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STABILIZATION OF WALLS FOR NANO-WIRES OF FINITE LENGTH

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ABSTRACT. We address the problem of control of the magnetic moment in a ferromagnetic nanowire of finite length by means of a magnetic field. Based on theoretical results for the 1D Landau-Lifschitz equation, we establish a stabilization result for the static solutions.

1. INTRODUCTION

This paper is concerned with stabilization of wall configurations in a mono-dimensional model of finite length nanowire. This kind of object can be found in nano electronic devices. The three dimensional model is the following (see [2], [10] and [17].) We denote by $m : \mathbb{R}_t^+ \times \Omega_x \longrightarrow \mathbb{R}^3$ the magnetic moment, defined on the ferromagnetic domain Ω . This unitary vector field links the magnetic induction B with the magnetic field H by the relation $B = H + \bar{m}$, where \bar{m} is the extension of m by zero outside Ω . The behaviour of m is governed by the Landau-Lifschitz equation:

$$(1) \quad \begin{cases} \frac{\partial m}{\partial t} = -m \wedge H_e - m \wedge (m \wedge H_e), \\ H_e = \varepsilon^2 \Delta m + h_d(m) + H_a, \\ \frac{\partial m}{\partial \mathbf{n}} = 0 \text{ on } \partial\Omega, \end{cases}$$

where the demagnetizing field $h_d(m)$ is characterized by

$$(2) \quad \text{curl } h_d(m) = 0 \text{ and } \text{div } (h_d(m) + \bar{m}) = 0 \text{ in } \mathbb{R}^3,$$

where H_a is an applied magnetic field and where \mathbf{n} is the outward unit normal on $\partial\Omega$.

Existence of weak solutions for (1) is established in [4], [12] and [16]. Existence of strong solutions is proved in [5] and [6]. Numerical simulations are performed

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in [13]. For thin domains, equivalent 2-d models are justified in [3], [1], [14]. For nanowires, 1-d models are discussed in [15], [7] and [8].

In this paper we deal with the following one-dimensional model of finite nanowires. After renormalization, the wire is assimilated to the segment $] -L/\varepsilon, L/\varepsilon[e_1$, where (e_1, e_2, e_3) is the canonical basis of \mathbb{R}^3 , where ε is the exchange length, and L is the length of the wire. The magnetic moment m is then defined on $\mathbb{R}_t^+ \times] -L/\varepsilon, L/\varepsilon[$. The equivalent demagnetizing field is given by

$$h_d(m) = -m_2e_2 - m_3e_3,$$

i.e. $-h_d$ is the orthogonal projection onto the plane orthogonal to the wire. In addition we assume that we apply a magnetic field in the direction of the wire axis. Therefore we consider the following system:

$$(3) \quad \begin{cases} \frac{\partial m}{\partial t} = -m \wedge H_e - m \wedge (m \wedge H_e), \\ H_e = \varepsilon^2 \frac{\partial^2 m}{\partial x^2} - m_2e_2 - m_3e_3 + he_1, \\ \frac{\partial m}{\partial x}(t, -L/\varepsilon) = \frac{\partial m}{\partial x}(t, L/\varepsilon) = 0. \end{cases}$$

This model will be justified by Γ -convergence arguments in Section 2.

Let us focus on the wall configurations, that is static solutions separating domains of almost constant magnetization. In [7] and [8], we studied wall configurations for an infinite nanowire. They were described by the "canonical" profile $(\text{th } x, 1/\text{ch } x, 0)$, and all its translations in x and rotations around the wire axis. We proved the asymptotic stability and the controllability for these configurations.

Here, for a finite wire, the situation is quite different. We call wall a configuration obtained by rotation around the wire axis of a profile $(\sin \theta_0, \cos \theta_0, 0)$, where $\theta_0 :] -L/\varepsilon, L/\varepsilon[\rightarrow] -\pi/2, \pi/2[$ satisfies $\theta_0(-\pi/2) < 0 < \theta_0(\pi/2)$ (in the infinite wire case, the canonical profile is obtained taking $\theta_0(x) = \text{Arcsin th}(x/2)$.) The walls exist if and only if the wire is longer enough compared to the exchange length:

Theorem 1.1. *If $L/\varepsilon > \pi$, there exists wall configurations. They are centered in the middle of the wire, that is $\theta_0(0) = 0$.*

Concerning the stability, we can prove that the wall configurations given by Theorem 1.1 are unstable.

Theorem 1.2. *Assume that $L/\varepsilon > \pi$. Let θ_0 given by Theorem 1.1. The static solution $M_0 = (\sin \theta_0, \cos \theta_0, 0)$ is linearly unstable for the Landau-Lifschitz equation (3) with $h = 0$.*

This phenomenon was expected. Let us consider a small translation of the centered wall. Without energetic cost, the Landau-Lifschitz equation induces then a displacement of the wall and pushes it outside the wire. Then the magnetic moment tends to $+\pi/2$ or $-\pi/2$ (i.e the minimizers of the ferromagnetism energy.) In the case of an infinite wire, obviously, this translation cannot make the wall disappear.

We prove now a stabilization result. We control the system by an applied field directed along the wire axis.

Theorem 1.3. *Let L and ε as in Theorem 1.1, and let $M_0 = (\sin \theta_0, \cos \theta_0, 0)$ be the canonical profile given by this theorem. We consider the following control:*

$$h(m(t, \cdot)) = \left[-\frac{\varepsilon}{2L} \int_{-L/\varepsilon}^{L/\varepsilon} m_1(t, s) ds \right].$$

Then M_0 is stable for the Landau Lifschitz equation controled with the applied field $H_a = h(m)e_1$.

Remark 1. The control given here is quite natural: when the wall is translated to the right hand side, the average of the profile first component in then negative and our applied field he_1 (with $h > 0$) pushes the wall to the left hand side.

The paper is organized as follows. In Section 2 we justify the one dimensional model by Gamma convergence arguments. In Section 3 we prove the existence of wall profiles. We address the unstability of these profiles in Section 4 by linearization of the Landau-Lifschitz equation. The last section is devoted to the stabilization of the walls by a convenient applied magnetic field.

2. MODELIZATION

In this article, we develop a model of nano-wires of finite length. The behavior of the magnetization in such wires is modeled by the following pde: for u_0 in $C^\infty(\cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot)$, find u in $C^\infty([0, T] \times \cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot)$

$$(4) \quad \begin{cases} \frac{\partial u}{\partial t} = -u \wedge H_e - u \wedge (u \wedge H_e), & x \in \cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot, \\ H_e = \frac{\partial^2 u}{\partial x^2} - u_2 e_2 - u_3 e_3 + h e_1, \\ \frac{\partial u}{\partial x}(-\frac{L}{\varepsilon}) = \frac{\partial u}{\partial x}(\frac{L}{\varepsilon}) = 0, \\ u(x, 0) = u_0(x), \quad \forall x \in \cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot, \end{cases}$$

this model is obtain as limit of a three dimensional model in the case of a wire of circular section.

The dynamical equation is build from the static equation in order to define the various contributions of the effective field H_e . In the finite wire, the static problem is the following:

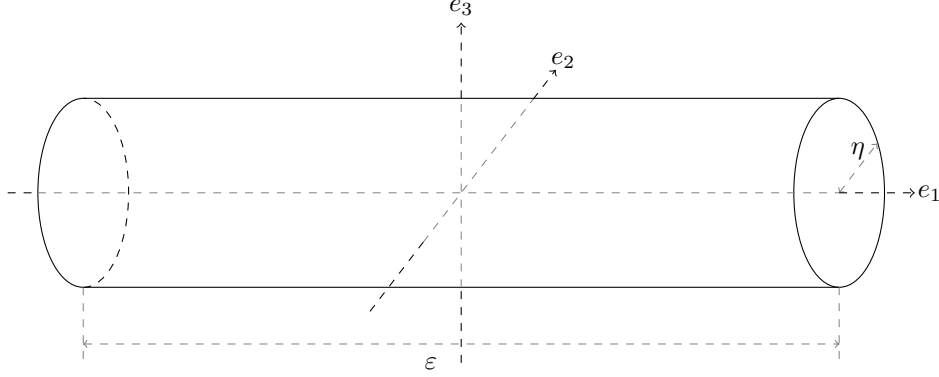
$$(5) \quad \begin{cases} \text{find } u \text{ in } H^1(\Omega_{\varepsilon, \eta}, S^2) \text{ such that} \\ \mathcal{E}_{\varepsilon, \eta}(u) = \min_{v \in H^1(\Omega_{\varepsilon, \eta}, S^2)} \mathcal{E}_{\varepsilon, \eta}(v), \end{cases}$$

where, for $B_d(x, r)$ the sphere of radius r and center x in \mathbb{R}^d

$$\Omega_{\varepsilon, \eta} = \cdot - \frac{L}{\varepsilon}, \frac{L}{\varepsilon} \cdot \times B_2(0, \eta),$$

and for all v in $H^1(\Omega_{\varepsilon, \eta}, \mathbb{R}^3)$ one has

$$\mathcal{E}_{\varepsilon, \eta}(v) = \frac{1}{2} \int_{\Omega_{\varepsilon, \eta}} |\nabla v|^2 dx + \frac{1}{2} \int_{\Omega_{\varepsilon, \eta}} |h_d(v)|^2 dx - h \int_{\Omega_{\varepsilon, \eta}} v \cdot e_1 \partial_x.$$

FIGURE 1. Cylinder $\Omega_{\varepsilon, \eta}$.

To apply a gamma convergence result, we introduce the following rescaling: for all v in $H^1(\Omega_{\varepsilon, \eta})$, we set $\underline{v} \circ l(\eta) = v$ where $l(\eta)$ is the stretching of ratio η in the direction of (e_2, e_3) :

$$\underline{v}(x, y, z) = v(x, \eta y, \eta z).$$

Clearly, \underline{v} is an element of $H^1(\Omega_{\varepsilon, 1}, \mathbb{R}^3)$. Then, one can introduce the following energy:

$$\underline{\mathcal{E}}_{\varepsilon, \eta}(\underline{v}) = \frac{1}{\eta^2} \mathcal{E}_{\varepsilon, \eta}(v \circ l(\eta))$$

Proposition 1. *For all $(\underline{v})_{\eta \in \mathbb{R}_*^+}$ in $H^1(\Omega_{\varepsilon, 1}, S^2)$, sequence of minimizers of (5), $(\underline{\mathcal{E}}_{\varepsilon, \eta}(\underline{v}_\eta))_{\eta \in \mathbb{R}_*^+}$ is a bounded sequence of \mathbb{R} .*

Proof. In order to exhibit an upper bound for $\underline{\mathcal{E}}_{\varepsilon, \eta}(\underline{v}_\eta)$, we write:

$$\forall \eta \in \mathbb{R}_*^+, \mathcal{E}_{\varepsilon, \eta}(\underline{v}_\eta \circ l(\eta)) \leq \mathcal{E}_{\varepsilon, \eta}(e_1),$$

then

$$\begin{aligned} \mathcal{E}_{\varepsilon, \eta}(\underline{v}_\eta \circ l(\eta)) &\leq -h\eta^2\pi\frac{L}{\varepsilon} - \frac{1}{2} \int_{\Omega_{\varepsilon, \eta}} h_d(e_1) \cdot e_1 \, dx \\ &\leq \eta^2\frac{L\pi}{\varepsilon}(2-h). \end{aligned}$$

Then, the lower bound is obtain by canceling the positive contributions of the energy and maximizing the external contribution:

$$-\eta^2h\frac{\pi L}{\varepsilon} \leq \mathcal{E}_{\varepsilon, \eta}(\underline{v}_\eta \circ l(\eta)).$$

So, we can conclude

$$-h\frac{\pi L}{\varepsilon} \leq \underline{\mathcal{E}}_{\varepsilon, \eta}(\underline{v}_\eta) \leq \frac{L\pi}{\varepsilon}(2-h).$$

□

Thanks to this proposition, one can write the following limit problem:

$$(6) \quad \begin{cases} \text{find } u \text{ in } \underline{H}^1(\Omega_{\varepsilon, 1}, S^2) \text{ such that} \\ \underline{\mathcal{E}}_{\varepsilon}(u) = \min_{v \in \underline{H}^1(\Omega_{\varepsilon, 1}, S^2)} \underline{\mathcal{E}}_{\varepsilon}(v), \end{cases}$$

where

$$\underline{H}^1(\Omega_{\varepsilon, 1}, \mathbb{R}^3) = \{u \in H^1(\Omega_{\varepsilon, 1}, \mathbb{R}^3) \text{ such that } \frac{\partial u}{\partial y} = \frac{\partial u}{\partial z} = 0\},$$

and for all v in $H^1(\Omega_{\varepsilon,1}, \mathbb{R}^3)$:

$$\underline{\mathcal{E}}_{\varepsilon}(v) = \frac{1}{2\pi} \int_{\Omega_{\varepsilon,1}} \left| \frac{\partial v}{\partial x} \right|^2 dx + \frac{1}{2\pi} \int_{\Omega_{\varepsilon}} |v \cdot e_1|^2 dx - \frac{h}{\pi} \int_{\Omega_{\varepsilon}} v \cdot e_1 dx,$$

and the problem (5) is rescaled

$$(7) \quad \begin{cases} \text{find } u \text{ in } H^1(\Omega_{\varepsilon,1}, S^2) \text{ such that} \\ \underline{\mathcal{E}}_{\varepsilon,\eta}(u) = \min_{v \in H^1(\Omega_{\varepsilon,1}, S^2)} \underline{\mathcal{E}}_{\varepsilon,\eta}(v), \end{cases}$$

Then, we state the following theorem

Theorem 2.1. $\underline{\mathcal{E}}_{\varepsilon,\eta}$ gamma-converges to $\underline{\mathcal{E}}_{\varepsilon}$ in sense of $H^1(\Omega_{\varepsilon,1}, \mathbb{R}^3)$ in $H^1(\Omega_{\varepsilon,1}, S^2)$, it is to say:

(i) (lower semi continuity) for all sequence $(v_{\eta})_{\eta \in \mathbb{R}_*^+}$ of $H^1(\Omega_{\varepsilon,1}, S^2)$ such that

$$\lim_{\eta \rightarrow 0} v_{\eta} = v_0 \text{ and } (\underline{\mathcal{E}}_{\varepsilon,\eta}(v_{\eta}))_{\eta \in \mathbb{R}_*^+} \text{ bounded,}$$

the limit v_0 is an element of $\underline{H}^1(\Omega_{\varepsilon,1}, B^2)$ such that:

$$\liminf_{\eta \rightarrow 0} \underline{\mathcal{E}}_{\varepsilon,\eta}(v_{\eta}) \geq \underline{\mathcal{E}}_{\varepsilon}(v_0),$$

(ii) (construction) for all u_0 , solution of (6), there exists $(v_{\eta})_{\eta \in \mathbb{R}_*^+}$, sequence of $H^1(\Omega_{\varepsilon,1}, S^2)$, such that:

$$\lim_{\eta \rightarrow 0} v_{\eta} = u_0,$$

and

$$\limsup_{\eta \rightarrow 0} \underline{\mathcal{E}}_{\varepsilon,\eta}(v_{\eta}) \leq \underline{\mathcal{E}}_{\varepsilon}(u_0).$$

Proof. (i) **Lower semi continuity:** Let $(u_{\eta})_{\eta \in \mathbb{R}_*^+}$, a sequence of $H^1(\Omega_{\varepsilon,1}, \mathbb{R}^3)$ of limit u_0 and such that the $(\underline{\mathcal{E}}_{\varepsilon,\eta}(v_{\eta}))_{\eta \in \mathbb{R}_*^+}$ is bounded. Then, one has :

$$\begin{aligned} \left| \frac{1}{\eta^2} \underline{\mathcal{E}}_{\varepsilon,\eta}(u_{\eta} \circ l(\eta)) \right| &= \left| \frac{1}{2} \int_{\Omega_{\varepsilon,1}} |\partial_x u|^2 dx + \frac{1}{2} \int_{\mathbb{R}^3} |h_d(u_{\eta} \circ l(\eta)) \circ l\left(\frac{1}{\eta}\right)|^2 dx \right. \\ &\quad \left. - \lambda \int_{\Omega_{\varepsilon,1}} e_1 \cdot u_{\eta} dx + \frac{1}{2\eta} \int_{\Omega_{\varepsilon,1}} |\nabla_X u_{\eta}|^2 dx \right|, \end{aligned}$$

where ∇_X is the gradient operator through the directions e_2 and e_3 . Then, we state that

$$\lim_{\eta \rightarrow 0} \|\nabla_X u_{\eta}\|_{0, \Omega_{\varepsilon,1}} = 0,$$

so, the limit u_0 only variate in the e_1 direction. Then, using the fact that h_d is a L^p multiplier of order 0, then,

$$\forall \eta \in \mathbb{R}_*^+, \frac{1}{\eta^2} \int_{\mathbb{R}^3} |h_d(u_{\eta} \circ l(\eta))|^2 dx < \infty$$

and, from the system (2) we get, in the sense of distributions

$$\eta(\partial_x h_{d,1} - \partial_x u_{\eta,1}) = -\nabla_X \cdot \tilde{h}_d(u_{\eta}) + \nabla_X \cdot \tilde{u}_{\eta},$$

where, for all Z in \mathbb{R}^3 , we set $\tilde{Z} = (Z \cdot e_2)e_2 + (Z \cdot e_3)e_3$. Then we have

$$\lim_{\eta \rightarrow 0} \|\nabla_X \cdot \tilde{h}_d(u_{\eta})\|_{0, \Omega_{\varepsilon,1}} = 0,$$

by the fact proved above, that

$$\lim_{\eta \rightarrow 0} \|\nabla_X \cdot \tilde{u}_\eta\|_{0, \mathbb{R}^3} = 0.$$

Then, we use that

$$\operatorname{curl}_X \tilde{h}_d(u_\eta) = 0,$$

and conclude that

$$\lim_{\eta \rightarrow 0} \|\tilde{h}_d(u_\eta)\|_{0, \mathbb{R}^3} = 0.$$

Then, u_0 is an element of $\underline{H}^1(\Omega_{\varepsilon,1}, S^2)$ and we can conclude, using the positivity of the difference between $\underline{\mathcal{E}}_{\varepsilon,\eta}$ and $\underline{\mathcal{E}}_\varepsilon$ that

$$\liminf_{\eta \rightarrow 0} \underline{\mathcal{E}}_{\varepsilon,\eta}(u_\eta) \geq \underline{\mathcal{E}}_\varepsilon(u_0)$$

(ii) Reconstruction In fact, this part is straightforward choosing, for a given solution u_0 of (6), the sequence $(u_\eta)_{\eta \in \mathbb{R}_*^+}$ constant equal to u_0 . In this case, trivially, one has

$$\limsup_{\eta \rightarrow 0} \underline{\mathcal{E}}_{\varepsilon,\eta}(v_\eta) \leq \underline{\mathcal{E}}_\varepsilon(u_0).$$

□

This theorem gives the behavior of minimizers for the nanowire. Then one can state that the limit energy is the following, for all u in $H^1(] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[, \mathbb{R}^3)$:

$$\mathcal{E}_\varepsilon(u) = 2\pi \underline{\mathcal{E}}_\varepsilon(u \cdot \chi_{B^2(0,1)}) = \int_{\Omega_{\varepsilon,1}} \left| \frac{\partial v}{\partial x} \right|^2 dx + \int_{\Omega_\varepsilon} |v \cdot e_1|^2 dx - h \int_{\Omega_\varepsilon} v \cdot e_1 dx,$$

then, in order to find out the effective field, we write (see [10]):

$$H_e = \frac{d\mathcal{E}_\varepsilon}{du},$$

it is to say

$$H_e = \frac{\partial^2 u}{\partial x^2} - u_2 e_2 - u_3 e_3 + h e_1.$$

Then, the limit dynamic system is obtain using the Landau Lifchitz combined with the new effective field H_e . This system is then given by (4) as we expected. The boundary conditions comes naturally from the fact that the solutions are in $H^1(] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[, \mathbb{R}^3)$.

3. EXISTENCE OF PARTICULAR EQUILIBRIUM STATES: THE WALLS

In this section we are interested in characterizing equilibrium states of the magnetization in a finite nano-wire when $h = 0$, it is to say when there is no external magnetic field. In this case, we look for solutions which can be written as follows:

$$M_0(x) = \begin{pmatrix} \sin \theta_0 \\ \cos \theta_0 \\ 0 \end{pmatrix}, \quad \forall x \in] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[,$$

where θ_0 is a map from $] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[, \mathbb{R}$ into \mathbb{R} such that M_0 is a stationary solution to (4). In fact, we want M_0 to verify:

$$-M_0 \wedge H_e - M_0 \wedge (M_0 \wedge H_e) = 0, \quad \forall x \in] - \frac{L}{\varepsilon}, \frac{L}{\varepsilon}[,$$

with

$$H_e = \frac{\partial^2 M_0}{\partial x^2} - \cos \theta_0 e_2,$$

then, one has the following relation

$$(8) \quad -\theta_0'' - \sin \theta_0 \cos \theta_0 = 0, \quad \forall x \in]-\frac{L}{\varepsilon}, \frac{L}{\varepsilon}[,$$

with, on the boundaries

$$(9) \quad \theta_0'(-\frac{L}{\varepsilon}) = \theta_0'(\frac{L}{\varepsilon}) = 0.$$

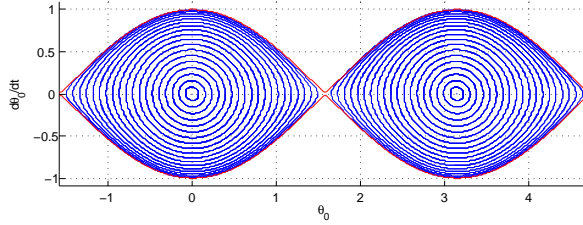


FIGURE 2. Phase portrait.

Setting $-\gamma_0 = \theta_0(0)$ ($\gamma_0 > 0$), we have, integrating the equation (8) and using (9):

$$(10) \quad (\theta_0')^2 + \sin^2 \theta_0 = \sin^2 \gamma_0.$$

The length of the nano-wire has to be such that the function θ_0 goes from $-\gamma_0$ to γ_0 . From formula (9), we deduce the length:

$$\ell(\gamma_0) = \int_{-\gamma_0}^{\gamma_0} \frac{d\theta}{\sqrt{\sin^2 \theta_0 - \sin^2 \theta}},$$

Then, using the length expression computed above, we deduce the following theorem

Theorem 3.1 (Existence of equilibrium states). *For every (ε, L) in \mathbb{R}_*^+ such that $\frac{L}{\varepsilon} > \pi$, there exists an equilibrium state for (4).*

Proof. In order to ensure that an equilibrium state as defined above, we must verify that the length given by the solution is at least greater to the effective length $2\frac{L}{\varepsilon}$. First of all, one has

$$\ell(\gamma_0) = \int_{-1}^1 \frac{\gamma_0}{\sin \gamma_0} \frac{d\theta}{\sqrt{1 - \frac{\sin^2 \theta}{\sin^2 \gamma_0}}},$$

we then see that

$$\lim_{\gamma_0 \rightarrow \frac{\pi}{2}} \ell(\gamma_0) = +\infty,$$

The first remark is that γ_0 is in the interval $]0, \frac{\pi}{2}[$. Now, computing the derivative of $\ell(\gamma_0)$ in γ_0 , we conclude that ℓ is strictly increasing on $]0, \frac{\pi}{2}[$. Then, the comparison

could be done at the limit $\gamma_0 = 0$:

$$\begin{aligned} \lim_{\gamma_0 \rightarrow 0} \ell(\gamma_0) &= \lim_{\gamma_0 \rightarrow 0} \int_{-1}^1 \frac{\gamma_0}{\sin \gamma_0} \frac{du}{\sqrt{1 - \frac{\sin^2(u \gamma_0)}{\sin^2 \gamma_0}}} \\ &= \int_{-1}^1 \frac{du}{\sqrt{1 - u^2}} \\ &= 2\pi, \end{aligned}$$

then, we require that $2\frac{L}{\varepsilon} > 2\pi$. □

4. UNSTABILITY OF WALLS WITHOUT APPLIED FIELD

In this section, we consider that L and ε given such that $\frac{L}{\varepsilon} > \pi$ and also a given wall θ_0 . We consider, as in [7] and [8], the mobile frame $(M_0(x), M_1(x), M_2)$ given by:

$$\forall x \in]-\frac{L}{\varepsilon}, \frac{L}{\varepsilon}[, M_0(x) = \begin{pmatrix} \sin \theta_0(x) \\ \cos \theta_0(x) \\ 0 \end{pmatrix}, M_1(x) = \begin{pmatrix} -\cos \theta_0(x) \\ \sin \theta_0(x) \\ 0 \end{pmatrix}, M_2(x) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Then, we describe the small perturbations of the static wall M_0 as follows:

$$\forall t \in \mathbb{R}^+, \forall x \in [-\frac{L}{\varepsilon}, \frac{L}{\varepsilon}], u(t, x) = r_1 M_1 + r_2 M_2 + \sqrt{1 - r_1^2 - r_2^2} M_0.$$

We denote $r = (r_1, r_2)$. In these coordinates, one can write:

$$\begin{aligned} H_e &= M_0 \left(2\partial_x r_1 \partial_x \theta_0 + r_1 \partial_{xx} \theta_0 - (\partial_x \theta_0)^2 - r_1 \sin \theta_0 \cos \theta_0 + \sin^2 \theta_0 \right) \\ &\quad + M_1 \left(\partial_{xx} r_1 - r_1 ((\partial_x \theta_0)^2 - \cos^2 \theta_0) - \partial_{xx} \theta_0 - \sin \theta_0 \cos \theta_0 \right) \\ &\quad + M_2 (\partial_{xx} r_2) + Q(r), \end{aligned}$$

where $Q(r)$ is the non linear part in r of H_e . Then, we can state that

$$\partial_t r = J \begin{pmatrix} (\mathcal{L} - \cos^2 \theta_0) r_1 \\ \mathcal{L} r_2 \end{pmatrix} + F(r, \partial_x r, \partial_{xx} r),$$

with $J = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$, and, $\mathcal{L}r = -\partial_{xx} r_2 + g_0 r_2$, where $g_0(x) = \sin^2 \theta_0 - (\theta'_0)^2$.

The linear unstability of the wall structure computed in the previous section is given by the study of the operator \mathcal{L} .

Proposition 2. \mathcal{L} is a linear, positive operator. Its first eigenvalue, 0, is associated to the eigenfunction $\cos \theta_0$ and its second eigenvalue, 1, is associated to the eigenfunction $\sin \theta_0$.

Proof. We set: $f = \theta'_0 \tan \theta_0$, then

$$\mathcal{L} = \ell^* \ell, \text{ where } \ell = \partial_x + f,$$

then, we can conclude that \mathcal{L} is a positive operator and that $\cos \theta_0$ is in the kernel of \mathcal{L} . Thus 0 is the first eigenvalue of \mathcal{L}_2 .

Furthermore, we have:

$$\mathcal{L}(\sin \theta_0) = \sin \theta_0,$$

it is to say that 1 is eigenvalue of \mathcal{L} associated to the eigenfunction $\sin \theta_0$. In addition we remark that $\sin \theta_0$ vanishes once in the domain, so by Sturm-Louville theorem, 1 is the second eigenvalue of \mathcal{L}_2 . \square

We can now prove Theorem 1.2.

Proof. From the previous proposition, since $-\cos^2 \gamma_0 < 0$, we conclude that $\mathcal{L} - \cos^2 \gamma_0$ has got one eigenvalue strictly negative, then, zero is unstable for the linearized of (3) around M_0 . \square

5. STABILIZATION OF WALLS

Now, we discuss the stabilization of M_0 by the command H_a (the applied field). We recall that we introduced the following command:

$$h(m) = \frac{-\varepsilon}{2L} \int_{-\frac{L}{\varepsilon}}^{\frac{L}{\varepsilon}} m_1(t, s) ds.$$

We want to prove that the profile M_0 is a stable stationary solution for the following system:

$$(11) \quad \begin{cases} \frac{\partial m}{\partial t} = -m \wedge H_e - m \wedge (m \wedge H_e), \\ \frac{\partial m}{\partial x}(-L/\varepsilon) = \frac{\partial m}{\partial x}(L/\varepsilon) = 0, \\ H_e = \frac{\partial^2 m}{\partial x^2} - m_2 e_2 - m_3 e_3 + h(m) e_1. \end{cases}$$

Proof. To start with, let us introduce $M_0 = (\sin \theta_0, \cos \theta_0, 0)$ given in Theorem 1.1. We recall that

$$\theta_0'' + \sin \theta_0 \cos \theta_0 = 0,$$

$$\theta'(-L/\varepsilon) = \theta'(L/\varepsilon) = 0,$$

Furthermore, on $[-L/\varepsilon, L/\varepsilon]$,

$$\cos^2 \theta_0 - (\theta_0')^2 = \cos^2 \gamma_0,$$

where $\gamma_0 = \theta_0(L/\varepsilon)$.

Since $h(M_0) = 0$, we remark that M_0 is a stationary solution of (11).

First step: moving frame.

As in the previous section, in the spirit of [7], we will describe the problem in the moving frame

$$(M_0(x), M_1(x), M_2(x)).$$

We write the solutions to (11) as:

$$m(t, x) = M_0(x) + r_1(t, x)M_1(x) + r_2(t, x)M_2(x) + \nu(r(t, x))M_0(x)$$

where $\nu(r) = \sqrt{1 - r_1^2 - r_2^2} - 1$. In this moving frame, we get

$$H_e = (g_0 + a_0)M_0 + (a_1 + \tilde{a}_1)M_1 + a_2M_2$$

with

$$\begin{aligned}
g_0(x) &= \sin^2 \theta_0 - (\theta'_0)^2 \\
a_0 &= 2\theta'_0 \partial_x r_1 + r_1 \theta''_0 - \cos \theta_0 \sin \theta_0 r_1 + \partial_{xx} \nu - \nu (\theta'_0)^2 + \nu \sin^2 \theta_0 + h(r) \sin \theta_0, \\
a_1 &= \partial_{xx} r_1 + \cos^2 \gamma_0 r_1 - S(r_1) \cos \theta_0, \\
\tilde{a}_1 &= -2\partial_x \nu \theta'_0 - \varphi(r) \cos \theta_0, \\
a_2 &= \partial_{xx} r_2.
\end{aligned}$$

where

$$S(r_1) = \frac{\varepsilon}{2L} \int_{-\frac{L}{\varepsilon}}^{\frac{L}{\varepsilon}} r_1 \cos \theta_0 ds, \quad \varphi(r) = -\frac{\varepsilon}{2L} \int_{-\frac{L}{\varepsilon}}^{\frac{L}{\varepsilon}} \nu(r) \sin \theta_0 ds, \quad \text{and } h(r) = S(r_1) + \varphi(r).$$

Using these coordinates in the Landau-Lifchitz equation (9) and projecting on M_1 and M_2 yield:

$$(12) \quad \partial_t r = \Lambda r + F(x, r, \partial_x r, \partial_{xx} r),$$

where

$$\Lambda r = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \tilde{\mathcal{L}}_1(r_1) \\ \mathcal{L}(r_2) \end{pmatrix}$$

with

$$\begin{aligned}
\mathcal{L} &= -\partial_{xx} + g_0, \\
\tilde{\mathcal{L}}_1 &= \mathcal{L} - \cos^2 \gamma_0 + \cos \theta_0 S,
\end{aligned}$$

and where the non linear part F is given by

$$F(x, r, \partial_x r, \partial_{xx} r) = F_1(r) \partial_{xx} r + F_2(r) (\partial_x r, \partial_x r) + F_3(x, r) \partial_x r + F_4(r) + F_5(r),$$

with

- $F_1 \in \mathcal{C}^\infty(\mathbb{R}^2; \mathcal{M}_2(\mathbb{R}))$:

$$\begin{aligned}
F_1(r) &= \begin{pmatrix} (r_2)^2 + (\nu(r))^2 + 2\nu(r) & \nu(r) - r_1 r_2 \\ r_1 r_2 - \nu(r) & (r_1)^2 + (\nu(r))^2 + 2\nu(r) \end{pmatrix} \\
&\quad + \begin{pmatrix} -r_1 - r_2 - \nu(r) r_1 \\ r_1 - r_2 - \nu(r) r_2 \end{pmatrix} \nu'(r),
\end{aligned}$$

- $F_2 \in \mathcal{C}^\infty(\mathbb{R}^2; \mathcal{L}_2(\mathbb{R}^2; \mathbb{R}^2))$:

$$F_2(r) (\partial_x r, \partial_x r) = \begin{pmatrix} -r_1 - r_2 - \nu(r) r_1 \\ r_1 - r_2 - \nu(r) r_2 \end{pmatrix} \nu''(r) (\partial_x r, \partial_x r),$$

- $F_3 \in \mathcal{C}^\infty([-L/\varepsilon, L/\varepsilon] \times \mathbb{R}^2; \mathcal{L}(\mathbb{R}^2; \mathbb{R}^2))$:

$$F_3(x, r) \partial_x r = 2\theta'_0(x) \begin{pmatrix} -r_1 - r_2 - \nu(r)r_1 \\ r_1 - r_2 - \nu(r)r_2 \end{pmatrix} \partial_x r_1 \\ - 2\theta'_0(x) \begin{pmatrix} (r_2)^2 + (\nu(r))^2 + 2\nu(r) + 1 \\ r_1 r_2 - \nu(r) - 1 \end{pmatrix} \nu'(r) (\partial_x r),$$

- $F_4(r) \in \mathcal{C}^\infty([-L/\varepsilon, L/\varepsilon] \times \mathbb{R}^2; \mathbb{R}^2)$:

$$F_4(r) = (\cos^2 \gamma_0 r_1) \begin{pmatrix} (r_2)^2 + (\nu(r))^2 + 2\nu(r) \\ r_1 r_2 - \nu(r) \end{pmatrix} - g_0 \nu(r) \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \\ + ((\theta_0'' - \cos \theta_0 \sin \theta_0) r_1 + g_0 \nu(r)) \begin{pmatrix} -r_1 - r_2 - \nu(r)r_1 \\ r_1 - r_2 - \nu(r)r_2 \end{pmatrix}.$$

- F_5 is given by

$$F_5(r) = -h(r) \cos \theta_0 \begin{pmatrix} (r_2)^2 + (\nu(r))^2 + 2\nu(r) \\ r_1 r_2 - \nu(r) \end{pmatrix} - \varphi(r) \cos \theta_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ + h(r) \sin \theta_0 \begin{pmatrix} -r_1 - r_2 - \nu(r)r_1 \\ r_1 - r_2 - \nu(r)r_2 \end{pmatrix}.$$

Remark 2. The command h makes the linear part of (12) positive. Indeed, on one hand, we know that $\mathcal{L} \geq 0$ with $\text{Ker } \mathcal{L} = \mathbb{R} \cos \theta_0$. On the other hand, $\tilde{\mathcal{L}}_1 = \mathcal{L} + \cos \theta_0 S - \cos^2 \gamma_0$. On $\mathbb{R} \cos \theta_0$, $\tilde{\mathcal{L}}_1 \cos \theta_0 = \alpha_0 \cos \theta_0$ with

$$(13) \quad \alpha_0 = \frac{\varepsilon}{2L} \int_{-L/\varepsilon}^{L/\varepsilon} \cos^2 \theta_0(x) dx - \cos^2 \gamma_0 > 0$$

since for $x \in]-L/\varepsilon, L/\varepsilon[$, $\cos^2 \theta_0(x) > \cos^2 \gamma_0$, and on $(\cos \theta_0)^\perp$,

$$\tilde{\mathcal{L}}_1|_{(\cos \theta_0)^\perp} = \mathcal{L} - \cos^2 \theta_0 \geq 1 - \cos^2 \theta_0$$

since $\mathcal{L} \geq 1$ on $(\cos \theta_0)^\perp$ (see Proposition 2).

Second step: new coordinates.

The Landau Lifschitz equation (11) is invariant by rotation around the wire axis, so we can build a family of static solutions.

For $\tau \in \mathbb{R}$ let us introduce the rotation around the x -axis given by

$$\rho_\tau = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \tau & -\sin \tau \\ 0 & \sin \tau & \cos \tau \end{pmatrix}.$$

We denote $M_\tau(x) = \rho_\tau M_0(x)$, and R_τ its projection on the moving frame:

$$R_\tau(x) = \begin{pmatrix} M_\tau(x) \cdot M_1(x) \\ M_\tau(x) \cdot M_2(x) \end{pmatrix} = \begin{pmatrix} \cos \theta_0(x) \sin \theta_0(x) (\cos \tau - 1) \\ \cos \theta_0(x) \sin \tau \end{pmatrix}.$$

Since M_τ is solution to (11), R_τ is a static solution of (12), that is

$$(14) \quad \Lambda R_\theta + F(x, R_\theta, \partial_x R_\theta, \partial_{xx} R_\theta) = 0.$$

Now in order to avoid the problems due to the zero eigenvalue of Λ , we describe r in the new coordinates (τ, σ, W) defined by

$$(15) \quad r(t, x) = R_{\tau(t)}(x) + \sigma(t) \begin{pmatrix} \cos \theta_0(x) \\ 0 \end{pmatrix} + W(t, x),$$

where $(\tau, \sigma) \in \mathcal{C}^1(\mathbb{R}^+; \mathbb{R}^2)$ and $W \in \mathcal{C}^1(\mathbb{R}^+; H^2)$ such that both coordinates of W are in $(\cos \theta_0)^\perp$.

Indeed, as in [11], we can prove that for a given $r \in H^2(-L/\varepsilon, L/\varepsilon)$ in a neighbourhood of 0, there exists a unique $(\tau, \sigma, W) \in \mathbb{R} \times \mathbb{R} \times \mathcal{W}$ such that

$$\rho = R_\theta + \sigma \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} + W,$$

where

$$\mathcal{W} = \left\{ W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} \in H^2, \int_{-L/\varepsilon}^{L/\varepsilon} W_1(x) \cos \theta_0(x) dx = \int_{-L/\varepsilon}^{L/\varepsilon} W_2(x) \cos \theta_0(x) dx = 0 \right\}.$$

Remark 3. On $(\cos \theta_0)^\perp$, the operators \mathcal{L} and $\mathcal{L} - \cos^2 \theta_0$ are non negative, so we introduce the following norms on \mathcal{W} , respectively equivalent to the H^2 and the H^3 norms:

$$\|W\|_{\mathcal{H}^2} = \left(\|\mathcal{L}^{\frac{1}{2}} (\mathcal{L} - \cos^2 \theta_0)^{\frac{1}{2}} W_1\|_{L^2}^2 + \|\mathcal{L} W_2\|_{L^2}^2 \right)^{\frac{1}{2}},$$

$$\|W\|_{\mathcal{H}^3} = \left(\|\mathcal{L}^{\frac{1}{2}} (\mathcal{L} - \cos^2 \theta_0) W_1\|_{L^2}^2 + \|\mathcal{L}^{\frac{3}{2}} W_2\|_{L^2}^2 \right)^{\frac{1}{2}}.$$

Plugging the decomposition (15) in (12) and using (14) yield the following equivalent form for the Landau-Lifschitz equation in the coordinates (τ, σ, W) , valid for little perturbations of M_0 . Indeed we have:

$$\partial_t r = \frac{d\tau}{dt} R'_\tau(x) + \frac{d\sigma}{dt} \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} + \partial_t W,$$

where

$$R'_\tau(x) = \begin{pmatrix} -\sin \tau \cos \theta_0 \sin \theta_0 \\ \cos \tau \cos \theta_0 \end{pmatrix}.$$

In addition,

$$\Lambda r = \Lambda R_\tau + \begin{pmatrix} -\alpha_0 \cos \theta_0 \\ \alpha_0 \cos \theta_0 \end{pmatrix} \sigma + \Lambda(W)$$

(see Remark 2 for the definition of α_0), and

$$F(x, r, \partial_x r, \partial_{xx} r) = F(x, R_\tau, \partial_x R_\tau, \partial_{xx} R_\tau) + G(x, \tau, \sigma, W, \partial_x W, \partial_{xx} W).$$

The last term G is obtained from F with the Taylor formula around R_τ :

$$\begin{aligned} G(x, \tau, \sigma, W, \partial_x W, \partial_{xx} W) = & F_1(r)(\partial_{xx} w) + \tilde{F}_1(r)(w)(\partial_{xx} R_\tau) \\ & + 2F_2(r)(\partial_x w, \partial_x R_\tau) + F_2(r)(\partial_x w, \partial_x w) \\ & + \tilde{F}_2(r)(w)(\partial_x R_\tau, \partial_x R_\tau) \\ & + F_3(x, r)(\partial_x w) + \tilde{F}_3(x, r)(w)\partial_x R_\tau + \tilde{F}_4(r)(w) \\ & + F_5(r), \end{aligned}$$

where $w = W + \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} \sigma$, and where for $i = 1..4$, $\tilde{F}_i(r) \in \mathcal{L}(\mathbb{R}^2; \mathbb{R}^2)$ is given by

$$\tilde{F}_i(r) = \int_0^1 F'_i(R_\tau + sw) ds.$$

From straightforward calculations, we see that:

$$F_1(r) = \mathcal{O}(|r|^2), \quad \tilde{F}_1(r) = \mathcal{O}(|r|^2)$$

$$F_2(r) = \mathcal{O}(|r|), \quad \tilde{F}_2(r) = \mathcal{O}(1)$$

$$F_3(x, r) = \mathcal{O}(|r|), \quad \tilde{F}_3(x, r) = \mathcal{O}(1)$$

$$\tilde{F}_4(r) \mathcal{O}(|r|).$$

In addition, $\partial_x R_\tau = \mathcal{O}(|r|)$. Concerning F_5 , on one hand we remark that

$$S(r_1) = S(W_1) + \sigma S(\cos \theta_0)$$

(since $S(R_\tau) = 0$.)

On the other hand, $\varphi(R_\tau) = 0$ and

$$\varphi(r) = -\frac{\varepsilon}{2L} \int_{-L/\varepsilon}^{L/\varepsilon} \tilde{\nu}(r)(w) \sin \theta_0,$$

where

$$\tilde{\nu}(r) = \int_0^1 \nu'(R_\tau + sw) ds = \mathcal{O}(|r|).$$

Therefore with all these estimates, if $\|r\|_{H^2}$ is sufficiently small, we have

$$(16) \quad \|G\|_{L^2} \leq K \|r\|_{L^\infty} [|\sigma| + \|W\|_{\mathcal{H}^2}],$$

$$\|\partial_x G\|_{L^2} \leq K \|r\|_{L^\infty} [|\sigma| + \|W\|_{\mathcal{H}^3}]$$

(see Remark 3.)

Hence, using (14), we have obtained

$$(17) \quad \frac{d\tau}{dt} R'_\tau + \frac{d\sigma}{dt} \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} + \partial_t W = \sigma \begin{pmatrix} -\alpha_0 \cos \theta_0 \\ \alpha_0 \cos \theta_0 \end{pmatrix} + \Lambda W + G(x, \tau, \sigma, W, \partial_x W, \partial_{xx} W).$$

In order to separate the unknowns τ , σ and W , we first take the inner product of (17) with $\begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix}$ and with $\begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix}$. We remark that both $\partial_t W$ and ΛW are orthogonal to these vectors, so that we obtain:

$$\rho_0 \sigma' = -\alpha_0 \rho_0 \sigma + \bar{G}_1,$$

$$g_\tau \tau' = \alpha_0 \rho_0 \sigma + \bar{G}_2,$$

where

$$\rho_0 = \int_{-L/\varepsilon}^{L/\varepsilon} \cos^2 \theta_0,$$

$$\bar{G}_1 = \int_{-L/\varepsilon}^{L/\varepsilon} G(x, \tau, \sigma W, \partial_x W, \partial_{xx} W) \cdot \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} dx,$$

$$g_\tau = \int_{-L/\varepsilon}^{L/\varepsilon} R'_\tau(x) \cdot \begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix} dx,$$

$$\bar{G}_2 = \int_{-L/\varepsilon}^{L/\varepsilon} G(x, \tau, \sigma W, \partial_x W, \partial_{xx} W) \cdot \begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix} dx.$$

By subtraction, we have:

$$\partial_t W = \Lambda W + \tilde{G}$$

with

$$\tilde{G} = G - \frac{\bar{G}_2}{g_\tau} R'_\tau - \frac{\bar{G}_1}{\rho_0} \begin{pmatrix} \cos \theta_0 \\ 0 \end{pmatrix} + \alpha_0 \sigma \left[\begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix} - \frac{\rho_0}{g_\tau} R'_\tau \right].$$

We are then led to study the following equation

$$(18) \quad \tau' = \alpha_0 \frac{\rho_0}{g_\tau} \sigma + \frac{1}{g_\tau} \bar{G}_2,$$

together with the system coupling:

$$(19) \quad \sigma' = -\alpha_0 \sigma + \frac{1}{\rho_0} \bar{G}_1,$$

with

$$(20) \quad \partial_t W = \begin{pmatrix} -(\mathcal{L} - \cos^2 \gamma_0) W_1 - \mathcal{L} W_2 \\ (\mathcal{L} - \cos^2 \gamma_0) W_1 - \mathcal{L} W_2 \end{pmatrix} + \tilde{G}.$$

From (16), with Remark 3 we have

$$(21) \quad \left| \frac{1}{\rho_0} \bar{G}_1 \right| (t) \leq K \|r\|_{L^\infty} \left[|\sigma(t)| + \|W(t)\|_{\mathcal{H}^3} \right].$$

In addition, since $g_\tau = \rho_0 + \mathcal{O}(\tau^2)$, since $R'_\tau = \begin{pmatrix} 0 \\ \cos \theta_0 \end{pmatrix} + \mathcal{O}(\tau)$, we get:

$$(22) \quad \|\tilde{G}\|_{H^1} \leq K \|r\|_{L^\infty} \left[|\sigma(t)| + \|W(t)\|_{\mathcal{H}^3} \right].$$

Taking the inner product of (20) with $\begin{pmatrix} \mathcal{L}(\mathcal{L} - \cos^2 \gamma_0)W_1 \\ \mathcal{L}^2 W_2 \end{pmatrix}$, we obtain using (22) that

$$\frac{d}{dt} (\|W\|_{\mathcal{H}^2}^2) + \|W\|_{\mathcal{H}^3}^2 \leq K \|r\|_{L^\infty} [|\sigma(t)| + \|W(t)\|_{\mathcal{H}^3}].$$

Multiplying (19) by σ , (21) yields:

$$\frac{d}{dt} \sigma^2 + \alpha_0 \sigma^2 \leq K \|r\|_{L^\infty} [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^3}^2].$$

Summing up the previous estimates, we have:

$$\frac{d}{dt} [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^2}^2] + \alpha_0 [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^3}^2] (1 - K \|r\|_{L^\infty}) \leq 0.$$

So there exists $\delta > 0$ such that while $\|r\|_{L^\infty} \leq \frac{1}{2K}$,

$$\frac{d}{dt} [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^2}^2] + \delta [|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^2}^2] \leq 0,$$

that is

$$[|\sigma(t)|^2 + \|W(t)\|_{\mathcal{H}^2}^2] \leq [|\sigma_0|^2 + \|W_0\|_{\mathcal{H}^2}^2] e^{-\delta t}.$$

Now, with equation (18), we have

$$|\tau'| \leq K |\sigma| + K [|\sigma(t)| + \|W(t)\|_{\mathcal{H}^2}]$$

so while $\|r\|_{L^\infty} \leq \frac{1}{2K}$,

$$|\tau| \leq |\tau_0| + K [|\sigma_0| + \|W_0\|_{\mathcal{H}^2}] e^{-\delta t/2}.$$

Therefore, if τ_0 , σ_0 and $\|W_0\|_{H^2}$ are small enough, we remain in the domain $\{\|r\|_{L^\infty} \leq \frac{1}{2K}\}$ and all the previous estimates remain valid for all times. This concludes the proof of Theorem 1.3. \square

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