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ADAPTIVE FILTER DESIGN FOR FAULT DETECTION AND ISOLATION OF NON LINEAR SYSTEMS BASED ON MULTIPLE MODEL APPROACH

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Abstract: In this paper, the fault diagnosis of non linear systems based on a adaptive filter is developed. The proposed method makes possible the fault detection, isolation and estimation for the whole operating range of the nonlinear system. Based on the assumption that the dynamic behavior of the nonlinear system is defined as a multiple model approach, a set of residual vector insensitive to faults is established and thus generating a global model exploited in an adaptive filter. The convergence and stability conditions of the adaptive filter are developed and its performances are tested in simulation.

Keywords: multi-model, FDI, stability, LMI, convex set.

1. INTRODUCTION

The role of a human operator is to preserve normal operating conditions according to several plant parameters, measurements and observations. But complex automated industrial systems are vulnerable to faults in instrumentation as sensors, actuators or controllers. With the growing complexity of modern engineering systems and ever increasing demand for safety and reliability, there has been great interest in the development of Fault Detection and Isolation (FDI) methods. Usually, faults and failures in systems are detected using analytical redundancy when comparing measured and estimated outputs of the system whatever the design used, the residual relations are elaborated so that it is possible to isolate any kind of fault. A large diversity of advanced methods for automated FDI already exists integrating the robustness against parameters uncertainties and model plant mismatches (Chen and Patton, 1999), (Frank and Ding, 1997), (Gertler,

1998). A short historical view on this development can be seen in (Isermann and Ballé, 1996) and current developments can be observed in (Frank et al., 2000). Rare are the methods dealing with the nonlinear systems, a great number is established around an operating point or requires the exact knowledge of the nonlinear system (Alcorta Garcia and Frank, 1997), (Hamouri et al., 1999) or as in the chapter nine of Chen and Patton (Chen and Patton, 1999), the nonlinear dynamic systems are described by a number of locally linearized models based on the idea of Tagaki-Sugeno fuzzy models. In the same idea, various studies, based on a multiple model method established through a bank of linear Kalman filters, have been developed in order to detect, isolate and estimate an accurate state of a system in presence of faults/failures when a model is defined around an operating point (Maybeck, 1999), (Rong, Li and Bar-Shalom, 1993) or the whole operating range of a nonlinear systems (Pan et al., 1997). This paper aims at investigating the FDI of nonlinear systems under

the assumption that the nonlinear system could be approximated by a finite number of interpolated Linear Time Invariant (L.T.I.) models (Johansen et al., 1998). Under stochastic assumption, the paper aims and contributes to the design of a fault detection and isolation adaptive filter where the distribution matrices, representatives of the nonlinear system, are obtained by probabilistically weighted matrices computation. This computation is based on the properties of residuals insensitive to faults and sensitive to some model errors. These residuals are generated according to a bank of filters (Keller, 1999) decoupled from faults.

In this paper, the magnitude, the time of occurrence and the localization of the faults are considered as unknown. The second section is devoted to the L.T.I. model selection in faulty case based on the use of a bank of decoupled Kalman filters. In this section, the design of the adaptive filter is developed. Section 3 is devoted to the simulation of a nonlinear system for various operating points and faults. The simulation results are then shown and commented. Finally, concluding remarks are given.

2. ADAPTIVE FILTER DESIGN

2.1 Positionnement du problème

Under actuator or sensor faults (following some transformations), a general discrete state space representation of MIMO system is described by:

$$\begin{aligned} X_{k+1} &= \sum_{i=1}^N \mu^i(\sigma_k) \left[A_i X_k + B_i U_k + F_i d_k + \Delta X_i \right] \\ Y_k &= \sum_{i=1}^N \mu^i(\sigma_k) \left[C_i X_k + \Delta Y_i \right] \end{aligned} \quad (1)$$

where $X_k \in \mathbb{R}^n$ is the state space vector, $U_k \in \mathbb{R}^p$ is the control input vector, $Y_k \in \mathbb{R}^m$ is the output vector, and $d_k \in \mathbb{R}^q$ is the fault vector to be detected. A_i , B_i and C_i are constant real matrices of appropriate dimensions and $F_i \in \mathbb{R}^{n \times q}$ is the fault matrix distribution.

The representation (1) can be considered as : - a conventional modelling approach for non linear smooth plant where $\mu^i(\sigma)$ is an appropriate weighting functions. The schedule variable $\mu^i(\sigma)$, which can be a vector or a scalar, embodies the nonlinearity of the plant (Leith and Leithead, 1999; Murray-Smith and Johansen, 1997). The N models should described the dynamic behaviour of the plant where N are the number of equilibrium points defined by (X_i^e, U_i^e, Y_i^e) allowing the determination of the matrices and as follows:

$$\begin{aligned} \Delta X_i &= -A_i X_i^e - B_i U_i^e \\ \Delta Y_i &= -C_i X_i^e + Y_i^e \end{aligned} \quad (2)$$

- a faulty linear parameter varying (LPV) system (Szaabo et al., 2002) when the terms $\sum_{i=1}^N \mu^i(\sigma)$ are time varying parameter dependent and $\Delta X_i = \Delta Y_i = 0$.
- an hybrid system when the terms are supposed defining as a system mode sequence with a transition probabilities (Rong, Li X. and Y. Bar-Shalom, 1993). The representation is generally used in linear context () where partial actuator or sensor failures are modelled by multiplying the appropriate column by a factor of effectiveness (Zhang Y. and X.R. Li., 1998).

In this context of N models should represent the evolution of nonlinear systems, the purpose is to isolate, estimate faults under knowledge of the equilibrium point where the system is representing at time t . An answer developped in (Adam-Medina.M et al., 2002) use a decoupling Kalman filter that generates two informations: one can show the equilibrium point, the other estimate faults and their magnitude.

In the state space representation (1), the interpolation of the matrix $F(p)$ is made as : $F(p) = \sum_{i=1}^N \mu^i(\sigma_k) F_i$ where matrix $F_i \in \mathbb{R}^{n \times q}$ are faulty matrix distribution of each model i . Faults events on the system are described into state space representation as : $(\sum_{i=1}^N \mu^i(\sigma_k) F_i) * d(k)$.

According to some previous conditions on direction of columns vectors if matrices F_i , this notation can be changed by following notations:

Definition 1. F_i^j and $\Im^j$ are defined respectively the j th column of the matrix F_i and $\Im$.

Theorem 1. If the rank of the i th matrix column $F_i \in \mathbb{R}^{n \times q}$ satisfies:

$$\forall j \in [1..q], \forall i \in [1..N], \quad \text{Rank}[F_1^j .. F_i^j .. F_N^j] = 1 \\ \text{with } \text{Rank}[F_i] = q$$

then: $(\sum_{i=1}^N \mu^i(\sigma_k) F_i) * d(k) = \Im * f(k)$ where $d(k) \in \mathbb{R}^q$

is the true faults vector in which $f(k) \in \mathbb{R}^q$ represents false faults, where $\Im \in \mathbb{R}^{n \times q}$ is a fixed matrix which columns vectors get direction of columns vectors of matrices F_i .

◊ **Proof** As $\text{Rank}[F_1^j .. F_i^j .. F_N^j] = 1$, the j th matrices columns F_i get same direction, this condition could be rewritten as $F_i^j = \alpha_i^j * \Im^j$ where α_i^j is a scalar corresponding to the j th matrix column F_i with $\Im$ a matrix composed by fixed elements. Thus, coefficients α_i^j can't be equal to zero otherwise rank conditions will not be true. By underlying colinearity of each column, faults in state space representation can be noted as:

$$\begin{aligned}
\left(\sum_{i=1}^N \mu^i(\sigma_k) F_i^j\right) * d^j(k) &= \left(\sum_{i=1}^N \mu^i(\sigma_k) (\alpha_i^j \mathfrak{S}^j)\right) * d^j(k) \\
&= \mathfrak{S}^j * \left(\sum_{i=1}^N \mu^i(\sigma_k) \alpha_i^j * d^j(k)\right) \\
&= \mathfrak{S}^j * f^j(k)
\end{aligned}$$

where $d^j(k)$ and $f^j(k)$ are j th elements of the column vector of the faults. This equality is repeated q times to get columns of the matrix $\mathfrak{S}$ as well as q elements of the vector $f(k)$. Thus, the given matrix $\mathfrak{S}$ is fixed, full column rank and false faults vector is noted $f(k) = [f^1(k) f^2(k) \dots f^q(k)]^T$ in which each elements $f^j(k)$ is an interpolation of the j th element of the true faults vector. Consequently, matrix $\mathfrak{S}$ and false faults $f(k)$ can be represented as:

$$\left(\sum_{i=1}^N \mu^i(\sigma_k) F_i\right) * d(k) = \mathfrak{S} * f(k) \quad (3)$$

with $\forall i$

$$\mathfrak{S} = \begin{bmatrix} \frac{1}{\alpha_i^1} F_i^1 & \frac{1}{\alpha_i^2} F_i^2 & \dots & \frac{1}{\alpha_i^q} F_i^q \end{bmatrix} \quad (4)$$

and

$$f(k) = \begin{bmatrix} \left(\sum_{i=1}^N \mu^i(\sigma_k) \alpha_i^1 * d^1(k)\right) & \dots & \left(\sum_{i=1}^N \mu^i(\sigma_k) \alpha_i^q * d^q(k)\right) \end{bmatrix} \quad (5)$$

Remark:

By its construction, matrix $\mathfrak{S}$ is not unic. The scalar α_i^j is not equal to zero because rank conditions would not be true. Secondly, true faults can be found by simple matrix pseudo inversion. If we get previously:

$$\left(\sum_{i=1}^N \mu^i(\sigma_k) F_i\right) * d(k) = \mathfrak{S} * f(k) \quad (6)$$

By using a pseudo inversion, we obtain:

$$d(k) = \left(\sum_{i=1}^N \mu^i(\sigma_k) F_i\right)^+ * \mathfrak{S} * f(k) \quad (7)$$

with $\left(\sum_{i=1}^N \mu^i(\sigma_k) F_i\right)^+$ pseudo-inverse matrix $\left(\sum_{i=1}^N \mu^i(\sigma_k) F_i\right)$. The equation

$$\left(\sum_{i=1}^N \mu^i(\sigma_k) F_i\right) * d(k) = \mathfrak{S} * f(k) \quad (8)$$

considering the following notation faults can be expressed as

$$\begin{aligned}
f(k) &= \begin{bmatrix} \sum_{i=1}^N \mu^i(\sigma_k) [\alpha_i^1 * d^1(k)] \\ \dots \\ \sum_{i=1}^N \mu^i(\sigma_k) [\alpha_i^q * d^q(k)] \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^N \mu^i(\sigma_k) g_i^1(k) \\ \dots \\ \sum_{i=1}^N \mu^i(\sigma_k) g_i^q(k) \end{bmatrix} \\
&= \sum_{i=1}^N \mu^i(\sigma_k) \begin{bmatrix} g_i^1(k) \\ \dots \\ g_i^q(k) \end{bmatrix} = \sum_{i=1}^N \mu^i(\sigma_k) g_i(k)
\end{aligned} \quad (9)$$

where $d^j(k)$ is the i th true fault element $d(k)$ and $g_i^j(k)$ is the j th element of the i th fault column vector $g_i(k)$ leading to the i th model. Conditions (8) and (9) lead to:

$$\left(\sum_{i=1}^N \mu^i(\sigma_k) F_i\right) * d(k) = \mathfrak{S} * \sum_{i=1}^N \mu^i(\sigma_k) g_i(k) \quad (10)$$

2.2 Adaptive filter design

The state space representation of the system with faults under a convex set Λ corresponds to:

$$\begin{cases} X_{k+1} = A_k X_k + B_k U_k + \mathfrak{S} f_k + \Delta X_k \\ Y_k = C_k X_k + \Delta Y_k \end{cases} \quad (11)$$

with

$$\begin{aligned}
A_k &= \sum_{i=1}^N \mu^i(\sigma_k) A_i, & B_k &= \sum_{i=1}^N \mu^i(\sigma_k) B_i \\
\Delta X_k &= \sum_{i=1}^N \mu^i(\sigma_k) \Delta X_i, & \Delta Y_k &= \sum_{i=1}^N \mu^i(\sigma_k) \Delta Y_i \\
C_k &= C
\end{aligned}$$

$$\Lambda = \{\mu \in \mathbb{R}^N : \mu^i(\sigma_k) \geq 0 \text{ for } i = 1, \dots, N \text{ and } \sum_{i=1}^N \mu^i(\sigma_k) = 1\}$$

By using a classic Kalman filter with a gain K_k that could be designed polytopically as matrix A_k , the estimation error $e(k) = X(k) - \hat{X}_k$ and a residual $r_k = (Y_k - \hat{Y}_k)$ can be expressed like

$$\begin{aligned}
e(k+1) &= (A_k - K_k C) e(k) + \mathfrak{S} f(k) \\
r(k) &= C e(k)
\end{aligned} \quad (12)$$

If we suppose that faults occur at time k_d ($k > k_d$), residual could be defined as (Liu.B and Si.J, 1997):

$$\begin{aligned}
r(k) &= \bar{r}(k) \\
&+ \rho_{k,k_d} [f(k_d) \ f(k_d+1) \ \dots \ f(k-1)]
\end{aligned} \quad (13)$$

with

$$\rho_{k,k_d} = C \begin{bmatrix} \Gamma^{k_d+1}(k) \mathfrak{S} \\ \Gamma^{k_d+2}(k) \mathfrak{S} \\ \dots \\ \mathfrak{S} \end{bmatrix} \quad (14)$$

where

$$\Gamma^{k_d, k_e}(k) = \prod_{\tau=k, k_e}^{k-1} L(\tau) \quad (15)$$

$$L(k) = (A_k - K_k C) \quad (16)$$

If the term $(A_k - K_k C)\Im$ equals to zero, we can obtain (Keller.J.Y, 1999) a sensitive residual to faults $r_k = (Y_k - \tilde{Y}_k)$ equals to $\bar{r}_k + C\Im f(k-1)$ where $\bar{r}_k$ is a residual without any fault and leads to a centered gaussian distribution.

Filter gain K_k is obtained under matrices assumptions like:

$$\Pi \bar{D} = I \text{ and } \Sigma \bar{D} = 0 \quad (17)$$

Adaptive filter generates two residuals:

$$\begin{bmatrix} \gamma_k \\ q_k \end{bmatrix} = \begin{bmatrix} \Sigma \\ \Pi \end{bmatrix} * r_k = \begin{bmatrix} \Sigma \bar{r}_k \\ \Pi \bar{r}_k + f(k-1) \end{bmatrix} \quad (18)$$

Residual $\gamma_k \in \mathbb{R}^{m-q}$ is insensitive to faults and residual $q_k \in \mathbb{R}^q$ is sensitive to faults.

Adaptive filter is described by the following representation:

$$\begin{cases} \tilde{X}_{k+1} = A_k \tilde{X}_k + B_k U_k + \bar{K}_k \gamma_{gk} \\ \quad + \omega_k q_k + \Delta X_k \\ \tilde{Y}_k = C \tilde{X}_k + \Delta Y_k \end{cases} \quad (19)$$

with $\tilde{X}_k$ and $\tilde{Y}_k$ are respectively the estimated state and the estimated output of the adaptive filter. The others matrices for the adaptive filter design are then defined as:

$$\begin{aligned} A_k &= \sum_{i=1}^N \mu^i(\sigma_k) A_i, \quad B_k = \sum_{i=1}^N \mu^i(\sigma_k) B_i, \quad C_k = C, \\ \Delta X_k &= \sum_{i=1}^N \mu^i(\sigma_k) \Delta X_i, \quad \Delta Y_k = \sum_{i=1}^N \mu^i(\sigma_k) \Delta Y_i, \\ K_k &= \omega_k \Pi_m + \bar{K}_k \Sigma, \quad \omega_k = A_k \Im, \\ \Pi &= \bar{D}^+, \quad \Sigma = \alpha(I_m - \bar{D} \Pi), \\ \bar{C} &= \Sigma C, \quad F_k = \Im, \quad \bar{D} = C \Im \end{aligned}$$

where F_k is the faulty matrix distribution at time k and α is an arbitrary matrix $\in \mathbb{R}^{m-q \times m}$ designed in order to get Σ full row rank. Gain $\bar{K}$ is the only free parameter of the filter and it is designed by eigenstructure assignment of $(\bar{A} - \bar{K} \bar{C})$.

We should remark in the adaptive filter design that faulty matrix distribution is fixed as the observation matrix C . These conditions will be very important for following study.

3. STABILITY ON A CONVEX HULL

In the study which leads to use a decomposition of the system evolution in some operating points, it

is worth mentioning the stability and convergence properties of used filters to get a good estimation of the system online. Properties established by (Liu.B and Si.J, 1997) for each filter is required to apply them on all range of the system.

3.1 Local stability

If we consider that a local filter designed for each model i th, its stability is assured by gain K^i so as to assure not only faults decoupling but also eigenvalues of $A^i - K^i C$ are lower than 1. Thus, the error estimation without fault is noted like:

$$\bar{e}_{k+1}^i = (A^i - K^i C) \bar{e}_k^i \quad (20)$$

As showed in the previous paragraph, decomposition of gain of each local filter generate that matrix $(A^i - K^i C)$:

$$\begin{aligned} (A_i - K_i C) &= A_i - (\omega \Pi + \bar{K}_i \Sigma) C \\ &= (A_i - \omega \Pi C) - \bar{K}_i \Sigma C \\ &= (A_i - A_i \Im \Pi C) - \bar{K}_i \bar{C} \\ &= \underbrace{A_i (I - \Im \Pi C)}_{\bar{A}_i} - \bar{K}_i \bar{C} \end{aligned} \quad (21)$$

with

$$\omega^i = A^i \Im, \quad \Pi = \bar{D}^+ = C \bar{\Psi} = C \Im \quad (22)$$

Gains $\bar{K}_i$ are established by eigenstructure assignment in order to matrix $\bar{A}^i - \bar{K}^i \bar{C}$ be Hurwitz, so error estimation without faults is described as follow:

$$\bar{e}_{k+1}^i = (\bar{A}^i - \bar{K}^i \bar{C}) \bar{e}_k^i \quad (23)$$

By using Lyapunov stability on a system like (23), estimation error (Oliveira.M.C.de *et al.*, 1999) leads to:

$$\begin{aligned} (\bar{A}^i - \bar{K}^i \bar{C})^T P (\bar{A}^i - \bar{K}^i \bar{C}) - P &< 0 \\ P &> 0 \end{aligned} \quad (24)$$

where P is a positive definite matrix with appropriated dimensions.

For adaptive filter design, polytopics matrices are defined as follow: $(A(\mu) = \sum_{i=1}^N \mu^i(\sigma_k) A_i)$ et $\bar{K}$: $(\bar{K}(\mu) = \sum_{i=1}^N \mu^i(\sigma_k) \bar{K}_i)$ generate directly:

$$\begin{aligned} \left(\sum_{i=1}^N \mu^i(\sigma_k) A_i \right) (I - \Im \Pi C) - \left(\sum_{i=1}^N \mu^i(\sigma_k) \bar{K}_i \right) \bar{C} &= \\ &= \sum_{i=1}^N \mu^i(\sigma_k) \bar{A}_i - \sum_{i=1}^N \mu^i(\sigma_k) \bar{K}_i \bar{C} \\ &= \bar{A}(\sigma) - \bar{K}(\sigma) \bar{C} \end{aligned} \quad (25)$$

with $\mathfrak{S}$ faulty matrix distribution of the interpolated model as represented in (11).

3.2 Robust stability of estimation error

In the adaptive filter, through state space representation of system (11) and adaptive filter (19) the estimation error is described by the following equations:

$$\bar{e}_{k+1} = (\bar{A}(\sigma) - \bar{K}(\sigma)\bar{C})\bar{e}_k \quad (26)$$

where the dynamics matrix $\bar{A}(\sigma)$ et $\bar{K}(\sigma)$ belong to convex polytopic set defined as:

$$\begin{aligned} \Lambda &= \{ \bar{A}(\sigma) : \sum_{i=1}^N \mu^i(\sigma_k) \bar{A}_i = 1, \sum_{i=1}^N \mu^i(\sigma_k) = 1, \mu^i(\sigma_k) \geq 0 \} \\ \Gamma &= \{ \bar{K}(\sigma) : \sum_{i=1}^N \mu^i(\sigma_k) \bar{K}_i = 1, \sum_{i=1}^N \mu^i(\sigma_k) = 1, \mu^i(\sigma_k) \geq 0 \} \end{aligned} \quad (27)$$

Lemma 1. Uncertain system (3.2) is robustly stable in uncertainty domain (3.2) if

$$(\bar{A}^i - \bar{K}^i \bar{C}^i)^T P (\bar{A}^i - \bar{K}^i \bar{C}^i) - P < 0 \quad (28)$$

$$P > 0$$

for all $i = 1..N$.

By using Schur complement, inequalities (1) can be described by an LMI as follow:

$$\begin{pmatrix} P & (\bar{A}_i - \bar{K}_i \bar{C})^T P \\ P(\bar{A}_i - \bar{K}_i \bar{C}) & P \end{pmatrix} > 0 \quad (29)$$

We can extend this result on a convex set defined in (27) and generate:

$$\begin{pmatrix} P & \sum_{i=1}^N \mu^i(\sigma_k) (\bar{A}_i - \bar{K}_i \bar{C})^T P \\ P \sum_{i=1}^N \mu^i(\sigma_k) (\bar{A}_i - \bar{K}_i \bar{C}) & P \end{pmatrix} > 0 \quad (30)$$

As we can see the estimation error checks the quadratic stability conditions depending on parameters. Each Lyapunov matrix is calculated for each vertex of the polytope so as to matrix $(\bar{a}_i - \bar{k}_i \bar{c})$ is stable.

Thus the adaptive filter synthesis extend local stability of each decoupling Kalman filter to a convex set λ representing whole parameters evolution μ . Then adaptive filter is stable on this convex set λ and generates robust residues allowing diagnosis of the system on all operating range.

4. EXEMPLE ILLUSTRATIF

La méthode proposée est appliquée à une simulation non linéaire discrète en boucle ouverte. Le processus non linéaire considéré, un système SIMO, se compose par un ensemble de trois systèmes stochastiques linéaires avec une dimension identique pour $X \in \mathbb{R}^4$, $U \in \mathbb{R}^1$ et $Y \in \mathbb{R}^3$. Le ième modèle linéaire représente le comportement dynamique du système non linéaire autour du ième point de fonctionnement et dépend de la grandeur d'entrée comme définie dans la table suivante :

Table 1. Définition des trois points de fonctionnement.

Point de Fonctionnement	$i = 1$ $U \in [0; 3[$	$i = 2$ $U \in [3; 6[$	$i = 3$ $U \in [6; 9[$
Y_i	3.2-5.0 2.58	5.82-8.54 4.88	9.88-15.62 8.34
U_i	2	4	7
Valeurs propres de A_i	0.15	0.01	0.20
	0.37	0.025	0.50
	0.60	0.04	0.80
	0.22	0.015	0.30

$$\forall i \ B_i = [1 \ 0 \ 0 \ 0]^T, \ C_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$R_i = 0.1 * I_{3 \times 3}.$$

Il est possible d'avoir des matrices de direction de défauts F_i différentes pour chaque modèle i sous les conditions requises et de prendre une seule matrice de direction de défauts $\mathfrak{S}$ pour le filtre adaptatif. Les matrices de direction de défauts sont:

$$F_1 = \begin{bmatrix} 0 & 0 \\ 0.9 & 0 \\ 0 & 2.8 \\ 0 & 0 \end{bmatrix}, \ F_2 = \begin{bmatrix} 0 & 0 \\ 2.4 & 0 \\ 0 & 1.4 \\ 0 & 0 \end{bmatrix},$$

$$F_3 = \begin{bmatrix} 0 & 0 \\ 3.2 & 0 \\ 0 & 3.7 \\ 0 & 0 \end{bmatrix}, \ \mathfrak{S} = \begin{bmatrix} 0 & 0 \\ 5 & 0 \\ 0 & 7 \\ 0 & 0 \end{bmatrix}.$$

Les 3 sorties Y et l'entrée U du système en présence de défauts se visualisent respectivement sur les figures (??) et (1). La figure (2) illustre l'évolution des probabilités des 3 modèles linéaires image du comportement dynamique du système non linéaire en présence de défauts: on remarquera que la sélection du modèle se fait correctement et indépendamment de la présence ou non de défauts.

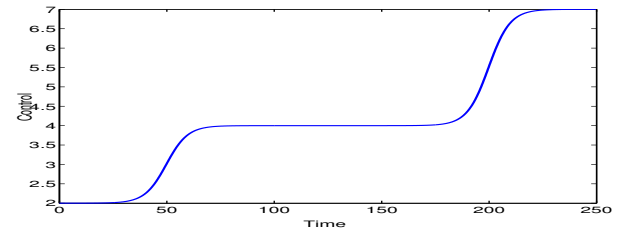


Fig. 1. Entry evolution

Les défauts $d(k) \in \mathbb{R}^2$, $\text{rang}(C_i * \mathfrak{S}) = 2 < 3$ (nombre de défauts inférieur au nombre de sortie)

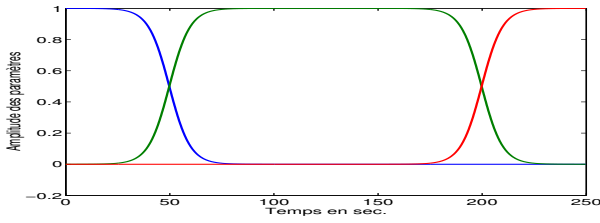


Fig. 2. Parameters evolution

apparaissant sur le système ont pour amplitude et temps d'apparition respectifs $d^1(k) = 2$ à $t = 50s$ et $d^2(k) = 5$ à $t = 150s$. En ayant plusieurs matrices de direction de défauts pour chaque modèle, les résidus générés par le filtre adaptatif sont des défauts fictifs fig(3).

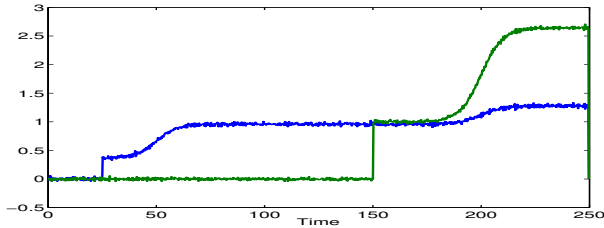


Fig. 3. Generated faults of adaptive filter

L'inversion matricielle qui s'en suit permet alors d'estimer les défauts vrais donnés sur la figure (4). On peut remarquer un léger pic lors du passage d'un modèle à l'autre mais il s'agit d'un seul échantillon dont la présence s'explique par le fait que l'on commute d'un modèle à l'autre sur l'entrée U et que l'évaluation s'effectue sur Y .

Remarque: On pourra vérifier que le résidu insensible fig(5) provenant du filtre adaptatif possède une distribution Gaussienne de moyenne nulle sur toute la plage de fonctionnement du système, assurant de ce fait la validité du modèle généré par celui-ci.

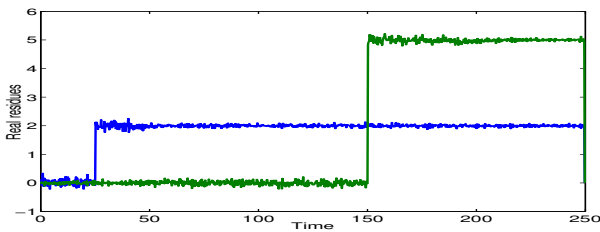


Fig. 4. Faults estimation of system

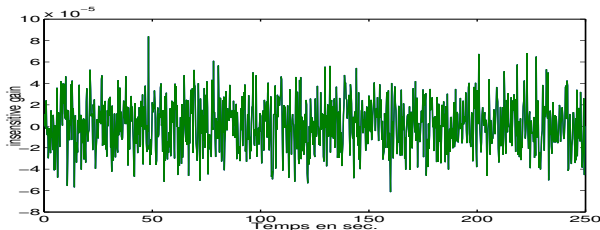


Fig. 5. Insensible residual of adaptive filter

En visualisant le gain $\bar{K}(p)$ du filtre adaptatif fig(6), on remarque son évolution en trois paliers correspondant chacun à l'un des trois modèles. Le gain $\bar{K}_k^i$ du i ème filtre est bien très vite stable car il faut un à deux échantillons pour converger vers la valeur finale du gain $\bar{K}^i$ du i ème filtre de Kalman découplant. Les valeurs de chaque gain peuvent être calculées hors-ligne pour pouvoir lancer l'algorithme du filtre adaptatif avec la valeur finale du gain de chaque filtre. On montre ainsi que reconstruire le gain du filtre adaptatif de manière algorithmique ou de manière polytopique (interpolée) est identique.

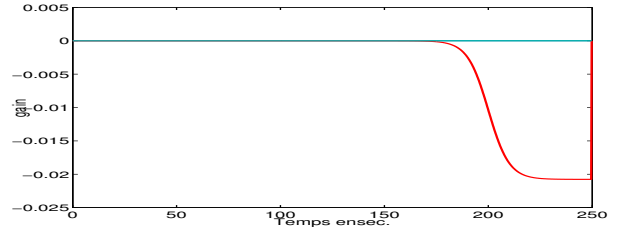


Fig. 6. Gain evolution of adaptive filter

Pour l'étude de stabilité de l'exemple, la résolution des équations (29) et (30) trouve la solution en la matrice $P > 0$ qui a pour valeur numérique:

$$P = \begin{pmatrix} 81.3347 & 0.0026 & 0.0026 & 0.0063 \\ 0.0026 & 82.7842 & 0 & 0.0038 \\ 0.0026 & 0 & 82.7842 & 0.0038 \\ 0.0063 & 0.0038 & 0.0038 & 79.5890 \end{pmatrix}$$

On notera la valeur des matrices

$$\bar{C} = [0 \ 0 \ 0 \ 1], \quad \bar{K}_1 = [0 \ 0 \ 0 \ 0.0535]^T, \quad \bar{K}_2 = [0 \ 0 \ 0 \ 0.0035]^T \text{ et } \bar{K}_3 = [0 \ 0 \ 0 \ 0.0731]^T.$$

5. CONCLUSION

Nous avons présenté une méthode de diagnostic de défauts sur des systèmes non linéaires représentés par un ensemble fini de modèles linéaires invariants dans le temps. Au travers l'utilisation de probabilités bayésiennes, les N filtres permettent alors de sélectionner le meilleur modèle estimant au mieux la dynamique du système non linéaire. Après avoir déterminé le meilleur modèle, la partie diagnostic proprement dite s'effectue au moyen de l'utilisation d'un filtre adaptatif construit avec les matrices polytopiques de la représentation d'état générée. La stabilité étant définie localement, elle est étendue sur toute la plage de fonctionnement par le recours aux LMI sous hypothèses de paramètres variants dans un ensemble convexe. Ce filtre génère alors un résidu insensible aux défauts tandis que l'autre sensible aux défauts permet la détection, l'isolation et l'estimation de défauts multiples. Il faut noter que les modèles ont été supposés représentant parfaitement le système quand celui-ci varie dans un point de fonctionnement et par conséquent que les erreurs de modélisation n'ont pas été prises en compte. Dans de futurs

travaux, il sera intéressant de prendre en compte les erreurs de modélisation ainsi que d'appliquer cette approche en boucle fermée.

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