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FAULT DIAGNOSIS FOR NONLINEAR SYSTEM BASED ON MULTI-MODEL REPRESENTATION: A ROBUST POLYTOPIC UNKNOWN INPUT OBSERVER SYNTHESIS

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Abstract: The main goal of this paper consists in fault diagnosis of nonlinear systems represented by a multi-model approach. Partial knowledge of the system representation around operating points must allow to obtain its dynamic behaviour under modeling errors consideration. The goal of this paper is to decouple modeling errors through a wide operating range by a dedicated polytopic Unknown Input Observer (*UIO*). The robust polytopic observer allows an optimization of modeling errors matrix distribution and permits to develop a fault diagnosis strategy on a wide operating range. Stability of the polytopic observer is guaranteed by pole assignment established through Linear Matrix Inequality (*LM**I*).

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Keywords: Fault diagnosis, Nonlinear system, Multi-model, LMI, Stability, Unknown Input Observer

1. INTRODUCTION

The increasing demands for good performance and higher standards of safety and reliability result in more emphasis on fault diagnosis. Fault detection and isolation (FDI) refers to the task of inferring the occurrence of faults in a process and finding the root causes of the faults. Fault diagnosis based on analytical models is developed for exact and uncertain linear mathematical description of the system, several books are dedicated to these topics: (Chen and Patton, 1999) and (Gertler, 1998). FDI on nonlinear systems remains a challenge due to the problem of discriminating between disturbances and faults through a wide range of operating conditions. Different techniques based on an exact knowledge of the nonlinear system allow to generate residuals insensitive to fault by specific decoupling methods (Alcorta-Garcia and Frank, 1997), (Kinnaert, 1999) or geometric approach (Persis and Isidori, 2001), (Hammouri *et al.*, 2001).

An attractive alternative to nonlinear modeling problem is to use a multi-linear model approach. This approach is successfully used for some nonlinear systems in control field (Ozkan *et al.*, 2003) but rarely for FDI, and consists in partitioning the

operating range of a system into separate regions in order to synthesize a global representation. The reader could refer to (Murray-Smith and Johansen, 1997) for a global review on multiple models strategy, and also for well developed identification method and modeling problems. Multiple Model Adaptive Estimation (MMAE) (Maybeck, 1999) or Interacting Multiple Model (IMM) developed by (Blom and Bar Shalom, 1988) introduce a multi-model approach for FDI, but these techniques are developed around an unique operating point (*OP*). Indeed, these methods consider a particular multi-model approach where each model is dedicated to a specified fault. A polytopic representation is sometimes used in multi-model and also for LTV systems (Angeli *et al.*, 2000) and (Ozkan *et al.*, 2003).

This paper addresses a more general method that could allow to detect both actuator or sensor fault in a nonlinear system. Time occurrence and fault magnitude are supposed to be unknown. Compared to methods that could detect both operating conditions evolution and detect, isolate and estimate faults in nonlinear system as pro-

posed by (Adam-Medina *et al.*, 2003), the paper considers time varying error modeling. Thus, the main contribution of this paper consists in the synthesis of a decoupled residual generator based on a Polytopic Unknown Input Observer (*UIO*) which generates robust residual against error modeling under fault occurrence. Another kind of *UIO* was developed in (Millerioux and Daafouz, 2004) for communication purposes but not for fault diagnosis. Consequently, a polytopic *UIO* is developed to make diagnosis on nonlinear systems. The paper is organized as follows:

Section II deals with the nonlinear system representation under a convex set through a multi-model approach with taking into account error modeling in the state space representation. So, stability of the *UIO* is performed by use of Linear Matrix Inequalities (*LMIs*) for observer gain synthesis. In the last section, the proposed technique is illustrated through a nonlinear CSTR simulation.

2. ROBUST POLYTOPIC UNKNOWN INPUT OBSERVER SYNTHESIS

2.1 Problem statement

Let assume that a nonlinear system can be represented on N operating points (*OP*) by the following state space representation:

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^N \rho_k^i [A_i x_k + B_i u_k + E_i d_k + F_i f_k + \Delta x_i] \\ y_k &= C x_k \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$ represents the state vector, $u \in \mathbb{R}^p$ is the input vector and $y \in \mathbb{R}^m$ is the output vector. $d_k \in \mathbb{R}^q$ and $f_k \in \mathbb{R}^f$ represent respectively the magnitude of modeling errors and faults. $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times p}$ and $C \in \mathbb{R}^{m \times n}$ are invariant matrices defined around the i^{th} operating point, and finally Δx_i represents the considered operating point. The modeling errors and fault distribution matrices are respectively noted as $E_i \in \mathbb{R}^{n \times q}$ and $F_i \in \mathbb{R}^{n \times f}$, both supposed to be full column rank for each operating point. The matrice F_i represents both actuator faults as sensor faults through an augmented state space representation (Park *et al.*, 1994).

As considered in (Murray-Smith and Johansen, 1997), a possible model that would be able to catch the full range of operation is defined from N weighting local models \mathcal{OP}_i by interpolation functions ρ_k^i . These interpolation functions $\rho_k^i(\cdot) \forall i \in [1, 2, \dots, N]$ lie in a convex set $\Phi = \{\rho_k^i \in \mathbb{R}^N, \rho_k = [\rho_k^1 \dots \rho_k^N]^T, \rho_k^i \geq 0 \text{ and } \sum_{i=1}^N \rho_k^i = 1\}$ and could be generated according to a study presented in (Adam-Medina *et al.*, 2003) and (Theilliol *et al.*, 2003), which permit to generate insensitive residual to faults and robust interpolation functions against faults.

In the following and without any reduction, the interpolation functions are supposed to be known and insensitive to faults. Multi-model approach could not provide good results due to the lack of adequation between model and system representation. Indeed, one way to achieve good robustness is to make disturbance de-coupling conditions hold true (in an optimal sense) for all modeling errors distribution matrices E_i . This can be done by using a single optimal disturbance distribution matrix E^* to approximate all of them as in (Rodrigues *et al.*, 2004) and (Chen and Patton, 1999). Consequently, we proposed to consider the following state space representation:

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^N \rho_k^i [A_i x_k + B_i u_k + E^* d_k + F_i f_k + \Delta x_i] \\ y_k &= C x_k \end{aligned} \quad (2)$$

So, in order to make robust diagnosis of complex systems represented by multiple operating points, based on equation 2, a robust polytopic unknown input observer against modeling errors is developed in the next section.

2.2 Polytopic *UIO* definition and robust residual generation

In a general way, the *UIO* allows to vanish undesirable information by decoupling unknown input. In a FDI scheme, the *UIO* can avoid some perturbation or modeling errors in the predictive state.

Definition 1. (Polytopic Unknown Input Observer) A polytopic observer is defined as a polytopic unknown input observer for the system described by (2) without fault ($f_k = 0$), if the estimation error tends asymptotically to zero despite unknown disturbances on the system. \diamond

This polytopic unknown input observer is defined such that:

$$\begin{aligned} z_{k+1} &= \sum_{i=1}^N \rho_k^i [S_i z_k + T B_i u_k + K_i y_k + \Delta z_i] \\ \hat{x}_{k+1} &= z_{k+1} + H^* y_{k+1} \end{aligned}$$

where \hat{x} is the state space estimation of x . The estimation error is equivalent to

$$\begin{aligned} e_{k+1} &= x_{k+1} - \hat{x}_{k+1} = x_{k+1} - (z_{k+1} + H^* y_{k+1}) \\ &= x_{k+1} - H^* (C x_{k+1}) - z_{k+1} = S(\rho) e_k \\ &\quad - [S(\rho) - (I - H^* C) A(\rho) - K^1(\rho) C] x_k \\ &\quad + (I - H^* C) E^* d_k - [\Pi(\rho) - S(\rho) H^*] y_k \\ &\quad - [T - (I - H^* C)] B(\rho) u_k + (I - H^* C) F(\rho) f_k \end{aligned}$$

where, with notation $S(\rho)$ stands for $\sum_{i=1}^N \rho_k^i S_i$, $K(\rho) = K^1(\rho) + \Pi(\rho)$ and $(I - H^* C) \Delta x(\rho) = \Delta z(\rho)$. $S(\rho)$, T , $K(\rho)$ and H^* should be designed so as to ensure the stability and the convergence of the estimation error $e_k = x_k - \hat{x}_k$ without fault on the system ($f_k = 0$). To obtain an exact decoupling, based on an optimal modeling error

distribution matrix E^* , the following conditions should be satisfied:

$$\begin{aligned} S(\rho) &= TA(\rho) - K^1(\rho)C & \Pi(\rho) &= S(\rho)H^* \\ E^* &= H^*CE^* & T &= I - H^*C \end{aligned} \quad (3)$$

The robust polytopic \mathcal{UIC} design is realized when equations (3) hold true and $S(\rho)$ is stable. So, estimation error without fault occurrence, denoted \bar{e}_k , tends asymptotically to zero if all these conditions (3) are satisfied. The justification of the gain design $K^1(\rho)$ is addressed in the next section. The necessary and sufficient conditions for the existence of a polytopic \mathcal{UIC} are:

- (i) $\text{Rank}(CE^*) = \text{Rang}(E^*)$
 - (ii) (TA_i, C) are detectable pairs, $\forall i \in [1, 2, \dots, N]$.
- These conditions are related in linear case (Chen and Patton, 1999).

If condition (i) is true, the synthesis of matrix H^* , which can permit to avoid unknown input effects, is performed by:

$$H^* = E^*(CE^*)^+ \quad (4)$$

It should be noted that the matrix E^* is a constant matrix $\forall i \in [1, 2, \dots, N]$. If conditions (3) hold true, the estimation error e_k and the residual r_k are described as:

$$\begin{aligned} e_{k+1} &= S(\rho)e_k + (I - H^*C)F(\rho)f_k \\ r_k &= Ce_k \end{aligned} \quad (5)$$

Decoupling method allows to avoid disturbances (like modeling errors) by cancelling the term $(I - H^*C)E^*$ in equation (3), whereas faults are represented with the term $(I - H^*C)F(\rho)$ in equation (5). It is important to take into account that fault distribution matrices F_i and modeling error distribution matrix E^* should have different directions. In order to give a specific dynamic to the polytopic observer in comparison with the system dynamic but also to ensure stability on the full operating range, next section is devoted to pole placement with Linear Matrix Inequality (\mathcal{LMI}).

3. GAIN SYNTHESIS BY \mathcal{LMI}

3.1 Introduction on \mathcal{LMI} region

Pole assignment by \mathcal{LMI} is dedicated for convex sets (Oliveira *et al.*, 1999). The main objective is to ensure, at first, polytopic observer stability and in a second time to constraint its poles in a specified region of the complex plane. The observer dynamic is performed by decomposing complex plane in \mathcal{LMI} region.

Definition 2. \mathcal{LMI} region (Chilali and Gahinet, 1996)

A subset \mathcal{D} of the complex plane is called an \mathcal{LMI} region if there exists a symmetric matrix $\alpha = [\alpha_{kl}] \in \mathbb{R}^{m \times m}$ and a matrix $\beta = [\beta_{kl}] \in \mathbb{R}^{m \times m}$, such that

$$\mathcal{D} = z \in \mathcal{C} : f_{\mathcal{D}}(z) < 0 \quad (6)$$

with

$$f_{\mathcal{D}}(z) = \alpha + z\beta + \bar{z}\beta^T = [\alpha_{kl} + z\beta_{kl} + \beta_{lk}\bar{z}]_{1 \leq k, l \leq m}$$

where $f_{\mathcal{D}}(z)$ is called the characteristic function of the \mathcal{LMI} region and $M = [\mu_{kl}]_{1 \leq k, l \leq m}$ means that M is an $m \times m$ matrix (respectively, bloc matrix) with generic entry (respectively bloc) μ_{kl} .

◇

Moreover, \mathcal{LMI} are convex and symmetric with respect to the real axis since for any $z \in \mathcal{D}$

$$f_{\mathcal{D}}(\bar{z}) = \overline{f_{\mathcal{D}}(z)} < 0 \quad (7)$$

Eigenvalues of a real matrix A lie in \mathcal{D} if and only if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$,

$$\begin{aligned} M_{\mathcal{D}}(A, P) &= \alpha_{kl} \otimes P + \beta_{kl} \otimes (AP) + \beta_{lk}^T \otimes (AP)^T < 0 \\ &= [\alpha_{kl}P + \beta_{kl}AP + \beta_{lk}PA^T]_{1 \leq k, l \leq m} \end{aligned} \quad (8)$$

This inequality underlines the fact that \mathcal{D} can be expressed as \mathcal{LMI} region with an unique matrix P for the intersection of all admissible value of $A(\rho)$. Clustering region is difficult to express with classical representations, but the use of function $f_{\mathcal{D}}(z)$ allows to consider a disk defined with a center $(-q, 0)$ and a radius r represented as:

$$f_{\mathcal{D}}(z) = \begin{pmatrix} -r & q + \bar{z} \\ q + z & -r \end{pmatrix} < 0 \quad (9)$$

So, (3.1) is expressed as:

$$\begin{pmatrix} -rP & qP + PA^T \\ qP + AP & -rP \end{pmatrix} < 0 \quad (10)$$

with $P > 0$. The \mathcal{LMI} (10) allows to define a prescribed region of complex plane and ensures a decay rate on the Lyapunov function (Chilali and Gahinet, 1996). Therefore, this circle region puts a lower bound on both decay rate and the damping ratio of the closed-loop response, and thus is very common in practical control design. Then, it is both possible to constraint poles of the observer to a prescribed region and ensure a quadratic stability.

3.2 Poles placement of the robust polytopic \mathcal{UIC} under \mathcal{LMI} constraints

According to (5), the estimation error \bar{e}_k , without fault, is expressed as

$$\begin{aligned} \bar{e}_{k+1} &= S(\rho)\bar{e}_k = (I - H^*C)A(\rho) - K^1(\rho)C\bar{e}_k \\ &= \bar{A}(\rho) - K^1(\rho)C\bar{e}_k \end{aligned} \quad (11)$$

Proposition 1. Let \mathcal{D} , a \mathcal{LMI} region defining a disk with a center $(-q, 0)$, and a radius r with $(q + r) < 1$ for defining pole assignment in the unit circle. The estimation error (11) is called quadratically \mathcal{D} -stabilizable (all the complex poles lie in \mathcal{LMI} region \mathcal{D}) for some observer gains K_i^1 if and only if there exists a positive symmetric matrix $X > 0$ such that $\forall i \in [1, 2, \dots, N]$:

$$\begin{pmatrix} -rX & qX + (\bar{A}_i - K_i^1 C)^T X \\ qX + X(\bar{A}_i - K_i^1 C) & -rX \end{pmatrix} < 0 \quad (12)$$

□

Proof: First, note that (3.1) can be multiplied both on the left and on the right with $P^{-1} = X > 0$. Secondly, by replacing the matrix A by $(\bar{A}_\rho - K_\rho^1 C)$ in (3.1) and by writing it at each vertex (\bar{A}_i, K_i^1, C) , we obtain:

$$\alpha_{kl}X + \beta_{kl}X(\bar{A}_i - K_i^1 C) + \beta_{lk}^T(\bar{A}_i - K_i^1 C)^T X < 0 \quad (13)$$

So, (13) could be expressed as the following \mathcal{LMI} :

$$\begin{pmatrix} -rX & qX + (\bar{A}_i - K_i^1 C)^T X \\ qX + X(\bar{A}_i - K_i^1 C) & -rX \end{pmatrix} < 0 \quad (14)$$

□

Note that if specific parameters as $q = 0, r = 1$ then \mathcal{LMI} in (14) expresses quadratic stability and with a Schür Complement, we find:

$(\bar{A}_i - K_i^1 C)^T X(\bar{A}_i - K_i^1 C) - X < 0$ and $X > 0$ that is the quadratic stability of estimation error. However, inequality (12) is not a convex \mathcal{LMI} due to the fact that the inequality is not linear with X and K_i^1 . A simple change $XK_i^1 = R_i$ (or $K_i^1 = X^{-1}R_i$) generates $\forall i \in [1, 2, \dots, N]$ the following \mathcal{LMI} :

$$\begin{pmatrix} -rX & qX + \bar{A}_i^T X - C^T R_i^T \\ qX + X\bar{A}_i - R_i C & -rX \end{pmatrix} < 0 \quad (15)$$

Remark: By weighting each \mathcal{LMI} defined in (12) by interpolation functions ρ^i defined in the previous section and summing all of them with $\sum_{i=1}^N \rho^i = 1$, (12) becomes $\forall i \in [1, 2, \dots, N]$ equivalent to

$$\begin{pmatrix} -rX & qX + \sum_{i=1}^N \rho_i (\bar{A}_i - K_i^1 C)^T X \\ qX + X \sum_{i=1}^N \rho_i (\bar{A}_i - K_i^1 C) & -rX \end{pmatrix} < 0 \quad (16)$$

with $K_i^1 = X^{-1}R_i$. We should note that using \mathcal{LMI} , observer gain synthesis is made through a polytopic form and estimation error (11) satisfies to the condition of quadratic \mathcal{D} -stability with $X > 0$ for the Lyapunov matrix. This proposed solution allows to ensure a quadratic \mathcal{D} -stability for the estimation error (without fault) and to fix the dynamic of the polytopic \mathcal{UIO} for the considered system. Poles constraints are specified with \mathcal{LMI} defining a prescribed region of the complex plan.

4. CSTR: CONTINUOUS STIRRED TANK REACTION

4.1 System description

The performance of the robust polytopic \mathcal{UIO} is illustrated through a Continuous Stirred Tank Reaction simulation (CSTR). A full description of the CSTR could be found in (Chen *et al.*, 1995). This system is a nonlinear chemical reactor that exhibits many interesting properties like input multiplicity, a gain sign change, asymmetric response and both minimum and non-minimum

phase behavior (Gatzke and Doyle, 1999). The so called van der Vusse reaction, is summarized by the following reaction scheme: $A \rightleftharpoons B$ with $B \rightleftharpoons C$ and $2A \rightleftharpoons D$.

The dynamic behaviour of the reactor is classically described by the following nonlinear differential equations :

$$\begin{cases} \dot{C}_A = \frac{\dot{V}_r}{V_r}(C_{A0} - C_A) - k_1(v)C_A - k_3(v)C_A^2 \\ \dot{C}_B = \frac{\dot{V}_r}{V_r}C_B + k_1(v)C_A - k_2(v)C_B \\ \dot{v} = \frac{\dot{V}_r}{V_r}(v_0 - v) - \frac{1}{\rho C_p} \left(k_1(v)C_A \Delta H_{RAB} + k_2(v)C_B \right. \\ \quad \left. \times \Delta H_{RBC} k_1(v) + k_3(v)C_A^2 \Delta H_{RAD} \right) + \frac{k_w A_r}{\rho C_p V_r} (v - v_k) \\ \dot{v}_k = \frac{1}{m_k C_{PK}} \times ((\dot{Q}_k - v_k) + k_w A_r (v - v_k)) \end{cases} \quad (17)$$

with $k_i(v) = k_{i0} e^{\frac{E_i}{v+273.15}}$.

The model has four states: concentration of A (C_A) with an initial concentration C_{A0} , concentration of B (C_B), temperature v with a fixed value for the feed temperature v_0 and cooling jacket temperature v_k . It is assumed that states are directly measurable except for v_k . The input variables are the flow rate normalized by the reactor volume $\frac{\dot{V}_r}{V_r}$ and the heat removal \dot{Q}_k .

4.2 Operating conditions

A specific condition, reported in the benchmark (Chen *et al.*, 1995), underlines the fact that concentration of B should be taken into the range $[0.8; 1.09] \text{ mol.l}^{-1}$ for manipulated variables $\frac{\dot{V}_r}{V_r}$ lying in $[3h^{-1}; 35h^{-1}]$ and $[-9000kJ.h^{-1}; 0]$ for \dot{Q}_k . The feed temperature v_0 is considered as a fixed value at $105^\circ C$ as proposed in (Chen *et al.*, 1995). In the proposed study, the CSTR is controlled in an open-loop way with multiple desirable concentrations of product B, each of them defining an operating point. Thus, CSTR could produce different concentrations C_B and the system is described around predefined operating points. By taking into account the static characteristic illustrated in figure (1) and in order to exploit the nonlinear system in a wide range, concentrations of B are C_{B1}, C_{B2}, C_{B3} with respectively 0.85, 1.00, 1.09 mol.L^{-1} . It can be noted

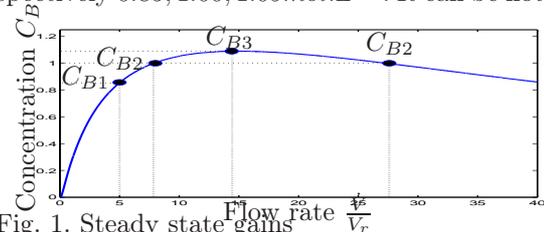


Fig. 1. Steady state gains

that concentration $C_{B2} = 1.00 \text{ mol.L}^{-1}$ could be obtained for two different control values $\frac{\dot{V}_r}{V_r}$: two operating points are associated to this concentration.

Figure (2) shows the dynamic evolution of the measurable states with a flow rate presented in figure (3) that permits to cover the wide range of operating conditions. It could be noticed that

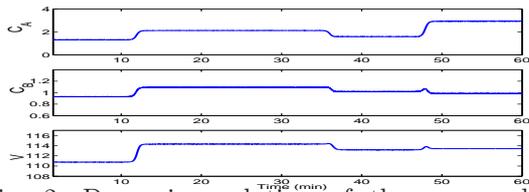


Fig. 2. Dynamic evolution of the measurable states

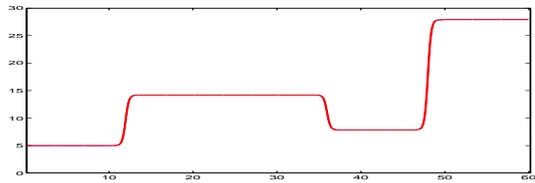


Fig. 3. Flow rate h^{-1} there is non-minimum phase behaviour appearing on concentrations of A and B during a transition.

4.3 Experiments and results

Based on the physical dynamic equation of the system, linear state space models were obtained around four \mathcal{OP} using a first order linearization. These four operating points developed are: $C_{B1} = 0.85 \text{ mol.L}^{-1}$, $C_{B3} = 1.09 \text{ mol.L}^{-1}$ and $C_{B2} = 1.00 \text{ mol.L}^{-1}$ with $\frac{\dot{V}_{21}}{V_r} = 7.8570 \text{ h}^{-1}$ and $C_{B2} = 1.00 \text{ mol.L}^{-1}$ with $\frac{\dot{V}_{22}}{V_r} = 27.90 \text{ h}^{-1}$. The nonlinear model has been discretized through a Tustin method in order to provide discrete linear models represented in equation (2). Disturbance matrices of modeling error E_i are directly obtained by a second order linearization and an unique matrix E^* is computed through an optimization technique based on Singular value Decomposition as proposed in (Chen and Patton, 1999). An unique actuator fault on the manipulated variables $\frac{\dot{V}_r}{V_r}$ is considered, fault matrices F_i are related with the column of B_i .

According to the four linear models and under the assumptions that the interpolation functions ρ_i are insensitive to faults, figure (4) shows the sequence of active models during all the experiment.

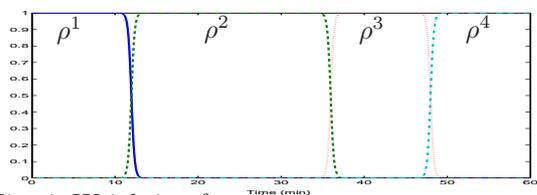


Fig. 4. Weighting functions

Based on the unique disturbance matrix $E^* \in \mathbb{R}^{4 \times 2}$, the polytopic observer gain synthesis is performed by pole placement developed in the previous paragraph (3) by a \mathcal{LMI} region defined with $r = 0.15$ and $q = 0.2$. The different matrices are given in the appendix.

In a first experiment, the polytopic \mathcal{UIO} is evaluated without fault with the control input il-

lustrated in figure (5). This residual norm is presented in figure 6. It can be observed that this norm is different from zero around operating points because polytopic \mathcal{UIO} is synthesized in order to obtain an exact decoupling against E^* , but this matrix is just an optimal approximation of modeling errors. During transitions, uncertainties are minimized and the norm is very closed to zero. The magnitude of the residual norm is almost constant through all operating points and represents the modeling errors due to approximations of modeling error representation. We should underline that an adaptive threshold could be taken into account the specificities of the polytopic \mathcal{UIO} synthesis in order to generate an efficient residual evaluation.

In a second experiment, an actuator fault is introduced as presented in figure (5). The faulty control input is sent to the system and the computed control input is exploited by the polytopic \mathcal{UIO} . This experiment simulates an actuator fault that would be stuck between \mathcal{OP}_1 and \mathcal{OP}_2 , then, a partial lost of an actuator fault is simulated during transitions of \mathcal{OP}_2 and \mathcal{OP}_3 . At the last change of operating point \mathcal{OP}_3 - \mathcal{OP}_4 , the actuator fault disappears.

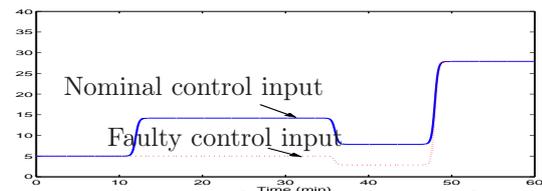


Fig. 5. Faulty control input and nominal control input

According to the synthesis of the robust polytopic \mathcal{UIO} , the residual norm is sensitive to the fault occurrence. As illustrated in figure (6), the residual norm of the polytopic \mathcal{UIO} is sensitive to actuator fault and robust to modeling errors. The actuator fault is detected easily due to the magnitude of the residual norm.

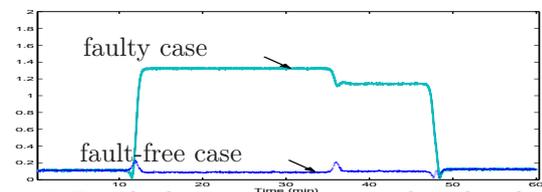


Fig. 6. Residual norms evolution with and without fault

5. CONCLUSION

In this paper, a new residual generator robust against modeling errors was developed. This residual generation is dedicated to fault diagnosis on nonlinear systems based on a multi-model approach. The modelling error approximation over a set of \mathcal{OP} allows an unique decoupling for all

the system range even during transitions. Our contribution is underlined by a polytopic *UIO* that generates decoupled residuals from modeling errors. The observer gain synthesis is realized by *LMI* so as to ensure both stability and pole assignment of the polytopic *UIO*. Fault diagnosis on nonlinear systems is made with both modeling error and actuator fault.

The illustration is made through a CSTR simulation with multi operating conditions. Future works will deal with analysis and adaptive threshold over a wide range of operating points.

APPENDIX

$$E^* = \begin{bmatrix} -0.8142 & 1.3831 \\ -0.0032 & -1.6090 \\ 11.7210 & 0.0956 \\ 0 & 0 \end{bmatrix} \quad H^* = \begin{bmatrix} 0.4289 & -0.4933 & -0.0398 & 0 \\ -0.4933 & 0.5739 & -0.0344 & 0 \\ -0.0398 & -0.0344 & 0.9972 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_1^1 = \begin{bmatrix} 0.4300 & 0.5161 & 0.0343 & 0 \\ 0.5439 & 0.2455 & 0.0296 & 0 \\ 0.0439 & 0.0359 & -0.1976 & 0 \\ 3.7878 & 3.2781 & 0.7970 & 0 \end{bmatrix} \quad K_2^1 = \begin{bmatrix} 0.3878 & 0.4710 & 0.0292 & 0 \\ 0.5079 & 0.2070 & 0.0253 & 0 \\ 0.0414 & 0.0332 & -0.1979 & 0 \\ 3.6401 & 3.2116 & 0.7846 & 0 \end{bmatrix}$$

$$K_3^1 = \begin{bmatrix} 0.4108 & 0.5010 & 0.0324 & 0 \\ 0.5271 & 0.2322 & 0.0279 & 0 \\ 0.0427 & 0.0351 & -0.1977 & 0 \\ 3.8198 & 3.2521 & 0.7925 & 0 \end{bmatrix} \quad K_4^1 = \begin{bmatrix} 0.3033 & 0.4148 & 0.0252 & 0 \\ 0.4342 & 0.1577 & 0.0217 & 0 \\ 0.0346 & 0.0285 & -0.1983 & 0 \\ 3.8482 & 3.1264 & 0.7560 & 0 \end{bmatrix}$$

$$X = 1000 \times \begin{bmatrix} 1.5003 & 0.2468 & 0.0199 & -0.0601 \\ 0.2468 & 1.4257 & 0.0171 & -0.0517 \\ 0.0199 & 0.0171 & 1.2148 & -0.0042 \\ -0.0601 & -0.0517 & -0.0042 & 0.0049 \end{bmatrix}$$

Matrices R_1, R_2, R_3, R_4 are not reported here because it could be done by the relation $K_i^1 = X^{-1}R_i$.

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