Adaptive Quantization: a comparative study

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***Abstract*—This paper investigates the comparison of two adaptive quantization algorithms for linear systems in the context of NCS[1]. The first algorithm that we call ZIZO[2] is due to Sharon and Liberzon and has been published in [10] and the second one that we call D-ZIZO[3] has been introduced in [7]. We show that D-ZIZO generates a smoother quantization noise than ZIZO algorithm and that D-ZIZO allows a faster convergence for same given properties. Moreover in the multivariable case, D-ZIZO allows us to use the smallest number of quantization bits in order to achieve stability.**

I. INTRODUCTION

In a networked controlled system, the output signals have to be digitalized before transmission. Our objective is then to use the minimum quantization bits necessary to maintain stability on the closed loop system, that is to say the minimum bandwidth.

Several quantization methods have already been proposed during the last decade. In [3], authors use a logarithm quantization of the signal which only ensures local stability and does not minimize the necessary channel bandwidth. In [1], authors propose an adaptive algorithm based on a Δ modulation and which allows a global stability. The key point of this method is to increase the value of the quantization steps when the signal is large ("Zoom In") and to decrease this value otherwise ("Zoom Out"). This method is improved in [10] in order to reduce the communication bandwidth. Thus, authors divide the "Zoom In" step into two steps ("Zoom In Escape" and "Zoom In Measurements"). We will call this method ZIZO in the rest of the paper. In [5], [11] authors prove that semi global stability is possible in a NCS provided that the communication rate R is greater than a minimum communication rate depending on the eigenvalues of the open loop matrix of the system. Recent works [2] and [6] based on Δ modulation provide local stability with a better characterization of the attraction domain than [5] and [11]. In [6], authors generalize their work to non diagonalizable systems. This also permits to tune the quantization steps in order to get a better signal reconstruction quality. In [7], we expose an adaptive algorithm which allows us to use the theoretical minimum bandwidth. Previously, in [4] authors have presented a quantization method based on a one-bit-adaptive Δ modulation. The interest of this technic is that it permits global stability in the scalar case provided that the open loop eigenvalue λ is such that: $|\lambda| < 1.3$. Nevertheless it can be shown in [11] that in this case (one bit quantization) it is theoretically possible to just have $|\lambda| < 2$. In [9] authors propose an adaptive quantization method robust to any bounded perturbation. This method also works for multivariable class of linear systems. However, the necessary bandwidth is generally greater than the minimum one exposed in [11], even if it may be very close in certain cases. At the contrary, the algorithm proposed in [10] needs a bandwidth which is always only slightly greater than, or even sometimes equal to, the minimum bandwidth. Nevertheless, this algorithm (ZIZO) does not guaranty that the reconstruction error signal is smooth, but it is subject to frequency pics. This issue is solved in [7] thanks to the introduction of a Dwell Time phase between Zoom In and Zoom Out. That is why we call this algorithm D-ZIZO. Moreover an improvement of the quantization itself permits to use the minimum bandwidth in the scalar case as well as in the multivariable one.

The paper is organized as follows. Firstly, we present the problem formulation in Section II. We go on in Section III with the description of the two Adaptive quantization algorithms. These two algorithms are described without and with bounded exogenous inputs. We continue with a comparison between the performances of ZIZO and D-ZIZO algorithms in Section IV. With some analysis and simulation results, we point out three results. Firstly, ZIZO generates high frequencies though the D-ZIZO signals are smoother. Secondly, for same constraints on regulation detection, D-ZIZO is faster than ZIZO. Moreover, with the same convergence rate, the regulation detection is faster for D-ZIZO. Finally, the change of coordinates enables D-ZIZO to achieve theoretical minimum channel bandwidth.

II. PROBLEM FORMULATION

The problem considered is the stabilization of a multivariable system in which sensor signals are centralized, and then transmitted through a digital communication link to the controller.

We assume the following:

- the coding process is centralized : a single encoder can be used to encode all the sensed states of the system,
- the encoded information is transmitted through a noiseless perfect transmission channel. Hence delay, errors due to the transmission are not considered,
- the encoder and decoder clocks are assumed to be synchronized, and samples are assumed to occur at each T_s.

The following notations will be used:

- $x_k = [x_k^1, \ldots, x_k^n]^T \in R^{(n\times 1)}$ is the n-dimensional sensed state vector at instant kT_s (each x_k^i corresponds to the $i-th$ sensor) ;

[1]Networked Controlled Systems
[2]Zoom In Zoom Out
[3]ZIZO with Dwell Time

- $u_k = [u_k^1, \ldots, u_k^m]^T \in R^{(m\times 1)}$, is m-dimensional control input vector at instant kT_s.
- M_i number of words by signal $\tilde{z}_k^i$.

The discretized system is described by:

$$x_{k+1} = \boldsymbol{A}x_k + \boldsymbol{B}u_k \tag{1}$$

$$u_k = -\boldsymbol{K}\hat{x}_k \tag{2}$$

with $\boldsymbol{K}$ such as $\boldsymbol{A} - \boldsymbol{BK}$ is Schur (eigenvalues have their module strictly inferior than 1). $\hat{x}_k$ is an estimation of x_k, and $\tilde{x}_k$ denotes the estimation error :

$$\tilde{x}_k = x_k - \hat{x}_k, \tag{3}$$

and, more generally, for a given signal s_k, $\hat{s}_k$ represents an estimated value of s_k and $\tilde{s}_k$ represents the error $s_k - \hat{s}_k$.

A. Architecture of ZIZO

In [10], the architecture of ZIZO is composed of two main components

- **The vector quantizer block** transforms the error $\tilde{x}_k$, into a finite codeword set, each signal is coded with the same granularity Δ.
- **The predictor**, that transforms back the codeword into a system state prediction $\hat{x}_k$ with the equation

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k \tag{4}$$

  This predictor uses input and signal information at the encoder: this hypothesis is less realistic than the one in [7] where only signal information is used at the encoder.

In ZIZO algorithm, we have the error equation:

$$\tilde{x}_{k+1} = A(\tilde{x}_k - \hat{\tilde{x}}_k) \tag{5}$$

B. Architecture of D-ZIZO

Figure 1 shows the architecture of the proposed differential coding algorithm. It is composed of three main components:

- **The vector quantizer block** transforms the error $\tilde{z}_k$, into a finite codeword set
- **The predictor** , that transforms back the codeword into a system state prediction $\hat{x}_k$
- **The rotation matrix** T_k transforms the estimation error $\tilde{x}_k$ between the signal x_k and its estimated (reconstructed) value $\hat{x}_k$ into a new set of coordinates $\tilde{z}_k$, i.e.

$$\tilde{z}_k = \boldsymbol{T}_k^{-1}\tilde{x}_k \tag{6}$$

In [6], we have proven that the change of coordinates with a dynamic matrix T_k permits to reduce the study to the following class of systems:

$$\tilde{z}_{k+1} = \underbrace{\begin{pmatrix} |\lambda| & 1 & 0 \\ 0 & |\lambda| & 1 \\ 0 & 0 & |\lambda| \end{pmatrix}}_{F(|\lambda|)\in R^{\mu*\mu}} (\tilde{z}_k - \hat{\tilde{z}}_k)$$

with

- $\hat{\tilde{z}}_k$ is the quantized signal of $\tilde{z}_k$ with its associated quantization steps Δ_k.
- μ is the readjusted size of $\tilde{z}_k$.
- $\Delta_k = [\Delta_k^1, \ldots, \Delta_k^\mu]^T$ are the quantization steps associated to $\hat{\tilde{z}}_k$

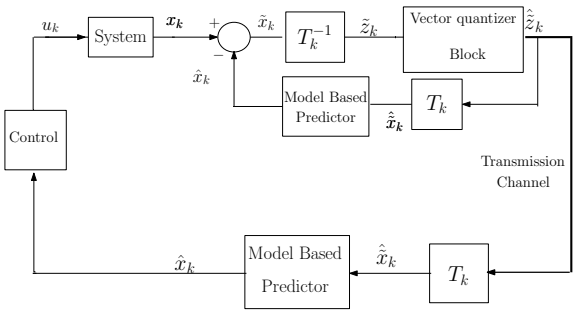


Fig. 1. Schematic representation of the system in our study.

III. Adaptive Delta modulation: Algorithm principles

A. Presentation of the algorithm introduced in [10]: ZIZO (Results without noise)

Let us begin with the following lemma:

Lemma 1: Important item to create ZIZO. Assuming that $\hat{\tilde{x}}_k$ is computed thanks to the quantization procedure given in [8], and suppose that

$$\tilde{z}_0 \in \Omega^{ext} = \{\tilde{z} \in \mathbf{R}^\mu : |\tilde{z}^i| \leqslant M\frac{\Delta}{2}, 1 \leqslant i \leqslant \mu\}$$

with Δ the quantization step. And M the number of words per signal chosen such that:

$$\|A\|_\infty \leqslant M$$

Then

i) Ω^{ext} is an invariant set

ii) $\tilde{z}_k \in \Omega^{\text{int}}$, $\forall k \geqslant 1$ where
$\Omega^{\text{int}} = \{\tilde{z} \in \mathbf{R}^\mu : \|\tilde{z}\|_\infty \leqslant \|A\|_\infty \Delta/2\}$

In the case of ZIZO and D-ZIZO algorithms, we introduce the set Ω_k^{ext} defined by

$$\Omega_k^{ext} = \{\tilde{z} \in \mathbf{R}^\mu : |\tilde{z}^i| \leqslant M_i\frac{\Delta_k^i}{2}, 1 \leqslant i \leqslant \mu\}$$

It is worthwhile noting that $\Delta^i = \Delta$ and $M_i = M$ $\forall 1 \leqslant i \leqslant \mu$. We emphasize the fact that contrary to the set Ω^{ext}, this set is dynamic. We will go on with the main phases of ZIZO algorithm:

- **"Zoom In measurement" strategy**: The quantization steps decrease during $p-1$ sample times.
- **"Zoom In escape" strategy**: After the "Zoom In measurement" process, quantization steps increase such that if $\tilde{x}_k \in \Omega_k^{ext}$ then $\|\tilde{x}_{k+1}\|_\infty \leqslant (M-2)\Delta_{k+1}/2$. So that $\tilde{x}_{k+1} \in \Omega_{k+1}^{ext}$. Else $\tilde{x}_k \notin \Omega_k^{ext}$ and we switch to a "Zoom Out" process.
- **"Zoom Out" strategy**: The signal is outside Ω^{ext}, the quantization steps increase with the target to ensure that the signal will be inside Ω^{ext} after a finite time.

To realize this strategy, ZIZO needs different coefficients such that $\Theta_{in,m}$, $\Theta_{in,e}$ and Θ_{out} and p which respectively are the compression ratio for "Zoom In measurement"

strategy, the expansion factor for "Zoom In escape" strategy and the expansion factor for "Zoom Out" strategy, p corresponds to the number of necessary sample times that ZIZO needs to prove that the signal belongs to Ω_k^{ext}.

Those parameters are constrained by

$$\begin{aligned} \frac{\|A\|_\infty}{M} &< \Theta_{in,m} < 1 && (7)\\ \Theta_{out} &> \|A\|_\infty && (8)\\ \Theta_{in,e} &> \frac{\|A\|_\infty}{M-2} && (9)\\ \Theta_{in,m}^{p-1}\Theta_{in,e} &< 1 && (10)\\ \Delta_{k+1} &= (\Theta_{in,m} \text{ or } \Theta_{in,e} \text{ or } \Theta_{out})\Delta_k && (11) \end{aligned}$$

With regards to these inequalities, the convergence depends on p. Bigger is p, faster is the convergence but worse is the regulation detection.

B. Presentation of the algorithm introduced in [7]:D-ZIZO (Results without noise)

Though in ZIZO, each signal is quantized with the same quantization step, in D-ZIZO each signal has its own quantization step but they are constrained by:

Lemma 2: Important item to obtain D-ZIZO. Assuming that $\hat{\tilde{z}}_k$ is computed thanks to the quantization procedure given in [6], and suppose that

$$\tilde{z}_0 \in \Omega^{ext} = \{\tilde{z} \in \mathbf{R}^\mu : |\tilde{z}^i| \leqslant M_i \frac{\Delta_0^i}{2}, 1 \leqslant i \leqslant \mu\}$$

and the quantization steps satisfy the equations

$$|\lambda| + \frac{\Delta_0^{i+1}}{\Delta_0^i} \leqslant M_i,\ 1 \leqslant i \leqslant \mu - 1 \qquad (12)$$

Then

$i)$ Ω^{ext} is an invariant set

$ii)$ $\tilde{z}_k \in \Omega^{\text{int}}$, $\forall k \geqslant 1$ where
$\Omega^{\text{int}} = \{\tilde{z} \in \mathbf{R}^\mu : |\tilde{z}^i| \leqslant |\lambda|\Delta_0^i/2 + \Delta_0^{i+1}/2$
$\forall i : 1 \leqslant i \leqslant \mu - 1$ and $|\tilde{z}^\mu| \leqslant |\lambda|\Delta_0^\mu/2\}$

Remark 1: In a semi-global stabilization context, item $ii)$ of 1 and 2 allows us to obtain the maximal convergence rate. In D-ZIZO algorithm (global stabilization), we can choose the quantization steps. If we choose $\Delta_0^{i+1}/\Delta_0^i$ close to 0, convergence rate $\max_{1\leqslant i\leqslant n-1}(|\lambda| + \Delta_0^{i+1}/\Delta_0^i)/M_i$ of D-ZIZO is near $|\lambda|/M$ that could be quantitatively least than the maximal shrinkage rate $\|A\|_\infty/M$ of ZIZO. Since in ZIZO there is no change of coordinates (with T_k matrix) and no quantization steps tuning methods, the necessary number of words to stabilize the system is also more important.

In what follows, we briefly explain the principle of the algorithm. Initially, the signal $\tilde{z}_0 \notin \Omega_0^{ext}$, the quantization steps have to increase faster than the signal to ensure that there exists a k_0 such that $\tilde{z}_{k_0} \in \Omega_{k_0}^{ext}$. Our aim is to determine whether $\tilde{z}_{k_0}$ belongs to $\Omega_{k_0}^{ext}$. When this situation is obtained, the quantization steps related to the set Ω_k^{ext} decrease.

The algorithm proposed in [7] is presented in 3 parts:

- **"Zoom Out" strategy**: $\tilde{z}_k \notin \Omega_k^{ext}$, the quantization steps increase with the target to ensure that the signal will belong to Ω_k^{ext} after a finite time.
- **Dwell time phase**: the algorithm does not have enough information to know if the signal $\tilde{z}_{k_0}$ at a precise moment k_0 belongs to $\Omega_{k_0}^{ext}$ or not. Quantization steps do not change.
- **"Zoom In" strategy**: $\tilde{z}_k \in \Omega_k^{ext}$, the quantization steps decrease.

The dwell time phase is the key point of the algorithm. In the scalar case, we define $m^\star$ the necessary dwell time to decide whether $|\tilde{z}_m| \leqslant M\Delta_m/2$: $m^\star$ is the smallest value of m verifying this constraint (considering that $m = 0$ at the beginning of the dwell time phase). After $m^\star$ iterations, a criterion exposed in [7] allows us to determine whether D-ZIZO enters in "Zoom Out" or "Zoom In" phase. To extend to multivariable class of linear systems, we use the cascade structure of $F(\lambda)$. Thus we use the same analysis but for $\tilde{z}^\mu$ and then $\tilde{z}^{\mu-1}$ and so on. To obtain the algorithm, we must introduce two scalars C_{out}, C_{in} which respectively correspond to the expansion factor for a "Zoom Out" procedure and the compression ratio for a "Zoom In" procedure. Those two parameters are constrained by

$$\begin{aligned} \max_{1\leqslant i\leqslant n-1}\left(\frac{|\lambda| + \frac{\Delta_0^{i+1}}{\Delta_0^i}}{M_i}\right) &< C_{in} < 1 && (13)\\ \max_{1\leqslant i\leqslant n-1}\left(|\lambda| + \frac{\Delta_0^{i+1}}{\Delta_0^i}\right) &< C_{out} && (14)\\ \Delta_{k+1} &= (C_{in} \text{ or } 1 \text{ or } C_{out})\Delta_k && (15) \end{aligned}$$

Remark 2: In ZIZO algorithm, authors do not use the change of coordinates exposed in the section concerning the architecture of D-ZIZO. To prove the stability results, the analysis is realized on $|\tilde{x}_k|$. The positivity of the eigenvalues and the cascade structure of $F(|\lambda|)$ provide some properties on the sign of each signal.

C. Results with noise

In the previous section, we have presented the case of noiseless systems. In what follows, we will extend this result to a bounded noisy systems. Equation (1) becomes, with noise:

$$\begin{aligned} x_{k+1} &= \boldsymbol{A}x_k + \boldsymbol{B}u_k + s_k && (16)\\ \hat{x}_{k+1} &= (A + BK)\hat{x}_{k+1} + A\hat{\tilde{x}}_k && (17)\\ \tilde{x}_{k+1} &= \boldsymbol{A}(\tilde{x}_k - \hat{\tilde{x}}_k) + s_k && (18) \end{aligned}$$

1) D-ZIZO:

$$\tilde{z}_{k+1} = T_{k+1}^{-1}AT_k(\tilde{z}_k - \hat{\tilde{z}}_k) + T_{k+1}^{-1}T_k s_k$$

Since $\|T_k\|_\infty$ is bounded, $T_{k+1}^{-1}T_k s_k$ is also bounded. We can then write, with w_k a bounded noise:

$$\tilde{z}_{k+1} = F(\lambda)(\tilde{z}_k - \hat{\tilde{z}}_k) + w_k \qquad (19)$$

The initialization of quantization steps is very important for D-ZIZO. When there are some exogenous inputs, the initialization of quantization steps in the algorithm depends on the maximal value of each exogenous input. Each initial value of quantization steps have to be tuned by a new method. Nevertheless in [10], the quantization steps of

each signal are the same, thus there is no condition on the quantization steps.

Let us define two dynamic scalars $C_{out}(k), C_{in}(k)$ which respectively correspond to the expansion factor for a "Zoom Out" procedure and the compression rate for a "Zoom In" procedure. Those two parameters are constrained by

$$\begin{aligned} \max_{1\leqslant i\leqslant n-1}\left(\frac{|\lambda|+\frac{\Delta_0^{i+1}}{\Delta_0^i}+\frac{2W^i}{\Delta_k^i}}{M_i}\right) &< C_{in}(k)<1 \\ \max_{1\leqslant i\leqslant n-1}\left(|\lambda|+\frac{\Delta_0^{i+1}}{\Delta_0^i}+\frac{2W^i}{\Delta_k^i}\right) &< C_{out}(k) \end{aligned}$$

The principle of the algorithm is the same except two rules.

- The value $m^{\star}_{i,k}(\frac{\Delta_k^i}{W_i})$ is dynamic
- In the Dwell Time phase, we fix a maximal time for this phase. If the dwell time is over this period, the algorithm enters in "Zoom Out" phase.

Remark 3: The smaller is Q, the bigger is the asymptotical value of quantization steps, so the set Ω_{∞}^{ext} will be larger. However regulation is faster. If the value of Q is big, asymptotical quantization steps are near their minimal value. But if a disturbance occurs, the system slowly regulates.

2) ZIZO: To cope with an exogenous input, ZIZO is constrained by a maximal value of quantization. This maximal value depends on the matrix A and the amplitude of the noise. The conception of this algorithm is simpler than D-ZIZO.

IV. Performance comparison between ZIZO and D-ZIZO

In this section, we will compare with some examples and by simulations those two strategies in term of regulation performances. We also show that D-ZIZO always can reach the minimum bit constraint though ZIZO can reach it only for special cases. The simulation results are presented here to emphasize the differences between the two algorithms.

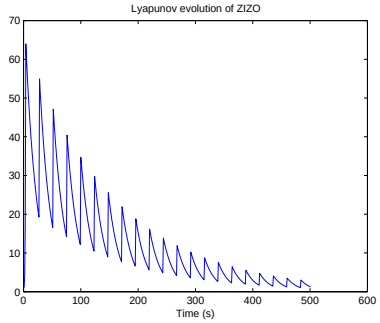
A. High frequency generation

1) Case study: Suppose the system (1) with $\boldsymbol{A} = 2.8, \quad \boldsymbol{B} = 0.2$. The controller gain is $K = 8$, so $A - BK$ is Schur. The initial conditions are $\tilde{z}_0^1 = 100$, $\hat{x}_0 = 0$, $M_1 = 3$. The initialization of the quantization step is $\Delta_0 = 0,6$. The parameters of ZIZO are $\Theta_{in,m} = 2.81/3$, $\Theta_{in,e} = 2.81$ and $\Theta_{out} = 3$ and $p = 22$. The parameters of D-ZIZO are $C_{out} = 3$,$C_{in} = 2.81/3$ and $m^{\star} = 6$. Those parameters are used for Figure 2 and Figure 3.

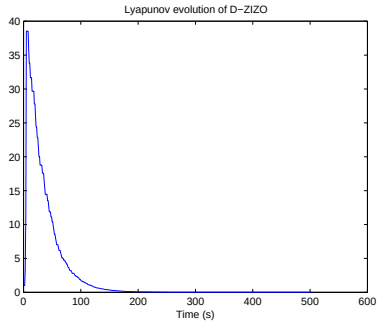
In this part, we highlight that for the "Zoom In" process the evolution of $\tilde{z}_k$ for D-ZIZO is smother than the evolution of $\tilde{x}_k$ with ZIZO. And we introduce the Lyapunov function $V_k = \tilde{z}_k^2$. Without loss of generalization, we assume that the initial condition is in $\Omega_{\text{ext}}(0)$. So $\tilde{z}_k$ is in $\Omega_{\text{ext}}(k)$, hence we have

$$\begin{aligned} V_k &= \tilde{z}_k^2 \\ V_k &\leqslant (\frac{M\Delta_k}{2})^2 = W_k \end{aligned}$$

In Figure 2, we see the evolution of V_k, the coefficients in the two algorithms are chosen such that $V_k = W_k$, we easily find out that the Lyapunov function is piecewise decreasing for ZIZO and that is smoothly decreasing for D-ZIZO. Since in that example $V_k = W_k$, the Lyapunov function depends on Δ_k^2. So the evolution of V_k is only driven by the algorithms ZIZO and D-ZIZO. In the "Zoom In Measurement" procedure of ZIZO, the quantization step begins to decrease and after a certain number of sample times p (coefficient relied to regulation detection), that depends on (7), algorithm enters in "Zoom In escape" process for only one sample time. In this procedure, quantization steps increase. This method is reproduced each p times. In D-ZIZO, quantization step only decreases. In Figure 3, we are interested in the spectral density of $\tilde{z}_k = \tilde{x}_k$. We remark that ZIZO generates a lot of frequency pics though D-ZIZO not. It is very important for closed loop system to be robust to noise generation (here, quantization noise). So D-ZIZO is better than ZIZO for the non-generation of frequency pics.



(a)



(b)

Fig. 2. Comparison between Lyapunov functions. Figure (a) is obtained with ZIZO Algorithm result and Figure (b) represents D-ZIZO algorithm.

B. Convergence rate

Moreover, in the noiseless adaptive process, we assure for the scalar case that the convergence to 0 is faster with same regulation performance $p = m^{\star}$. Let us define the convergence coefficient C_R with an initialization at $\tilde{z}_0 \in \Omega_0^{\text{ext}}$

$$\begin{aligned} C_R &= \lim_{n\to\infty}\prod_{i=1}^{n}\sqrt[n]{\frac{\Delta^i}{\Delta^{i-1}}} \\ &= \lim_{n\to\infty}\sqrt[n]{\frac{\Delta^n}{\Delta^0}} \end{aligned}$$

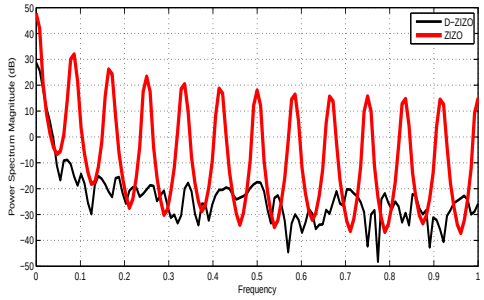


Fig. 3. Comparison realized with the power signal densities of quantization noise with $T_s = 1s$.

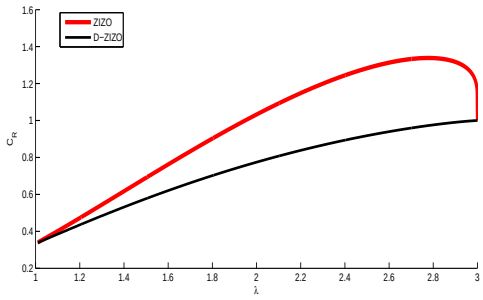


Fig. 4. Comparison of C_R between ZIZO and D-ZIZO with $p = m^\star$ (same constraints on regulation detection). We see that if $\lambda > 2$, $C_R > 1$ so the algorithm can not be used because it diverges. Moreover with $\lambda < 2$ we can use only 2 words.

For processes which are p cyclic, we have $C_R = \sqrt[p]{\frac{\Delta^p}{\Delta^0}}$. ZIZO is p cyclic so we have $C_R = \sqrt[p]{\Theta_{in,m}^{p-1}\Theta_{in,e}}$. When these coefficients are close to their minimal constraints exposed in (7), we obtain

$$C_R = \sqrt[p]{\frac{|\lambda|}{M}^{p-1}\frac{|\lambda|}{M-2}}$$

with p such that $C_R < 1$.

In D-ZIZO, when coefficients are close tho their minimal constraints exposed in (13), we have

$$C_R \leqslant (\frac{|\lambda|}{M})^{\frac{\ln(|\lambda|)}{\ln(\frac{M+2-|\lambda|}{M-|\lambda|})}}$$

In that follows, we interest in the closed loop system equations. The following analysis is realized with $a_c = a - bK$ and when $|\tilde{x}_k| \leqslant M\Delta_k$. Then we obtain

$$x_{k+1} = a_c^n x_0 + bK\sum_{i=0}^{k} a_c^{i-1}\tilde{x}_{k-i}$$

$$|x_{k+1}| \leqslant |a_c|^k|x_0| + M|bK|\sum_{i=0}^{k}|a_c|^{i-1}\Delta_{k-i}$$

$$|x_{k+1}| \leqslant |a_c|^k|x_0| + M|bK|\frac{C_R^k}{|a_c|}\frac{1-(\frac{|a_c|}{C_R})^{k+1}}{1-\frac{|a_c|}{C_R}}\Delta_0$$

In Fig 4, we present the convergence behavior represented by C_R for the regulation constraint on detection $p = m^\star$. If there is an impulse disturbance or a change of initial conditions, algorithms react at the same moment, i.e algorithm enters in a "Zoom Out" process.

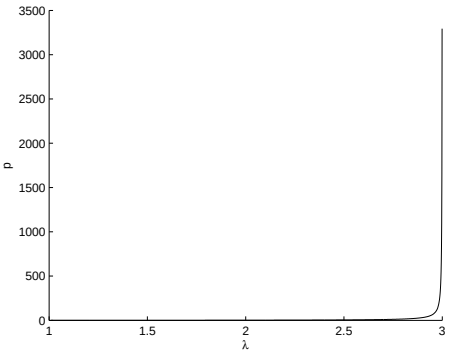
In Figure 5, we present for same mean convergence rate (same C_R), the evolution of detection rapidity. That means the necessary number of samples to decide to enter in a "Zoom Out" process while an impulsive disturbance occurs. It is related to $m^\star$ for D-ZIZO and p for ZIZO.

C. Distance to theoretical stabilization rate bound

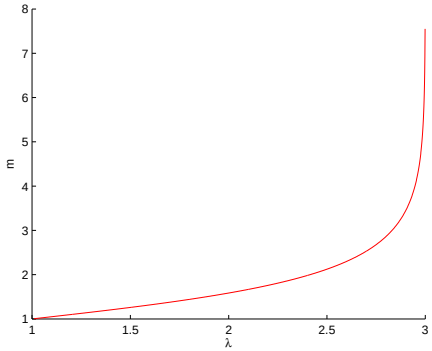
Firstly, we introduce the theorem given in [11]

Theorem 1: Suppose the system (1) with the pair $(\boldsymbol{A}, \boldsymbol{B})$ controllable. And a channel rate R bounded by

$$\prod_{l=1,|\lambda_l|>1}^{n} \lceil|\lambda_l|\rceil < 2^R$$



(a)



(b)

Fig. 5. Comparison of p between ZIZO (a) and D-ZIZO (b) with the same C_R for $\lambda \in [1, 2.999]$.

Then, there exists a coding structure ensuring that the system (1) is globally stable.

With D-ZIZO, the bound is achieved for all linear systems. ZIZO could verify this theorem for certain class of systems. If the matrix A of the system is diagonal and its eigenvalues are greater than 3, ZIZO verifies the theorem. However, since this algorithm needs at least 3 words $M = 3$. That means for $|\lambda| < 2$, there is a loss of one word by signal (though D-ZIZO needs 2 words). If the matrix A is not diagonalizable, each signal has the same quantization step Δ in ZIZO. The analysis realized in [10] is too much conservative to achieve the bound exposed in the theorem. The algorithm ZIZO needs more words by

TABLE I
COMPARISON RESULT WITH OPTIMAL CODING, D-ZIZO AND ZIZO. A*B MEANS THAT THE COMPONENT 1 NEEDS A WORDS AND THE COMPONENT 2 NEEDS B WORDS.

condition	Optimal result	ZIZO	D-ZIZO
$a < 2$	2	3	2
$3 < a < 4$	4	4	4
$\boldsymbol{A} = \begin{pmatrix} a & 10 \\ 0 & a \end{pmatrix}$ with $3 < a < 4$	4*4	$14 * 14$	$4 * 4$
$\boldsymbol{A} = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$ with $3 < a < 4$	4*4	$5 * 5$	$4 * 4$

signal than the minimum number of words exposed in the theorem. With D-ZIZO, the tuning of quantization steps permits to achieve the constraint for all linear systems.

In the table IV-C, we present the necessary number of words by signal to assure global stability for ZIZO and D-ZIZO .

V. CONCLUDING REMARKS

In this paper, we have firstly recalled two recent algorithms of adaptive quantization namely ZIZO and D-ZIZO. Those two algorithms permit to obtain global stability. When the signal is too important it needs that the quantization steps increase otherwise, quantization steps decrease. In "Zoom In Measurements" and "Zoom In Escape" (ZIZO) phases, quantization step respectively decrease and increase though, only for the "Zoom In" phase (D-ZIZO) quantization steps decrease and remain identical in Dwell Time period. Even if the essence of the algorithms are the same, it seems that the dwell time phase permits to have some improvements. This comparison could be summarized with these following items.

- This change of coordinates and the quantization steps tuning method enable D-ZIZO to obtain the minimum bandwidth on the communication channel for all linear systems whereas ZIZO *may* sometimes reach the theoretical limit for certain classes of linear systems.
- ZIZO generates high frequencies on the quantization noise though this signal is smoother for D-ZIZO.
- For same regulation detection, D-ZIZO is faster than ZIZO. Moreover, with the same convergence rate, the detection is faster for D-ZIZO.
- One drawback for D-ZIZO is the generalization to non-linear systems that seems difficult. Without the change of coordinates which is the key point of D-ZIZO, it is impossible to obtain an adaptive algorithm in D-ZIZO case. With ZIZO, the issue is solved.

In future works, we will investigate the study of uncertain models with fixed and adaptive quantization.

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