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Magnetization Identification Problem Illustration of an effective approach

Olivier Chadebec*, Jean-Louis Coulomb*, Gilles Cauffet, Jean-Paul Bongiraud**, Sébastien Guérin****

* Laboratoire d'Electrotechnique de Grenoble, LEG/ENSIEG, BP46, 38402 St Martin d'Hères, France

** Laboratoire du Magnétisme du Navire, LMN/ENSIEG, BP46, 38402 Saint Martin d'Hères, France
chadebec@leg.ensieg.inpg.fr

Abstract—This paper deals with the problem of magnetization identification. We consider a ferromagnetic body placed in an inductor field. The goal of this work is, from static magnetic field measurements taken around the device, to get a fine model of its magnetization. This inverse problem is usually ill-posed and its solution is non unique. It's then necessary to use mathematical regularization applied to such case. These methods require the choice of a priori parameters which weight the regularization. This choice is generally uneasy and is strongly geometry and measurements dependent. Instead of trying to solve this ill-posed inverse, we prefer to change it in a better posed one by injecting our physical knowledge of the problem. No choice of regularization parameter is then necessary and a simple resolution leads to a faithful solution. Our approach is tested on the magnetization's identification of a real ferromagnetic shell.

Keywords -- inverse problem, magnetization identification, moments method, regularization techniques, static magnetic field.

I. INTRODUCTION

Forward modeling is now well-known many scientific domains. The procedure consists in finding physical laws which rule the behavior of a system, in solving it and then in making predictions as to the result of measurements. In the last three decades, the theory and the practice of inverse problem have become more and more important in the area of modeling. This approach, which can be opposed to forward modeling, consists in finding the causes by knowing the effect. The most widely used inverse application is called identification problem. This approach uses the result of some measurements to infer the actual behavior of a physical system. It can be summed up by the determination of the sources from measurements.

In this paper, we are particularly interested in the identification problem of the magnetization (or magnetostatic inverse problem). Indeed, a ferromagnetic body has a magnetization. The goal of this work is to determine it. In fact, the problem is not so simple as it seems. The magnetization of such ferromagnetic body can be divided in two parts, with two different causes. The first one, called reversible induced magnetization, is due to the reaction of the material to an inductor field. This field can be created by coils or magnets placed in (or around) the system or can simply be the earth's magnetic field. The calculus of this kind of magnetization is now well known and has led to numerous works. Numerical methods like finite element method [1], boundary element method [2] or moment method [3] gives very satisfying results. The second magnetization that produces a ferromagnetic body is called remanent magnetization and is due to the magnetic history of the material. It depends on past inductor field, on mechanical and thermal constraint and on the complex magnetic material behavior which is the hysteresis (magnetostriction, villari effect,...). In most cases, the users have absolutely no idea of this magnetic past and even if they had such knowledge, existing models would be too complex to apply to real devices. So, if we want to obtain an exact model of the magnetization of a ferromagnetic body, it is then necessary to use measurement

of the static magnetic field on sensors placed around the device. We propose to solve an inverse problem (determination of the magnetization from discrete measurements of radiating field in air region).

Three fundamental steps are necessary to solve an inverse problem. The first one is the parametrization of the system which consists in determining a minimal set of parameters whose value completely characterizes the system. The second step is to solve the forward problem, by finding a forward relation which links the sources to the measurements. The first part of this paper deals with the resolution of these two first points. We will see that the forward identification problem can be summed up by the writing of a linear system of equations. The last step is then to inverse the forward relation obtained. This resolution is not trivial and the main difficulty comes from the lack of uniqueness of the solution. The problem is said to be ill-posed. Usually, to overcome this difficulty, regularization or probabilistic technics are used. However, we will see that these approaches are very difficult to apply to complex 3D geometries and the choice of regularization parameters is not trivial. A quick overview of these technics will be made in the second section. Then, in the third section, we propose a new method which avoids using classical regularization or probabilistic technics. By injecting physical knowledge about the magnetic behavior of the sources, we then obtain a valid a linear system inside the device. This system is then associated with the linear system given by measurements and the global resolution gives a unique solution. This enhanced knowledge of the behavior of the material allows fewer sensors to be used. The problem can then be said to be well-posed. Our approach has been recently published in [4], for the magnetization identification of a complex ship's hull. In this paper, in order to be more explicit, we proposed to illustrate it with a simple rectangular steel where an artificial local magnetization is created. Classical inversion and our method can then be compared and its effectiveness is demonstrated.

II. FORWARD FORMULATION

Let us consider a ferromagnetic device defined by a volume V placed in an inductor field \mathbf{H}^0 . This inductor field can be uniform or created by coils. The device has an inhomogeneous magnetization \mathbf{M} which can be divided in two parts, the induced \mathbf{M}^{ind} one and the remanent \mathbf{M}^{rem} one such that :

$$\mathbf{M} = \mathbf{M}^{\text{ind}} + \mathbf{M}^{\text{rem}} \quad (1)$$

The field in the whole space is the sum of the inductor field \mathbf{H}^0 and the reduced field \mathbf{H}^{red} which is created by the magnetization of the volume V .

$$\mathbf{H} = \mathbf{H}^0 + \mathbf{H}^{\text{red}} \quad (2)$$

The relation between the magnetization in V and the induction in the air region is then [5]:

$$\mathbf{B}(\mathbf{P}) = \mu_0(\mathbf{H}^0(\mathbf{P}) + \frac{1}{4\pi} \iiint_V \left(\frac{3(\mathbf{M}(\mathbf{Q}) \cdot \mathbf{r})}{r^5} \mathbf{r} - \frac{1}{r^3} \mathbf{M}(\mathbf{Q}) \right) dV) \quad (3)$$

where μ_0 is the permeability of air region, \mathbf{P} is a point in the air region, \mathbf{Q} the integration point of V and \mathbf{r} is the \mathbf{PQ} vector. Let us mesh the volume V in n bulk element. For simplicity, let us assume that magnetization is constant on each of them. If we place m tri-axis magnetic sensors in the surrounding air region, we obtain a vector linear equation which links the measured induction on sensor number j and the unknown components \mathbf{M}_i of the magnetization on each element i (Fig. 1.).

$$\mathbf{B}_j^{\text{mes}} = \mu_0 \left(\mathbf{H}_j^0 + \frac{1}{4\pi} \sum_i^n \iiint_{V_i} \left(\frac{3(\mathbf{M}_i \cdot \mathbf{r})}{r^5} \mathbf{r} - \frac{1}{r^3} \mathbf{M}_i \right) dV_i \right) \quad (4)$$

take in Figure 1

By taking into account all sensors, we can write a linear system of equations.

$$[\mathbf{A}] \{\mathbf{M}\} = \{\mathbf{b}\} \quad (5)$$

where $[\mathbf{A}]$ is the linear relation which links sources to measurements, $\{\mathbf{M}\}$ is the vector of unknown magnetization and $\{\mathbf{b}\}$ is the vector of reduced measured field (inductor field

subtracted from the measured field). $[\mathbf{A}]$ is a $(3m \times 3n)$ matrix, $\{\mathbf{M}\}$ is a $(3n \times 1)$ vector (3 unknowns per element) and $\{\mathbf{b}\}$ is a $(3m \times 1)$ vector (3 measurement per sensor).

We have explained the first fundamental part of the identification problem of the magnetization which consists in writing the forward problem. Now, the inverse problem must be solved, i.e. to find the best distribution of \mathbf{M} which fits magnetic measurements.

III. ILL-POSED PROBLEMS

A. Least-square resolution and SVD

In the general case, the number of sensors is different from the number of bulk element of the mesh then $[\mathbf{A}]$ is not a square matrix. It is then impossible to solve (5) by a simple inversion. The usual solution consists in solving the system with a least square method and then to:

$$\text{Find } \mathbf{M} \text{ such } \|\mathbf{A}\mathbf{M} - \mathbf{b}\| \text{ is minimal}$$

By writing the previous norm and by deriving it, we obtain a new matrix system:

$$[\mathbf{A}]^T[\mathbf{A}] \{\mathbf{M}\} = [\mathbf{A}]^T \{\mathbf{b}\} \quad (6)$$

where $[\mathbf{A}]^T$ is the transposed matrix of $[\mathbf{A}]$. Let us note that $[\mathbf{A}]^T[\mathbf{A}]$ is now a square matrix and can be simply inverted. The solution of the inverse problem is then:

$$\{\mathbf{M}\} = ([\mathbf{A}]^T[\mathbf{A}])^{-1} [\mathbf{A}]^T \{\mathbf{b}\} \quad (7)$$

To solve (5), another approach can be applied, by directly using a single value decomposition (SVD). \mathbf{A} is then decomposed in the product of two orthogonal matrix and a diagonal one. These three matrixes can then be easily inverted [6]. This method is now privileged in most work on inverse problems. It possesses the advantage of avoiding the inversion of a matrix (saving of calculation time). Furthermore, the matrix $[\mathbf{A}]$ is often better conditioned than $[\mathbf{A}]^T[\mathbf{A}]$ (see following section).

However, most inverse problems are said ill-posed and their resolution is not obvious.

B. Ill-posed problem

A well-posed problem verifies the three following points [7]:

- The solution exists.
- The solution is unique.
- The solution depends continuously on the initial data (stability problem).

On the contrary, a problem which does not verify one of the previous points is said to be ill-posed. We can then distinguish two principal facts that make our magnetization identification problem ill-posed [8].

(i) Rank-deficient problem

If we have less information than unknowns ($m < n$), (5) is purely underdetermined. Indeed, every linear combination of vectors from the kernel of \mathbf{A} is not observable and an infinite number of solutions \mathbf{M} exactly fit the data. The problem is then said to be rank-deficient. The solution returned by the SVD is the one with the minimum norm [6]. This solution, called pseudo-solution, can be not physically acceptable. Let us note that most identification problems of the magnetization are rank-deficient, the number of sensors used being limited and the number of elements being high to obtain a fine model of the magnetization (high mesh density).

(ii) Problem with poor condition number

Generally, several different distributions of sources can create a field very close to the measured one. It is then very difficult to favor a solution, the measurements being noisy. In that sense the solution is not unique either. This fact is due to the presence of quasi-contradictory equations in the system. The resolution of (5) leads then to an unacceptable divergent solution. The parallelism between equations can be evaluated by the condition number of matrix \mathbf{A} (ratio between the highest and smallest single values of matrix \mathbf{A}). If the condition number is close to 1, equations are strongly independent, on the contrary, a high

condition number leads to amplification of measurement inaccuracies and to an unstable solution. Let us note that a high number of measurements increase the phenomena, by multiplying equations and increasing the condition number too, especially for the case $m > n$.

Inverse problems are generally ill-posed. Some of them can even verify (i) and (ii). Solving identification problems consists in selecting a good solution from the diversity of acceptable solutions.

IV. REGULARIZATION THEORY

For more than two decades, a lot of scientific work has dealt with the resolution of inverse problems. In most, this work favors applications with lots of measurements to improve the knowledge of the problem. Resolutions of rank-deficient matrix writing with few measurements are often avoided, because they do not give a full description of the problem. These methods lead then to systems with poor condition number and associated unstable solutions. Then, it is necessary to regularize to ensure the stability. This part of the paper gives a quick overview of these regularization technics.

A. *Truncated SVD* [6]

This method is the easiest approach to use. It consists in truncating one part of the spectrum of the single value decomposition of \mathbf{A} . Some of smallest single values are set to zero. The condition number of the system is then improved, by suppressing contradictory equations. This method can lead to quite good solutions for very simple cases but the choice of the truncation level remains a problem.

B. *Tikhonov's regularization*

The most widely used regularization method was developed by Tikhonov [8]. To avoid divergences, [8] proposes to replace the problem by:

$$\text{Find } x \text{ such } \|\mathbf{Ax} - \mathbf{b}\| + \alpha \|\mathbf{Lx}\| \text{ is minimal}$$

where \mathbf{L} is called regularization matrix and α defines level of the regularization. The expression of the solution is then:

$$\{\mathbf{M}\} = ([\mathbf{A}]^T [\mathbf{A}] + \alpha [\mathbf{L}]^T [\mathbf{L}])^{-1} [\mathbf{A}]^T \{\mathbf{b}\} \quad (8)$$

If \mathbf{L} equal \mathbf{I}_d (i.e. the identity matrix), the method tries to fit the distribution to the measurements by keeping a minimal norm for \mathbf{M} (0-order regularization). \mathbf{L} can also be chosen as the derivative matrix (1st-order regularization) or as the laplacian one (2nd-order regularization). Generally, higher the order is, better the regularization is. For the identification problem of the magnetization 1-order regularization has already been tested on a 2D problem [5]. However, these approaches are difficult to apply to 3D complex geometry. The geometrical complexity of such devices and vector basis makes matrix \mathbf{L} very difficult to compute. Moreover, the choice of α parameter, even if some very efficient methods like L-curve approach have been developed [10], remains a major problem.

C. Iterative regularization

Iterative regularization is a method to compute a stable solution to an ill-posed problem using a classic iterative method complemented by a stopping rule for the iteration number. For a large iteration number, the solution is equivalent to that obtained by (7), but by stopping the minimization process as soon as the residual becomes lower than the level of measurement errors, we generally obtain a stable solution. Particularly well adapted for non-linear inverse problem, the method can be successfully adapted to linear ones [11], but the choice of the iteration stopping level still remains a difficulty.

D. Bayesian approach [12]

This kind of approach is certainly the most highly evolved and it is based on the evaluation of all uncertainties and inconsistencies of the model and of the measurement. It needs a very good knowledge of the probabilistic behavior of the measurement system (sensors) and an a-

priori knowledge of the behavior of the sources. We have not tested it on our problem but it still requires the determination of parameters.

E. Conclusions on regularization theory

As we said, the classical approach for solving inverse problems and, in particular, magnetization identification problems is to make a lot of measurements around the device. This leads to contradictory equations in the resulting system and to a poor condition number. It is then necessary to regularize. Even if the existing methods can lead to efficient results, they all need the choice of a regularization parameter or an a-priori coefficient. Most of the time, the choice of this coefficient is not easy, measurement dependent and must change for each application. On the contrary, if few measurements are taken, the condition numbers are generally good (if all the sensors are far enough away from each other) but the lack of information leads to severely underdetermined systems. Their solutions are not unique and then the sources close to the sensors are favored to the detriment of others. We then propose a different way to perceive the identification problem of the magnetization, by reducing the number of acceptable solutions (i.e. the dimension of the kernel of the system). By adding an equation based on our knowledge of sources behavior, we improve the rank of the system to inverse.

V. ADDITION OF AN INTERNAL EQUATION SYSTEM

As we said, classical approaches favor applications with lots of sensors. This is generally due to the lack of information about the behavior of sources. Thus, classical approaches prefer to obtain very rich measurement information, even it is then necessary to apply to it particular mathematical processing. In fact, the magnetization identification problem is very specific in comparison to other applications. One fundamental physical characteristic of a ferromagnetic body is to channel the induction with respect to Maxwell's laws. The behavior of

magnetization in such a body is closely linked to the physics and this link can lead to a rich knowledge of the problem.

A. An internal physical system

Let us consider a very general equation which is representative of the behavior of the ferromagnetic material. In volume V , we have:

$$\mathbf{B} = \mu\mathbf{H} + \mathbf{B}^{\text{rem}} \quad (9)$$

where μ is reversible linear permeability. We have also:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}^{\text{ind}} + \mathbf{M}^{\text{rem}}) \quad (10)$$

$$\mathbf{B}^{\text{rem}} = \mu_0\mathbf{M}^{\text{rem}} \quad (11)$$

By combining (9), (10) and (11), we obtain:

$$\mathbf{M}^{\text{ind}} = (\mu_r - 1)\mathbf{H} \quad (12)$$

where μ_r is the reduced reversible permeability of the material. Let us note that \mathbf{H} is created by the total magnetization \mathbf{M} of the volume V and the inductor field \mathbf{H}^0 . By writing an equation similar to (3), we obtain:

$$\mathbf{M}^{\text{ind}} = (\mu_r - 1)(\mathbf{H}^0(\mathbf{P}) + \frac{1}{4\pi} \iiint_V \left(\frac{3(\mathbf{M}(\mathbf{Q}) \cdot \mathbf{r})}{r^5} \mathbf{r} - \frac{1}{r^3} \mathbf{M}(\mathbf{Q}) \right) dV) \quad (13)$$

Let us now consider that the device is meshed into n bulk elements like in the previous section and that magnetization is assumed to be constant on each of them. We can write (13), at the barycenter of each element (collocation method), the following equation:

$$\begin{aligned} \mathbf{M}_{\mathbf{k}}^{\text{ind}} = & (\mu_r - 1) \left(\mathbf{H}_{\mathbf{k}}^0 + \frac{1}{4\pi} \sum_i^n \iiint_{V_i} \left(\frac{3(\mathbf{M}_i^{\text{ind}} \cdot \mathbf{r})}{r^5} \mathbf{r} - \frac{1}{r^3} \mathbf{M}_i^{\text{ind}} \right) dV_i + \right. \\ & \left. \sum_i^n \iiint_{V_i} \left(\frac{3(\mathbf{M}_i^{\text{rem}} \cdot \mathbf{r})}{r^5} \mathbf{r} - \frac{1}{r^3} \mathbf{M}_i^{\text{rem}} \right) dV_i \right) \end{aligned} \quad (14)$$

We obtain then a system of n vector equations which links the unknown magnetization.

$$[\mathbf{C} + \mathbf{I}_d \quad \mathbf{C}] \begin{bmatrix} \mathbf{M}^{\text{ind}} \\ \mathbf{M}^{\text{rem}} \end{bmatrix} = [\mathbf{d}] \quad (15)$$

where \mathbf{I}_d is the identity matrix and \mathbf{d} depends essentially on the reduced permeability of the material and on the inductor field \mathbf{H}^0 .

System (15) is composed of $6 \times n$ unknowns for $3 \times n$ equations. It is then rank-deficient and its solution is not unique. This situation could be predicted because we did not add any knowledge about the magnetic past of the material, so lots of combinations of magnetization are possible. In fact, (15) minimizes the global energy of the system by ensuring coherent behavior of sources between themselves. It has the outstanding property of owning a good condition number due to the collocation method employed to obtain it. Moreover, its solution obtained by a single value decomposition leads to the pseudo-solution (see section II.B.), i.e. the solution with the minimal norm. This pseudo solution leads to the calculus of the induced magnetization (only created by \mathbf{H}^0) and forces remanent magnetization to zero. This solution is the most credible solution in absence of measurements.

B. Towards a better posed problem

We now consider an application with few measurements taken around the device. The sensor location is chosen to ensure good condition number. In particular, they are far away from each others to obtain strong decoupled equations. On the contrary, (5) is severely rank-deficient. The kernel of the system has then a high dimension, and a high number of distributions fit measurements. To obtain a pertinent solution, it is then necessary to reduce its dimension. It can be done by adding some other equations, in particular, the system (15). We are then going to solve the following system:

$$\begin{bmatrix} \mathbf{C} + \mathbf{I}_d & \mathbf{C} \\ \mathbf{A} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{M}^{\text{ind}} \\ \mathbf{M}^{\text{rem}} \end{bmatrix} = \begin{bmatrix} \mathbf{d} \\ \mathbf{b} \end{bmatrix} \quad (16)$$

The solution returned by the resolution of (16) is now the distribution that fits the

measurements and moreover that takes into account the behavior of the material (according to Maxwell's laws). Our knowledge of the problem is now much better than before. Moreover, as we said (5) and (15) have both good condition numbers, therefore it will also be the case for (16), so it is then useless to regularize it.

VI. EXAMPLE OF APPLICATION: MAGNETIZATION'S IDENTIFICATION OF A FERROMAGNETIC SHELL

In this section, we propose to apply this approach to the magnetization's identification of a ferromagnetic shell.

A. Preliminary remarks

Ferromagnetic shells have particular mechanical properties and mechanical constraints which lead to high magnetostriction effects. It is then can be very interesting to obtain their magnetization for applications like shielding, for example.

The thickness of the shell is obviously small is comparison to other dimensions of the device. It is then usual to assume that the magnetization of the shell is tangential to it [13]. The number of unknowns to identify is then reduced and becomes two per element.

B. Description of measurement process

We consider a rectangular steel shell of 500mm×900mm with a thickness of 1.4mm. Its reversible relative permeability is $\mu_r=96$. The shell is placed on a railway above 3 tri-axis fluxgate magnetic sensors. We can then obtain measurements of the field created by the plate on several points in the surrounding space. In this process 27 points are selected at 74 mm above the plate. The shell is placed in the homogenous earth's magnetic field (see Fig.2.).

take in Figure 2

In the first step, the shell is degaussed to reduce its remanent magnetization. Then, we create a strong magnetic anomaly in the shell with a SmCo permanent magnet. We try to

localize this magnetization in a square (see Fig.3).

take in Figure 3

C. Magnetization's identification of the shell with simple single value decomposition.

The shell is meshed in 480 square surface elements. The number of unknowns to determine is then 1920 (2×480 for \mathbf{M}^{ind} and 2×480 for \mathbf{M}^{rem}). The magnetization being tangential to the shell, we only consider tangential components of the measured magnetic field (x and y components). We then obtain two equations per sensor. By writing (5), we obtain a 54×1920 system with a condition number equal to:

$$\xi(5) = 7.27$$

The condition number is then good. In particular, there is no contradiction between equations and the solution will to be stable (a small measurement error will not lead to a divergent solution). However, the system is severely rank-deficient and lots of distributions will fit the measurements. Let us solve this system with a simple single value decomposition, the solution obtained is then the pseudo-solution i.e. with the minimal norm (see Fig.4.).

take in Figure 4

The distribution obtained is obviously not satisfying. In particular, the magnetization is not continuous on the shell. On one hand, elements directly above sensors are favored and get a strong magnetization, on the other hand, elements away from sensors get a small one. Some neighboring elements have even opposite magnetization direction.

D. Magnetization's identification of the shell with internal physical system.

Now, let us write (15), the size of the system obtained is 960×1920 and its condition number is:

$$\xi(15) = 6.53$$

As we said, the resolution by a single value decomposition of (15) leads to the calculus of induced magnetization (created by the earth's magnetic field \mathbf{H}_0) and then to the following

magnetization's distribution.

take in Figure 5

Let us now add the measurements system and solve (16), we then obtain a solution which takes into account physical property of the material, inductor field and which fits measurements. System (16) is still rank-deficient (1014×1920) but the dimension of the kernel is highly reduced in comparison with (5). Its condition number is good too:

$$\xi(16) = 5.45$$

The pseudo-solution returned by the single value decomposition is then:

take in Figure 6

The solution obtained is then more continuous than the previous one (Fig.4.). Moreover, the anomaly is localized with quite good accuracy.

VII. CONCLUSIONS

We have then explained a new approach to pass from an ill-posed magnetization's identification problem to a much better one. Instead of following the classical approach which consists in taking lots of measurements and in regularizing the solution, we prefer to favor our knowledge of the behavior of sources and in the panel of solutions which minimize energy of the system, to choose the one which fits the measurements. Systems which can be called well (or better) posed are then solved by simple single value decomposition. There are two fundamental advantages of our approach. Firstly, there is no choice of an a-priori parameter in this method and this choice is still a problem in inverse problems. Secondly, the number of sensors can be considerably reduced, the measurements process complexity and its cost are then reduced too. Moreover, we think that our method by injecting physical knowledge of the behavior of sources is very general, and could be applied to other kinds of inverse problems like thermal applications, for example.

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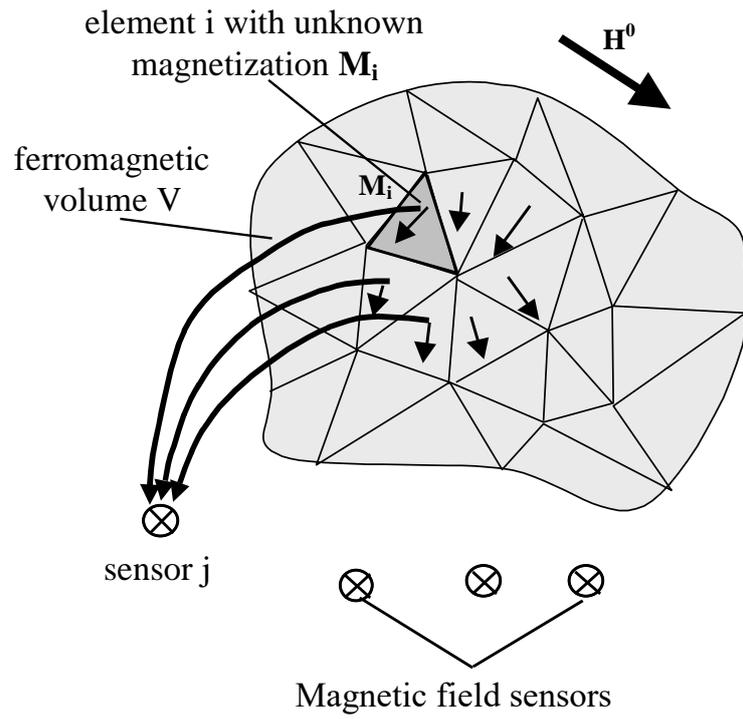


Fig.1. Ferromagnetic device with mesh, sensors and inductor field

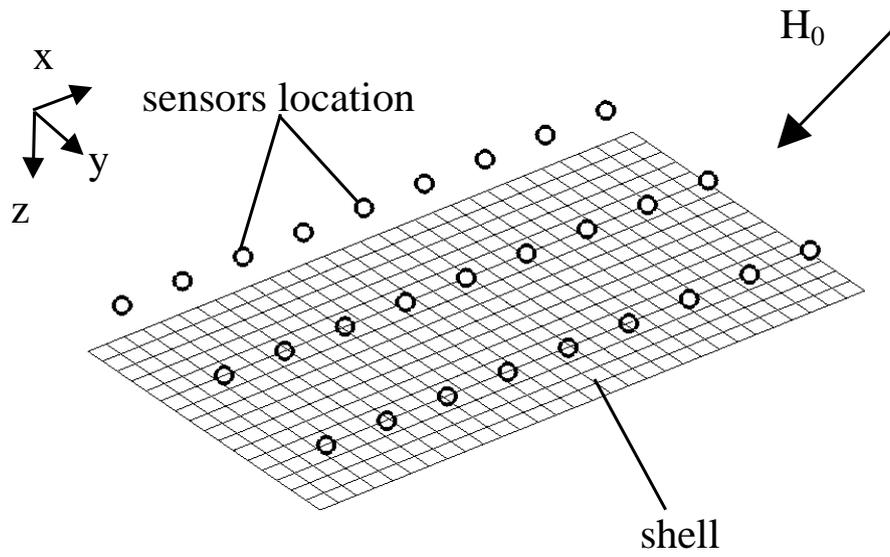
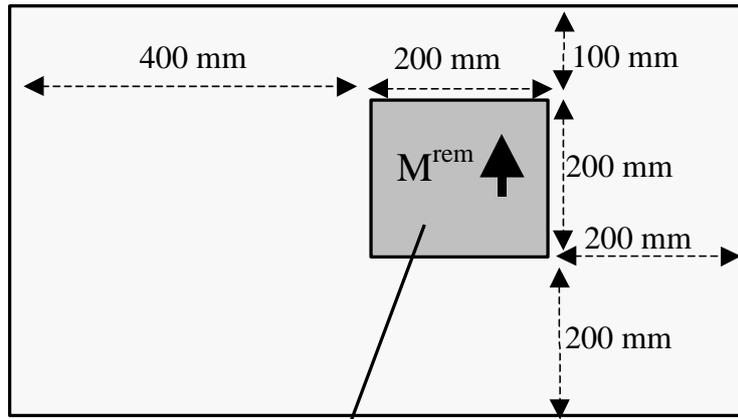


Fig.2. Ferromagnetic shell $900 \times 500 \times 1.4$ mm, $\mu_r = 96$ with mesh, sensors location and inductor field ($H_{0x} = -22649$ nT, $H_{0y} = -421$ nT, $H_{0z} = 40671$ nT)



Area where the magnet
has been moved at the
proximity of the shell

Fig.3. Localization and direction of the magnetization anomaly created by the SmCo magnet.

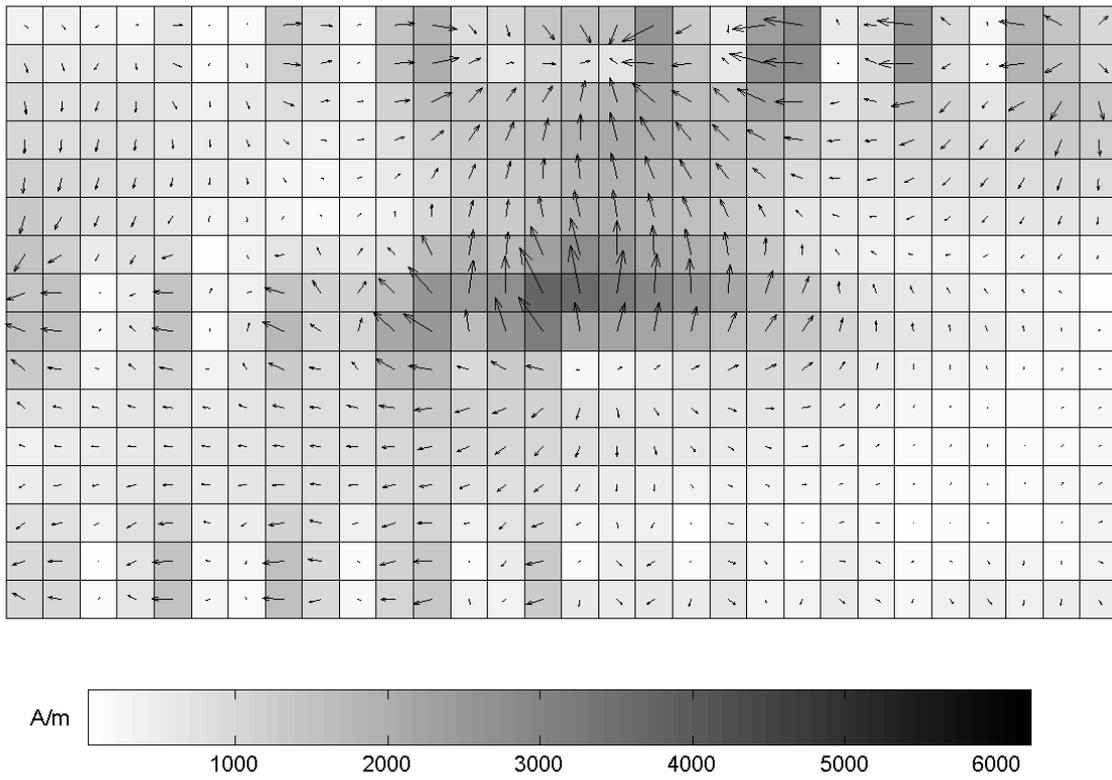


Fig.4. Identification of the total magnetization of the shell obtained by resolution of (5) with a single value decomposition.

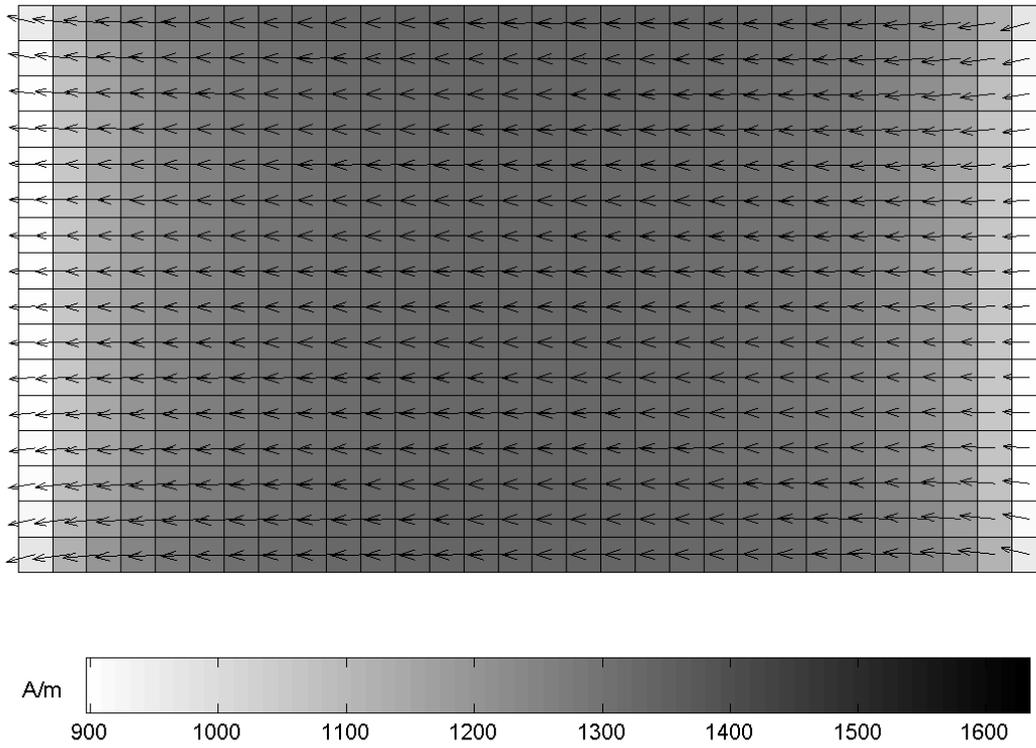


Fig.5. Induced magnetization obtained by resolution of system (15).

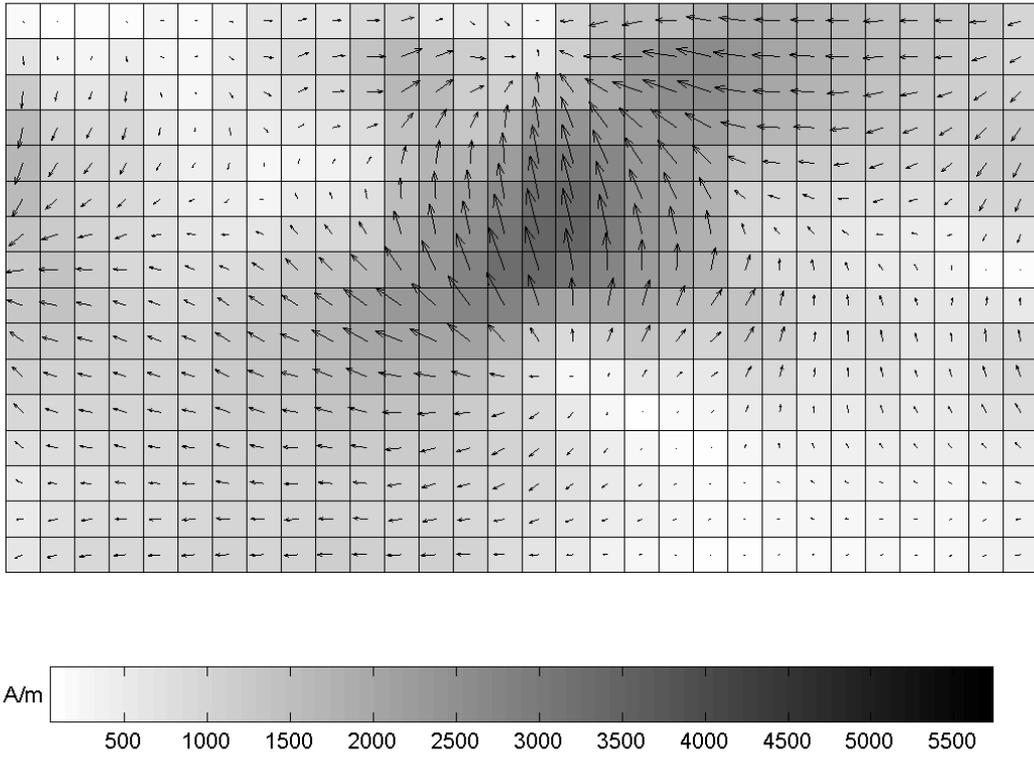


Fig.6. Identification of the total magnetization of the shell obtained by resolution of (16) with a single value decomposition.