frac{1}{2}} W_1\|_{L^2}^2 + \|\mathcal{L} W_2\|_{L^2}^2 \right)^{\frac{1}{2}},$$

$$\|W\|_{\mathcal{H}^3} = \left(\|\mathcal{L}^{\frac{1}{2}} (\mathcal{L} - \cos^2 \theta_0) W_1\|_{L^2}^2 + \|\mathcal{L}^{\frac{3}{2}} W_2\|_{L^2}^2 \right)^{\frac{1}{2}}.$$

Plugging the decomposition (15) in (12) and using (14) yield the following equivalent form for the Landau-Lifschitz equation in the coordinates (τ, σ, W) , valid for little perturbations of M_0 . Indeed we have:

$$\partial_t r = \frac{d\tau}{dt} R'_\tau(x) + \frac{d\sigma}{dt} \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} + \partial_t W,$$

where

$$R'_\tau(x) = \begin{pmatrix} -\sin \tau \cos \theta_0 \sin \theta_0 \\ \cos \tau \cos \theta_0 \end{pmatrix}.$$

In addition,

$$\Lambda r = \Lambda R_\tau + \begin{pmatrix} -\alpha_0 \cos \theta_0 \\ \alpha_0 \cos \theta_0 \end{pmatrix} \sigma + \Lambda(W)$$

(see Remark 2 for the definition of α_0), and

$$F(x, r, \partial_x r, \partial_{xx} r) = F(x, R_\tau, \partial_x R_\tau, \partial_{xx} R_\tau) + G(x, \tau, \sigma, W, \partial_x W, \partial_{xx} W).$$

The last term G is obtained from F with the Taylor formula around R_τ :

$$\begin{aligned} G(x, \tau, \sigma, W, \partial_x W, \partial_{xx} W) = & F_1(r)(\partial_{xx} w) + \tilde{F}_1(r)(w)(\partial_{xx} R_\tau) \\ & + 2F_2(r)(\partial_x w, \partial_x R_\tau) + F_2(r)(\partial_x w, \partial_x w) \\ & + \tilde{F}_2(r)(w)(\partial_x R_\tau, \partial_x R_\tau) \\ & + F_3(x, r)(\partial_x w) + \tilde{F}_3(x, r)(w)\partial_x R_\tau + \tilde{F}_4(r)(w) \\ & + F_5(r), \end{aligned}$$

where $w = W + \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} \sigma$, and where for $i = 1..4$, $\tilde{F}_i(r) \in \mathcal{L}(\mathbb{R}^2; \mathbb{R}^2)$ is given by

$$\tilde{F}_i(r) = \int_0^1 F'_i(R_\tau + sw) ds.$$

From straightforward calculations, we see that:

$$F_1(r) = \mathcal{O}(|r|^2), \quad \tilde{F}_1(r) = \mathcal{O}(|r|^2)$$

$$F_2(r) = \mathcal{O}(|r|), \quad \tilde{F}_2(r) = \mathcal{O}(1)$$

$$F_3(x, r) = \mathcal{O}(|r|), \quad \tilde{F}_3(x, r) = \mathcal{O}(1)$$

$$\tilde{F}_4(r) \mathcal{O}(|r|).$$

In addition, $\partial_x R_\tau = \mathcal{O}(|r|)$. Concerning F_5 , on one hand we remark that

$$S(r_1) = S(W_1) + \sigma S(\cos \theta_0)$$

(since $S(R_\tau) = 0$.)

On the other hand, $\varphi(R_\tau) = 0$ and

$$\varphi(r) = -\frac{\varepsilon}{2L} \int_{-L/\varepsilon}^{L/\varepsilon} \tilde{\nu}(r)(w) \sin \theta_0,$$

where

$$\tilde{\nu}(r) = \int_0^1 \nu'(R_\tau + sw) ds = \mathcal{O}(|r|).$$

Therefore with all these estimates, if $\|r\|_{H^2}$ is sufficiently small, we have

$$(16) \quad \|G\|_{L^2} \leq K \|r\|_{L^\infty} [|\sigma| + \|W\|_{\mathcal{H}^2}],$$

$$\|\partial_x G\|_{L^2} \leq K \|r\|_{L^\infty} [|\sigma| + \|W\|_{\mathcal{H}^3}]$$

(see Remark 3.)

Hence, using (14), we have obtained

$$(17) \quad \frac{d\tau}{dt} R'_\tau + \frac{d\sigma}{dt} \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} + \partial_t W = \sigma \begin{pmatrix} -\alpha_0 \cos \theta_0 \\ \alpha_0 \cos \theta_0 \end{pmatrix} + \Lambda W + G(x, \tau, \sigma, W, \partial_x W, \partial_{xx} W).$$

In order to separate the unknowns τ , σ and W , we first take the inner product of (17) with $\begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix}$ and with $\begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix}$. We remark that both $\partial_t W$ and ΛW are orthogonal to these vectors, so that we obtain:

$$\rho_0 \sigma' = -\alpha_0 \rho_0 \sigma + \bar{G}_1,$$

$$g_\tau \tau' = \alpha_0 \rho_0 \sigma + \bar{G}_2,$$

where

$$\rho_0 = \int_{-L/\varepsilon}^{L/\varepsilon} \cos^2 \theta_0,$$

$$\bar{G}_1 = \int_{-L/\varepsilon}^{L/\varepsilon} G(x, \tau, \sigma W, \partial_x W, \partial_{xx} W) \cdot \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} dx,$$

$$g_\tau = \int_{-L/\varepsilon}^{L/\varepsilon} R'_\tau(x) \cdot \begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix} dx,$$

$$\bar{G}_2 = \int_{-L/\varepsilon}^{L/\varepsilon} G(x, \tau, \sigma W, \partial_x W, \partial_{xx} W) \cdot \begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix} dx.$$

By subtraction, we have:

$$\partial_t W = \Lambda W + \tilde{G}$$

with

$$\tilde{G} = G - \frac{\bar{G}_2}{g_\tau} R'_\tau - \frac{\bar{G}_1}{\rho_0} \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} + \alpha_0 \sigma \left[\begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix} - \frac{\rho_0}{g_\tau} R'_\tau \right].$$

We are then led to study the following equation

$$(18) \quad \tau' = \alpha_0 \frac{\rho_0}{g_\tau} \sigma + \frac{1}{g_\tau} \bar{G}_2,$$

together with the system coupling:

$$(19) \quad \sigma' = -\alpha_0 \sigma + \frac{1}{\rho_0} \bar{G}_1,$$

with

$$(20) \quad \partial_t W = \begin{pmatrix} -(\mathcal{L} - \cos^2 \gamma_0) W_1 - \mathcal{L} W_2 \\ (\mathcal{L} - \cos^2 \gamma_0) W_1 - \mathcal{L} W_2 \end{pmatrix} + \tilde{G}.$$

From (16), with Remark 3 we have

$$(21) \quad \left| \frac{1}{\rho_0} \bar{G}_1 \right| (t) \leq K \|r\|_{L^\infty} \left[|\sigma(t)| + \|W(t)\|_{\mathcal{H}^3} \right].$$

In addition, since $g_\tau = \rho_0 + \mathcal{O}(\tau^2)$, since $R'_\tau = \begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix} + \mathcal{O}(\tau)$, we get:

$$(22) \quad \|\tilde{G}\|_{H^1} \leq K \|r\|_{L^\infty} \left[|\sigma(t)| + \|W(t)\|_{\mathcal{H}^3} \right].$$

Taking the inner product of (20) with $\begin{pmatrix} \mathcal{L}(\mathcal{L} - \cos^2 \gamma_0)W_1 \\ \mathcal{L}^2 W_2 \end{pmatrix}$, we obtain using (22) that

$$\frac{d}{dt} (\|W\|_{\mathcal{H}^2}^2) + \|W\|_{\mathcal{H}^3}^2 \leq K \|r\|_{L^\infty} [|\sigma(t)| + \|W(t)\|_{\mathcal{H}^3}].$$

Multiplying (19) by σ , (21) yields:

$$\frac{d}{dt} \sigma^2 + \alpha_0 \sigma^2 \leq K \|r\|_{L^\infty} [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^3}^2].$$

Summing up the previous estimates, we have:

$$\frac{d}{dt} [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^2}^2] + \alpha_0 [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^3}^2] (1 - K \|r\|_{L^\infty}) \leq 0.$$

So there exists $\delta > 0$ such that while $\|r\|_{L^\infty} \leq \frac{1}{2K}$,

$$\frac{d}{dt} [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^2}^2] + \delta [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^2}^2] \leq 0,$$

that is

$$[|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^2}^2] \leq [|\sigma_0|^2 + \|W_0\|_{\mathcal{H}^2}^2] e^{-\delta t}.$$

Now, with equation (18), we have

$$|\tau'| \leq K |\sigma| + K [|\sigma(t)| + \|W(t)\|_{\mathcal{H}^2}]$$

so while $\|r\|_{L^\infty} \leq \frac{1}{2K}$,

$$|\tau| \leq |\tau_0| + K [|\sigma_0| + \|W_0\|_{\mathcal{H}^2}] e^{-\delta t/2}.$$

Therefore, if τ_0 , σ_0 and $\|W_0\|_{H^2}$ are small enough, we remain in the domain $\{\|r\|_{L^\infty} \leq \frac{1}{2K}\}$ and all the previous estimates remain valid for all times. This concludes the proof of Theorem 1.3. \square

REFERENCES

- [1] François Alouges, Tristan Rivière, and Sylvia Serfaty. Néel and cross-tie wall energies for planar micromagnetic configurations. *Control, Optimisation and Calculus of Variations* **8** (2002), 31–68.
- [2] W.F. Brown, *Micromagnetics*, Interscience Publisher, John Wiley and Sons, New York 1963.
- [3] Gilles Carbou. Thin layers in micromagnetism. *Math. Models Methods Appl. Sci.* **11** (2001), no. 9, 1529–1546.
- [4] Gilles Carbou and Pierre Fabrie. Time average in micromagnetism. *J. Differential Equations* **147** (1998), no. 2, 383–409.
- [5] Gilles Carbou and Pierre Fabrie. Regular solutions for Landau-Lifschitz equation in a bounded domain. *Differential Integral Equations* **14** (2001), no. 2, 213–229.
- [6] Gilles Carbou, Pierre Fabrie and Olivier Guès. On the ferromagnetism equations in the non static case. *Comm. Pure Appl. Anal.* **3** (2004), 367–393.
- [7] Gilles Carbou and Stéphane Labbé. Stability for static walls in ferromagnetic nanowires. *Discrete Contin. Dyn. Syst. Ser. B* **6** (2006), no. 2, 273–290.
- [8] Gilles Carbou, Stéphane Labbé and Emmanuel Trélat. Control of travelling walls in a ferromagnetic nanowire. *Discrete Contin. Dyn. Syst.* (2007), suppl.
- [9] Antonio DeSimone, Robert V. Kohn, Stefan Müller, and Felix Otto. Magnetic microstructures—a paradigm of multiscale problems. In *ICIAM 99 (Edinburgh)*, pages 175–190. Oxford Univ. Press, Oxford, 2000.
- [10] Laurence Halpern and Stéphane Labbé. Modélisation et simulation du comportement des matériaux ferromagnétiques. *Mataphi* **66** (2001), 70–86.

- [11] Todd Kapitula. Multidimensional stability of planar travelling waves. *Trans. Amer. Math. Soc.* **349** (1997) no. 1, 257–269.
- [12] Stéphane Labbé, Simulation numérique du comportement hyperfréquence des matériaux ferromagnétiques. Thèse de l'Université Paris 13 (1998).
- [13] Stéphane Labbé and Pierre-Yves Bertin. Microwave polarisability of ferrite particles with non-uniform magnetization. *Journal of Magnetism and Magnetic Materials* **206** (1999), 93–105.
- [14] Tristan Rivière and Sylvia Serfaty. Compactness, kinetic formulation, and entropies for a problem related to micromagnetics. *Comm. Partial Differential Equations* **28** (2003), no. 1-2, 249–269.
- [15] David Sanchez, Méthodes asymptotiques en ferromagnétisme, Thèse Université Bordeaux 1 (2004).
- [16] A. Visintin. On Landau Lifschitz equation for ferromagnetism. *Japan Journal of Applied Mathematics* **1**, 69-84 (1985).
- [17] H. Wynled. *Ferromagnetism*. Encyclopedia of Physics, Vol. XVIII / 2, Springer Verlag, Berlin (1966).

E-mail address: `carbou@math.u-bordeaux1.fr`

E-mail address: `stephane.labbe@imag.fr`