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Torque in PM Couplings: Comparison of Uniform and Radial Magnetization

R. Ravaud, G. Lemarquand, V. Lemarquand and C. Depollier

Abstract

We present a three-dimensional study of the torque transmitted between tile permanent magnets uniformly magnetized. All this study is based on the Coulombian model. The torque is calculated semi-analytically by considering all the surface densities that appear on the tiles. In addition, no simplifying assumptions are done in the expressions given in this paper. Consequently, the evaluation of the torque is very accurate and allows us to show the drawbacks of using tile permanent magnets uniformly magnetized instead of using tile permanent magnets radially magnetized. Such an approach is useful because it allows us to realize easily parametric studies.

Index Terms

Magnetic coupling, Tile permanent magnet, Torque, Uniform magnetization, Radial magnetization, Three-dimensional calculation.

I. INTRODUCTION

MAGNETIC couplings are often realized with tile permanent magnets radially or uniformly magnetized. Tile permanent magnets radially magnetized allow us to obtain great couplings but they are difficult to fabricate. Consequently, an alternative experimental method consists in using tiles uniformly magnetized. Indeed, they are simpler to fabricate than tiles radially magnetized and thus cheaper. Unfortunately, they are also less efficient and can lower the quality of transmission between tiles located on the stator and tiles located on the rotor. Therefore, it is interesting to predict theoretically the

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19 way the torque is transmitted between tile permanent magnets uniformly magnetized. Such a model allows
20 us to realize easily parametric studies and thus to know when the use of tile permanent magnets radially
21 magnetized is necessary or not. Indeed, the angular width has a great influence on the radial field created by
22 a tile permanent magnet uniformly magnetized. As this radial field is not perfectly symmetrical, the torque
23 transmitted is modified. Therefore, the aim of this paper is to determine precisely with a three-dimensional
24 model how the torque transmitted between tiles changes according to the way the tiles are magnetized.
25 Many authors have studied magnetic couplings [1]-[4] and structures using permanent magnets [5]-[10].

26 Historically, we can say that the first models used to study magnetic couplings were the two-dimensional
27 models [11]-[19]. The main reason lies in the fact that a two-dimensional approach is fully analytical and
28 allows us to make an easy parametric optimization of the permanent magnet dimensions. However, this
29 2D-approach is not valid when we determine the magnetic field far from one magnet [20].

30 Three-dimensional approaches seem to be more difficult to realize parametric studies because they are
31 not fully analytical. In fact, the calculation of the radial and axial magnetic field components created by
32 tile permanent magnets is not strictly analytical but is necessary based on special functions [21]- [40].

33 More generally, all the semi-analytical or analytical approaches used by several authors allow manu-
34 facturers to optimize devices using permanent magnets [41]-[48].

35 This paper presents a semi-analytical approach based on the Coulombian model for calculating pre-
36 cisely the torque transmitted between tile permanent magnets uniformly magnetized. This semi-analytical
37 approach is three-dimensional. We explain in the second section how this problem can be solved. Then,
38 we present a semi-analytical expression of the torque transmitted between two tile permanent magnets
39 uniformly magnetized. Eventually, we present the main drawbacks of tile permanent magnets uniformly
40 magnetized.

41 II. MODELING TILE PERMANENT MAGNETS WITH THE COULOMBIAN MODEL

42 This section presents the geometry studied and the model used for the modeling of the torque transmitted
43 between two tile permanent magnets uniformly magnetized.

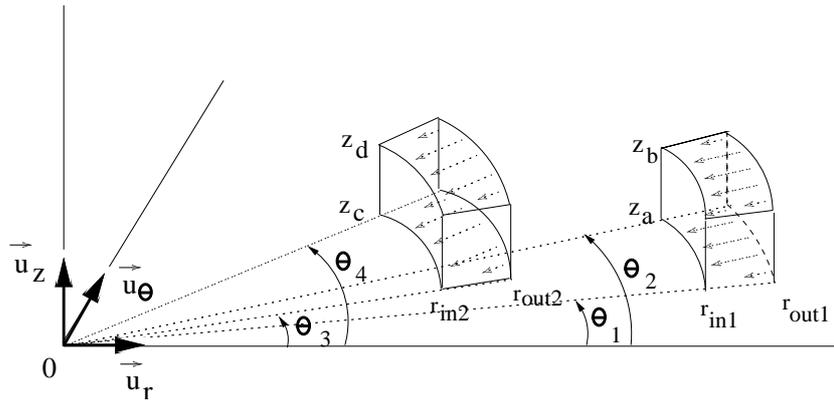


Fig. 1. Representation of the geometry considered in three-dimensions : two tile permanent magnets uniformly magnetized

44 A. Notation and geometry

45 The geometry considered and the related parameters are shown in Fig 1. A two-dimensional repre-
 46 sentation of the geometry is shown in Fig 2. We consider here two tile permanent magnets uniformly
 47 magnetized. The inner radius of the outer tile is r_{in1} and its outer one is r_{out1} . Its height is $z_b - z_a$ and
 48 its angular width is $\theta_2 - \theta_1$. The inner radius of the inner tile is r_{in2} and its outer one is r_{out2} . Its height
 49 is $z_d - z_c$ and its angular width is $\theta_4 - \theta_3$. The two magnetic polarization vectors \vec{J}_1 and \vec{J}_2 are expressed
 50 as follows:

$$\vec{J}_1 = -\cos\left(\frac{\theta_2 + \theta_1}{2}\right)\vec{u}_r - \sin\left(\frac{\theta_2 + \theta_1}{2}\right)\vec{u}_\theta \quad (1)$$

51

$$\vec{J}_2 = -\cos\left(\frac{\theta_4 + \theta_3}{2}\right)\vec{u}_r - \sin\left(\frac{\theta_4 + \theta_3}{2}\right)\vec{u}_\theta \quad (2)$$

52 The magnetic field created by tile permanent magnets can be obtained by using the Coulombian Model.
 53 Indeed, a permanent magnet can be represented by a magnetic pole surface density that is located on
 54 the faces of the magnet and a magnetic pole volume density that is located inside the magnet. In the
 55 configuration presented in Fig 1, the magnetic pole volume density is 0 because the magnetic polarizations
 56 of the magnets are uniform. Consequently, each tile permanent magnet is modelled by its magnetic pole
 57 surface density, which is determined as follows: by denoting \vec{n}_{ij} , the unit normal vector of the face i of

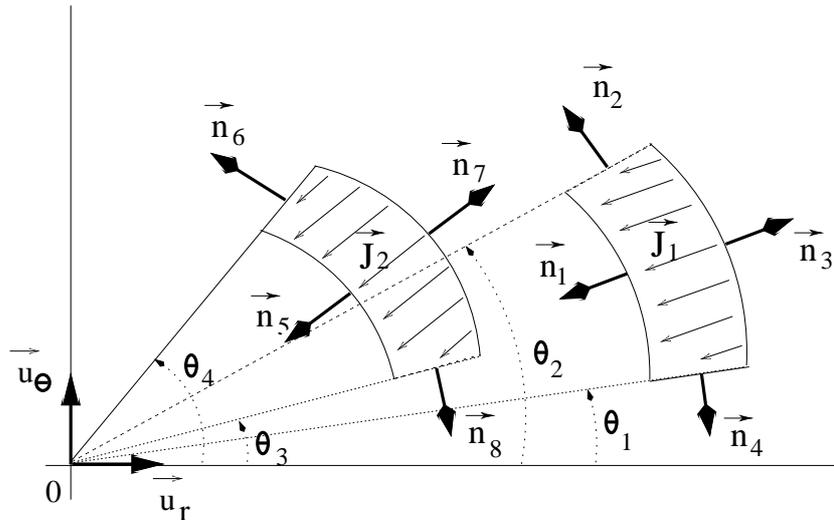


Fig. 2. Representation of the geometry considered in two dimensions : two tile permanent magnets uniformly magnetized

Surface densities	Scalar Product	Expression
σ_{11}^*	$\vec{J}_1 \cdot \vec{n}_1$	$J_1 \cos(\theta - \frac{\theta_1 + \theta_2}{2})$
σ_{12}^*	$\vec{J}_1 \cdot \vec{n}_2$	$J_1 \sin(\frac{\theta_2 - \theta_1}{2})$
σ_{13}^*	$\vec{J}_1 \cdot \vec{n}_3$	$-J_1 \cos(\theta - \frac{\theta_1 + \theta_2}{2})$
σ_{14}^*	$\vec{J}_1 \cdot \vec{n}_4$	$J_1 \sin(\frac{\theta_2 - \theta_1}{2})$
σ_{25}^*	$\vec{J}_2 \cdot \vec{n}_5$	$J_2 \cos(\theta - \frac{\theta_3 + \theta_4}{2})$
σ_{26}^*	$\vec{J}_2 \cdot \vec{n}_6$	$J_2 \sin(\frac{\theta_4 - \theta_3}{2})$
σ_{27}^*	$\vec{J}_2 \cdot \vec{n}_7$	$-J_2 \cos(\theta - \frac{\theta_3 + \theta_4}{2})$
σ_{28}^*	$\vec{J}_2 \cdot \vec{n}_8$	$J_2 \sin(\frac{\theta_4 - \theta_3}{2})$

TABLE I

DEFINITION OF THE MAGNETIC POLE SURFACE DENSITIES LOCATED ON THE MAGNETS

58 the magnet j , the corresponding surface density σ_{ij}^* is determined with (3).

$$\sigma_{ij}^* = \vec{J}_j \cdot \vec{n}_{ij} \quad (3)$$

59 Thus, four faces of the two magnets are charged with a magnetic pole surface density. All the surface
60 density calculations are represented in Table I. The torque transmitted between two tile permanent magnets
61 can be determined in two steps. The first step consists in calculating the magnetic field created by one tile
62 permanent magnet. The second step consists in integrating the magnetic field created by the first tile on

63 the second one. Let us first consider the tile permanent magnet located on the right of Fig 1. By using the
64 Coulombian approach, we can write that the azimuthal field created by this tile is expressed as follows:

$$\begin{aligned}
H_\theta(r, \theta, z) = & \frac{J_1}{4\pi\mu_0} \int_{\theta_1}^{\theta_2} \int_{z_a}^{z_b} \cos\left(\theta - \frac{\theta_1 + \theta_2}{2}\right) \frac{\vec{u}(r_{in1})}{|\vec{u}(r_{in1})|^3} r_{in1} d\tilde{\theta} d\tilde{z} \\
& - \frac{J_1}{4\pi\mu_0} \int_{\theta_1}^{\theta_2} \int_{z_a}^{z_b} \cos\left(\theta - \frac{\theta_1 + \theta_2}{2}\right) \frac{\vec{u}(r_{out1})}{|\vec{u}(r_{out1})|^3} r_{out1} d\tilde{\theta} d\tilde{z} \\
& + \frac{J_1}{4\pi\mu_0} \int_{r_{in1}}^{r_{out1}} \int_{z_a}^{z_b} \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \frac{\vec{v}(\theta_2)}{|\vec{v}(\theta_2)|^3} d\tilde{r} d\tilde{z} \\
& + \frac{J_1}{4\pi\mu_0} \int_{r_{in1}}^{r_{out1}} \int_{z_a}^{z_b} \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \frac{\vec{v}(\theta_1)}{|\vec{v}(\theta_1)|^3} d\tilde{r} d\tilde{z}
\end{aligned} \tag{4}$$

65 where

$$\vec{u}(x) = (r - x \cos(\theta - \tilde{\theta}))\vec{u}_r - x \sin(\theta - \tilde{\theta})\vec{u}_\theta + (z - \tilde{z})\vec{u}_z \tag{5}$$

66 and

$$\vec{v}(y) = (r - \tilde{r} \cos(\theta - y))\vec{u}_r - \tilde{r} \sin(\theta - y)\vec{u}_\theta + (z - \tilde{z})\vec{u}_z \tag{6}$$

67 The next step is thus to express the torque transmitted to the second tile permanent magnet uniformly
68 magnetized (as shown in Fig 1). By using (4), the torque T_θ can be determined as follows:

$$\begin{aligned}
T_\theta = & \frac{J_1 J_2}{4\pi\mu_0} \int_{\theta_3}^{\theta_4} \int_{z_c}^{z_d} r_{in2} H(r_{in2}, \tilde{\theta}, \tilde{z}) r_{in2} \cos\left(\tilde{\theta} - \frac{\theta_3 + \theta_4}{2}\right) d\tilde{\theta} d\tilde{z} \\
& - \frac{J_1 J_2}{4\pi\mu_0} \int_{\theta_3}^{\theta_4} \int_{z_c}^{z_d} r_{out2} H(r_{out2}, \tilde{\theta}, \tilde{z}) r_{out2} \cos\left(\tilde{\theta} - \frac{\theta_3 + \theta_4}{2}\right) d\tilde{\theta} d\tilde{z} \\
& + \frac{J_1 J_2}{4\pi\mu_0} \int_{r_{in2}}^{r_{out2}} \int_{z_c}^{z_d} \tilde{r} H(\tilde{r}, \theta_4, \tilde{z}) \sin\left(\frac{\theta_4 - \theta_3}{2}\right) d\tilde{r} d\tilde{z} \\
& + \frac{J_1 J_2}{4\pi\mu_0} \int_{r_{in2}}^{r_{out2}} \int_{z_c}^{z_d} \tilde{r} H(\tilde{r}, \theta_3, \tilde{z}) \sin\left(\frac{\theta_4 - \theta_3}{2}\right) d\tilde{r} d\tilde{z}
\end{aligned} \tag{7}$$

69 B. Semi-analytical Expression of the Torque

70 The torque transmitted between two tile permanent magnets can be written as follows:

$$T_\theta = \frac{J_1 J_2}{4\pi\mu_0} \left(\int_{\theta_1}^{\theta_2} \int_{\theta_3}^{\theta_4} dT_\theta^{(1)} + T_\theta^{(2)} + \int_{z_a}^{z_b} dT_\theta^{(3)} + \int_{\theta_1}^{\theta_2} dT_\theta^{(4)} \right) \tag{8}$$

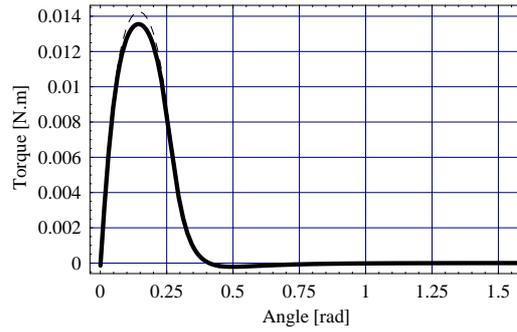


Fig. 3. Representation of the torque transmitted between two tiles ; thick line: the magnetization is uniform, dashed lines: the magnetization is radial; $\theta_4 - \theta_3 = \theta_2 - \theta_1 = \frac{\pi}{12}$, $r_{in1} = 0.025\text{m}$, $r_{out1} = 0.028\text{m}$, $r_{in2} = 0.021\text{m}$, $r_{out2} = 0.024\text{m}$, $z_d - z_c = 0.003\text{m}$, $z_b - z_a = 0.003\text{m}$, $z_a = 0.001\text{m}$, $r = 0.024\text{m}$, $J_1 = J_2 = 1\text{T}$

71 where $dT_\theta^{(1)}$, $T_\theta^{(2)}$, $dT_\theta^{(3)}$ and $dT_\theta^{(4)}$ are given in the appendix. Strictly speaking, this expression is three-
 72 dimensional and we take into account all the contributions between the two tiles. However, it is noted that
 73 all the contributions have not the same weight and the interaction between the surface densities located
 74 on the inner and outer faces of each tile permanent magnet is in fact the most important. Furthermore, it
 75 is noted that this expression could probably still be simplified and led to incomplete elliptical integrals of the
 76 first, second and third kind. In addition, we use the Cauchy principal value for determining the singular
 77 cases that appear for example when one tile permanent magnet is exactly in front of the other one.

78 III. COMPARISON OF THE TORQUE TRANSMITTED BETWEEN TWO TILES RADIALLY MAGNETIZED 79 AND TWO TILES UNIFORMLY MAGNETIZED

80 Tiles uniformly magnetized are generally less efficient than tiles radially magnetized for magnetic
 81 couplings. However, this loss of torque depends greatly on the tile angular width. Therefore, it is interesting
 82 to determine this loss of torque for different tile angular widths. For this purpose, we represent in the same
 83 figure the torque transmitted between two tiles radially magnetized and the torque transmitted between
 84 two tiles uniformly magnetized. On Figure 3, the angular width of each tile is the same ($\frac{\pi}{12}$). Then,
 85 we represent in Figs 4, 5 and 6 the torque transmitted between tiles whose angular widths are $\frac{\pi}{6}$, $\frac{\pi}{4}$
 86 and $\frac{\pi}{3}$. Figs 3, 4, 5 and 6 show clearly that the more the tile angular width is important, the less
 87 the magnetic torque between tiles uniformly magnetized is important. Consequently, we deduce that a
 88 manufacturer should use tiles uniformly magnetized only if their widths are small. Furthermore, Figs 3,

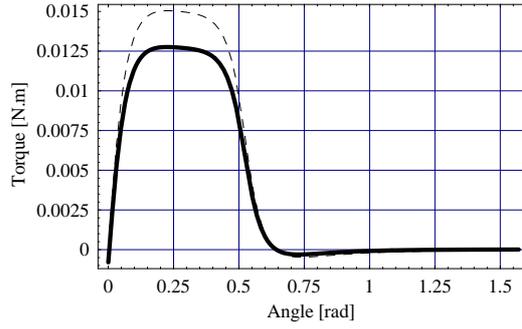


Fig. 4. Representation of the torque transmitted between two tiles ; thick line: the magnetization is uniform, dashed lines: the magnetization is radial; $\theta_4 - \theta_3 = \theta_2 - \theta_1 = \frac{\pi}{6}$, $r_{in1} = 0.025\text{m}$, $r_{out1} = 0.028\text{m}$, $r_{in2} = 0.021\text{m}$, $r_{out2} = 0.024\text{m}$, $z_d - z_c = 0.003\text{m}$, $z_b - z_a = 0.003\text{m}$, $z_a = 0.001\text{m}$, $r = 0.024\text{m}$, $J_1 = J_2 = 1\text{T}$

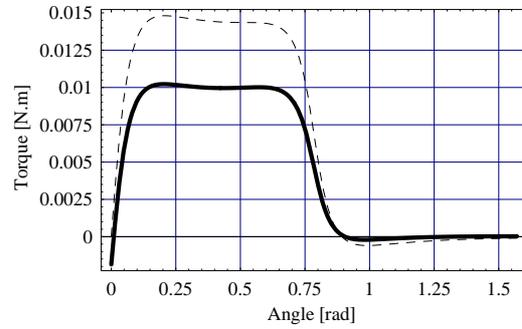


Fig. 5. Representation of the torque transmitted between two tiles ; thick line: the magnetization is uniform, dashed lines: the magnetization is radial; $\theta_4 - \theta_3 = \theta_2 - \theta_1 = \frac{\pi}{4}$, $r_{in1} = 0.025\text{m}$, $r_{out1} = 0.028\text{m}$, $r_{in2} = 0.021\text{m}$, $r_{out2} = 0.024\text{m}$, $z_d - z_c = 0.003\text{m}$, $z_b - z_a = 0.003\text{m}$, $z_a = 0.001\text{m}$, $r = 0.024\text{m}$, $J_1 = J_2 = 1\text{T}$

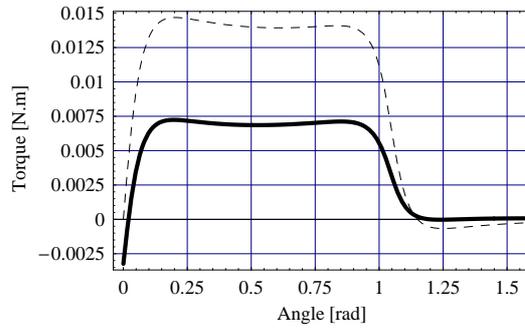


Fig. 6. Representation of the torque transmitted between two tiles ; thick line: the magnetization is uniform, dashed lines: the magnetization is radial; $\theta_4 - \theta_3 = \theta_2 - \theta_1 = \frac{\pi}{3}$, $r_{in1} = 0.025\text{m}$, $r_{out1} = 0.028\text{m}$, $r_{in2} = 0.021\text{m}$, $r_{out2} = 0.024\text{m}$, $z_d - z_c = 0.003\text{m}$, $z_b - z_a = 0.003\text{m}$, $z_a = 0.001\text{m}$, $r = 0.024\text{m}$, $J_1 = J_2 = 1\text{T}$

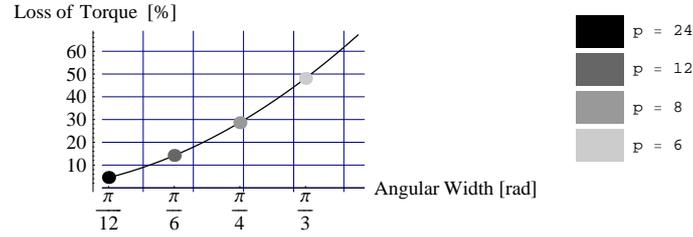


Fig. 7. Representation of the mean loss of efficiency of the torque transmitted between two tiles uniformly magnetized versus the angular width of the tiles. This loss of torque is calculated in comparison with the torque transmitted between two tiles radially magnetized. The points correspond to the figures 3, 4, 5 and 6. We have : $r_{in1} = 0.025\text{m}$, $r_{out1} = 0.028\text{m}$, $r_{in2} = 0.021\text{m}$, $r_{out2} = 0.024\text{m}$, $z_b - z_a = z_d - z_c = 0.003\text{m}$, $J_1 = J_2 = 1\text{T}$.

89 4, 5 and 6 show that the torque transmitted between two tile permanent magnets radially magnetized
 90 does not depend very much on their angular widths whereas it decreases greatly with the increase in the
 91 angular width when the tiles are uniformly magnetized.

92 We can also represent this decrease by calculating its rate of loss versus the angular width of tiles (Fig
 93 7).

94 Figure 7 is consistent with Figs 3, 4, 5 and 6. Indeed, when the angular width of the tile permanent
 95 magnets tends to zero, we can use either tiles radially magnetized or tiles uniformly magnetized. This is
 96 the only case in which the choice has not a great importance. Consequently, as tiles uniformly magnetized
 97 are cheaper to fabricate, their use is more judicious. However, when the angular width of the tiles used
 98 becomes greater, the choice is certainly more difficult and other considerations must be taken into account.
 99 For example, if the first property required in a coupling is really the value of the torque transmitted, the
 100 magnetization of the tiles should be radial and not uniform.

101 IV. APPLICATION : ALTERNATE MAGNET STRUCTURES USING TILES RADially OR UNIFORMLY 102 MAGNETIZED

103 We can illustrate the expression established in the previous section by studying the torque transmitted
 104 in an alternate magnet structure with 8 tile permanent magnets on each rotor. For this purpose, we use
 105 the principle of superposition with (7) for the calculation of the torque transmitted between the leading

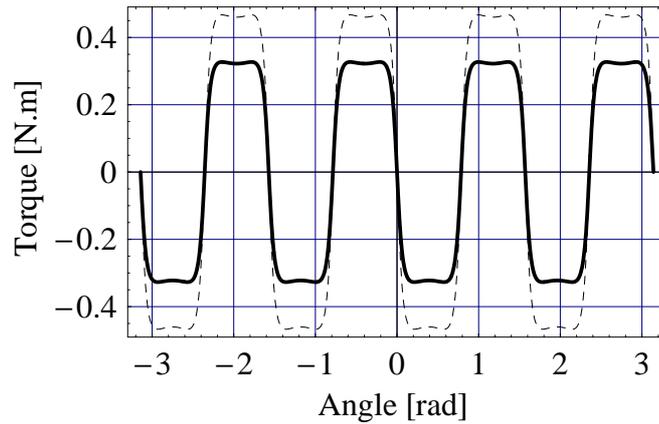


Fig. 8. Representation of the total torque transmitted between an eight tile rotor and an eight tile stator versus the angle θ . The tiles are either radially magnetized (dashed line) or uniformly magnetized (thick line) $r_{in1} = 0.025\text{m}$, $r_{out1} = 0.028\text{m}$, $r_{in2} = 0.021\text{m}$, $r_{out2} = 0.024\text{m}$, $z_b - z_a = z_d - z_c = 0.003\text{m}$, $J_1 = J_2 = 1\text{T}$

106 rotor and the lead rotor. We represent this torque versus the shifting angle θ in Fig 8.

107 Figure 8 is consistent with the previous representations of the torque transmitted between two tiles
 108 radially or uniformly magnetized. First, we note that the torque transmitted in the alternate magnet structure
 109 is sixteen times greater than the one transmitted between only two tile permanent magnets. Then, we note
 110 that the torque transmitted between tiles uniformly magnetized is smaller than the one transmitted between
 111 tiles radially magnetized, which is still consistent with Fig 4.

112 V. CONCLUSION

113 This paper has presented a three-dimensional expression of the torque transmitted between two tile
 114 permanent magnets uniformly magnetized. Then, we have compared the torque transmitted between two
 115 tiles uniformly magnetized with two tiles radially magnetized. An alternate magnet structure has been
 116 studied to illustrate the three-dimensional expression of the torque transmitted between tiles radially or
 117 uniformly magnetized. Some theoretical results have been analytically determined. First, tiles uniformly
 118 magnetized are less interesting than tiles radially magnetized for realizing couplings. Indeed, the more the
 119 tile angular widths are important, the less the torque transmitted between two tiles uniformly magnetized
 120 is important in comparison with the torque transmitted between two tiles radially magnetized. However,
 121 the cost of the magnets must be taken into account. Tiles uniformly magnetized are easier to fabricate

122 than tiles radially magnetized (and thus cheaper...).

123

APPENDIX

124 We give the expressions of the parameters used for calculating the torque transmitted between two
125 tile permanent magnets uniformly magnetized. The torque can be expressed in terms of semi-analytical
126 expressions using one or two numerical integrations or not.

$$T_\theta = \frac{J_1 J_2}{4\pi\mu_0} \left(\int_{\theta_1}^{\theta_2} \int_{\theta_3}^{\theta_4} dT_\theta^{(1)} + T_\theta^{(2)} + \int_{z_a}^{z_b} dT_\theta^{(3)} + \int_{\theta_1}^{\theta_2} dT_\theta^{(4)} \right) \quad (9)$$

127 where

$$\begin{aligned} dT_\theta^{(1)} = & + \cos\left(\tilde{\theta} - \frac{\theta_1 + \theta_2}{2}\right) \cos\left(\tilde{\theta} - \frac{\theta_3 + \theta_4}{2}\right) \sin(\tilde{\theta} - \tilde{\theta}) A[r_{in1}, r_{in2}] d\tilde{\theta} d\tilde{\theta} \\ & - \cos\left(\tilde{\theta} - \frac{\theta_1 + \theta_2}{2}\right) \cos\left(\tilde{\theta} - \frac{\theta_3 + \theta_4}{2}\right) \sin(\tilde{\theta} - \tilde{\theta}) A[r_{out1}, r_{in2}] d\tilde{\theta} d\tilde{\theta} \\ & - \cos\left(\tilde{\theta} - \frac{\theta_1 + \theta_2}{2}\right) \cos\left(\tilde{\theta} - \frac{\theta_3 + \theta_4}{2}\right) \sin(\tilde{\theta} - \tilde{\theta}) A[r_{in1}, r_{out2}] d\tilde{\theta} d\tilde{\theta} \\ & + \cos\left(\tilde{\theta} - \frac{\theta_1 + \theta_2}{2}\right) \cos\left(\tilde{\theta} - \frac{\theta_3 + \theta_4}{2}\right) \sin(\tilde{\theta} - \tilde{\theta}) A[r_{in2}, r_{out1}] d\tilde{\theta} d\tilde{\theta} \end{aligned} \quad (10)$$

128

$$\begin{aligned} T_\theta^{(2)} = & + \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \sin\left(\frac{\theta_4 - \theta_3}{2}\right) B[\theta_2, \theta_4] \\ & + \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \sin\left(\frac{\theta_4 - \theta_3}{2}\right) B[\theta_1, \theta_4] \\ & + \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \sin\left(\frac{\theta_4 - \theta_3}{2}\right) B[\theta_2, \theta_3] \\ & + \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \sin\left(\frac{\theta_4 - \theta_3}{2}\right) B[\theta_1, \theta_3] \end{aligned} \quad (11)$$

129

$$\begin{aligned} dT_\theta^{(3)} = & \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \cos\left(\frac{\theta_2 - \theta_1}{2} - \frac{\theta_4 - \theta_3}{2}\right) C[\theta_2, r_{in2}] d\tilde{z} \\ & + \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \cos\left(\frac{\theta_2 - \theta_1}{2} - \frac{\theta_3 + \theta_4}{2}\right) C[\theta_1, r_{in2}] d\tilde{z} \\ & - \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \cos\left(\frac{\theta_2 - \theta_1}{2} - \frac{\theta_3 + \theta_4}{2}\right) C[\theta_2, r_{out2}] d\tilde{z} \\ & - \sin\left(\frac{\theta_2 - \theta_1}{2}\right) \cos\left(\frac{\theta_2 - \theta_1}{2} - \frac{\theta_3 + \theta_4}{2}\right) C[\theta_1, r_{out2}] d\tilde{z} \end{aligned}$$

(12)

130

$$\begin{aligned}
dT_{\theta}^{(4)} = & + \cos(\tilde{\theta} - \frac{\theta_1 + \theta_2}{2}) \sin(\frac{\theta_4 - \theta_3}{2}) D[\theta_4, r_{in1}] d\tilde{\theta} \\
& - \cos(\tilde{\theta} - \frac{\theta_1 + \theta_2}{2}) \sin(\frac{\theta_4 - \theta_3}{2}) D[\theta_4, r_{out1}] d\tilde{\theta} \\
& + \cos(\tilde{\theta} - \frac{\theta_1 + \theta_2}{2}) \sin(\frac{\theta_2 - \theta_1}{2}) D[\theta_3, r_{in1}] d\tilde{\theta} \\
& - \cos(\tilde{\theta} - \frac{\theta_1 + \theta_2}{2}) \sin(\frac{\theta_2 - \theta_1}{2}) D[\theta_3, r_{out1}] d\tilde{\theta}
\end{aligned} \tag{13}$$

131 The calculation of the torque expression can be done as follows. Eqs (4) and (7) show four fundamental
132 expressions, written $A[r_1, r_2]$, $B[\theta_i, \theta_j]$, $C[\theta_i, r_i]$ and $D[\theta_j, r_j]$ that are necessary for calculating the torque
133 T_{θ} . These expressions are expressed as follows:

$$A[r_1, r_2] = \int_{z_a}^{z_b} \int_{z_c}^{z_d} \frac{-r_1^2 r_2^2}{(r_1^2 + r_2^2 - 2r_1 r_2 \cos(\tilde{\theta} - \tilde{\theta}) + (\tilde{z} - \tilde{z})^2)^{\frac{3}{2}}} d\tilde{z} d\tilde{z} \tag{14}$$

134

$$B[\theta_i, \theta_j] = \int_{r_{in2}}^{r_{out2}} \int_{z_c}^{z_d} \int_{r_{in1}}^{r_{out1}} \int_{z_a}^{z_b} \frac{-\tilde{r} \tilde{r} \sin(\theta_j - \theta_i)}{(\tilde{r}^2 + \tilde{r}^2 - 2\tilde{r} \tilde{r} \cos(\theta_j - \theta_i) + (\tilde{z} - \tilde{z})^2)^{\frac{3}{2}}} d\tilde{r} d\tilde{z} d\tilde{r} d\tilde{z} \tag{15}$$

135

$$C[\theta_i, r_i] = \int_{r_{in1}}^{r_{out1}} \int_{z_a}^{z_b} \int_{z_c}^{z_d} \frac{-r_i^2 \tilde{r} \sin(\tilde{\theta} - \theta_i)}{(\tilde{r}^2 + r_i^2 - 2\tilde{r} r_i \cos(\tilde{\theta} - \theta_i) + (\tilde{z} - \tilde{z})^2)^{\frac{3}{2}}} d\tilde{r} d\tilde{z} d\tilde{z} \tag{16}$$

136

$$D[\theta_j, r_j] = \int_{r_{in2}}^{r_{out2}} \int_{z_c}^{z_d} \int_{z_a}^{z_b} \frac{-r_j^2 \tilde{r} \sin(\theta_j - \theta_i)}{(\tilde{r}^2 + r_j^2 - 2r_j \tilde{r} \cos(\theta_j - \tilde{\theta}) + (\tilde{z} - \tilde{z})^2)^{\frac{3}{2}}} d\tilde{z} d\tilde{r} d\tilde{z} \tag{17}$$

137 The torque transmitted between two tile permanent magnets could be calculated directly by numerical
138 means with (8). However, the computational cost would be too long. Therefore, we give here four reduced
139 semi-analytical expressions of $A[r_1, r_2]$, $B[\theta_i, \theta_j]$, $C[\theta_i, r_i]$ and $D[\theta_j, r_j]$. We obtain:

$$A[r_1, r_2] = A^{(1)}[z_a, z_c] - A^{(1)}[z_b, z_c] - A^{(1)}[z_a, z_d] + A^{(1)}[z_b, z_d] \tag{18}$$

140 with

$$A^{(1)}[z_i, z_j] = \frac{r_1^2 r_2^2 \sqrt{r_1^2 + r_2^2 + (z_i - z_j)^2 - 2r_1 r_2 \cos(\tilde{\theta} - \tilde{\theta})}}{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\tilde{\theta} - \tilde{\theta})} \tag{19}$$

141

$$\begin{aligned}
B[\theta_i, \theta_j] &= \int_{r_{in2}}^{r_{out2}} \left(-B^{(1)}[z_a - z_c, r_{out1}] + B^{(1)}[z_b - z_c, r_{out1}] \right) d\tilde{r} \\
&+ \int_{r_{in2}}^{r_{out2}} \left(+B^{(1)}[z_a - z_d, r_{out1}] - B^{(1)}[z_b - z_d, r_{out1}] \right) d\tilde{r} \\
&+ \int_{r_{in2}}^{r_{out2}} \left(B^{(1)}[z_a - z_c, r_{in1}] - B^{(1)}[z_b - z_c, r_{in1}] \right) d\tilde{r} \\
&+ \int_{r_{in2}}^{r_{out2}} \left(-B^{(1)}[z_a - z_d, r_{in1}] + B^{(1)}[z_b - z_d, r_{in1}] \right) d\tilde{r}
\end{aligned} \tag{20}$$

142 with

$$\begin{aligned}
B[y, r_i] &= \tilde{r} \sqrt{\tilde{r}^2 - 2\tilde{r}\tilde{r}x + y} + \tilde{r}^2 x \log \left[\tilde{r} - \tilde{r}x + \sqrt{\tilde{r}^2 - 2\tilde{r}\tilde{r}x + y} \right] \\
&+ \frac{\tilde{r}i(-x + \sqrt{-1 + x^2})\sqrt{\tilde{r}^2 - y}}{2\sqrt{-1 + x^2}} \log[A] \\
&+ \frac{\tilde{r}i(x + \sqrt{-1 + x^2})\sqrt{\tilde{r}^2 - y}}{2\sqrt{-1 + x^2}} \log[B]
\end{aligned} \tag{21}$$

143

$$\begin{aligned}
A &= \frac{2i(\tilde{r}\tilde{r}(-1 + x^2) + \tilde{r}^2(x - x^3 + x^2\sqrt{-1 + x^2}))}{(-x + \sqrt{-1 + x^2})(\tilde{r} + \tilde{r}(-x + \sqrt{-1 + x^2}))(\tilde{r}^2 - y)^{\frac{3}{2}}} \\
&+ \frac{2i\sqrt{-1 + x^2}(-y + i\sqrt{\tilde{r}^2 - y}\sqrt{\tilde{r}^2 - 2\tilde{r}\tilde{r}x + y})}{(-x + \sqrt{-1 + x^2})(\tilde{r} + \tilde{r}(-x + \sqrt{-1 + x^2}))(\tilde{r}^2 - y)^{\frac{3}{2}}}
\end{aligned} \tag{22}$$

144

$$\begin{aligned}
B &= \frac{2i\tilde{r}\tilde{r}(-1 + x^2) - 2i\tilde{r}^2 x(-1 + x^2 + x\sqrt{-1 + x^2})}{(x + \sqrt{-1 + x^2})(-\tilde{r} + \tilde{r})(x + \sqrt{-1 + x^2})(\tilde{r}^2 - y)^{\frac{3}{2}}} \\
&+ \frac{2\sqrt{-1 + x^2}(iy + \sqrt{\tilde{r}^2 - y}\sqrt{\tilde{r}^2 - 2\tilde{r}\tilde{r}x + y})}{(x + \sqrt{-1 + x^2})(-\tilde{r} + \tilde{r}(x + \sqrt{-1 + x^2}))(\tilde{r}^2 - y)^{\frac{3}{2}}}
\end{aligned} \tag{23}$$

145

$$x = \cos(\theta_i - \theta_j) \tag{24}$$

$$C[\theta_i, r_i] = \int_{r_{in1}}^{r_{out1}} C^{(1)}[\theta_i, r_i] d\tilde{r} \quad (25)$$

$$\begin{aligned} C^{(1)}[\theta_i, r_i] = & r_i \sqrt{r_i^2 + \tilde{r}^2 + (z_c - \tilde{z})^2 - 2r_i \tilde{r} \cos(\theta_3 - \theta_i)} \\ & - r_i \sqrt{r_i^2 + \tilde{r}^2 + (z_d - \tilde{z})^2 - 2r_i \tilde{r} \cos(\theta_3 - \theta_i)} \\ & - r_i \sqrt{r_i^2 + \tilde{r}^2 + (z_c - \tilde{z})^2 - 2r_i \tilde{r} \cos(\theta_4 - \theta_i)} \\ & + r_i \sqrt{r_i^2 + \tilde{r}^2 + (z_d - \tilde{z})^2 - 2r_i \tilde{r} \cos(\theta_4 - \theta_i)} \\ & + r_i (z_c - \tilde{z}) \log \left[-z_c + \tilde{z} + \sqrt{r_i^2 + \tilde{r}^2 + (z_c - \tilde{z})^2 - 2r_i \tilde{r} \cos(\theta_3 - \theta_i)} \right] \\ & - r_i (z_d - \tilde{z}) \log \left[-z_d + \tilde{z} + \sqrt{r_i^2 + \tilde{r}^2 + (z_d - \tilde{z})^2 - 2r_i \tilde{r} \cos(\theta_4 - \theta_i)} \right] \\ & + r_i (z_d - \tilde{z}) \log \left[-z_d + \tilde{z} + \sqrt{r_i^2 + \tilde{r}^2 + (z_d - \tilde{z})^2 - 2r_i \tilde{r} \cos(\theta_3 - \theta_i)} \right] \\ & - r_i (z_c - \tilde{z}) \log \left[-z_c + \tilde{z} + \sqrt{r_i^2 + \tilde{r}^2 + (z_c - \tilde{z})^2 - 2r_i \tilde{r} \cos(\theta_3 - \theta_i)} \right] \end{aligned} \quad (26)$$

$$D[\theta_j, r_j] = \int_{r_{in2}}^{r_{out2}} \left(D^{(2)}(z_b, z_c) - D^{(2)}(z_a, z_c) + D^{(2)}(z_a, z_d) - D^{(2)}(z_b, z_d) \right) d\tilde{r} \quad (27)$$

146 with

$$D^{(2)}(x, y) = \frac{\sin(\theta_i - \theta_j) r_j^2 \tilde{r} \sqrt{r_j^2 + \tilde{r}^2 + (x - y)^2 - 2r_j \tilde{r} \cos(\theta_j - \theta_i)}}{r_j^2 + \tilde{r}^2 - 2r_j \tilde{r} \cos(\theta_j - \theta_i)} \quad (28)$$

147

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