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# A Superstabilizing $\log(n)$ -Approximation Algorithm for Dynamic Steiner Trees

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## Abstract

In this paper we design and prove correct a fully dynamic distributed algorithm for maintaining an approximate Steiner tree that connects via a minimum-weight spanning tree a subset of nodes of a network (referred as Steiner members or Steiner group). Steiner trees are good candidates to efficiently implement communication primitives such as publish/subscribe or multicast, essential building blocks for the new emergent networks (e.g. P2P, sensor or adhoc networks).

The cost of the solution returned by our algorithm is at most  $\log |S|$  times the cost of an optimal solution, where  $S$  is the group of members. Our algorithm improves over existing solutions in several ways. First, it tolerates the dynamism of both the group members and the network. Next, our algorithm is self-stabilizing, that is, it copes with nodes memory corruption. Last but not least, our algorithm is *superstabilizing*. That is, while converging to a correct configuration (i.e., a Steiner tree) after a modification of the network, it keeps offering the Steiner tree service during the stabilization time to all members that have not been affected by this modification.

## 1 Introduction

The design of efficient distributed applications in the newly distributed emergent networks such as MANETs, P2P or sensor networks raises various challenges ranging from models to fundamental services. These networks face frequent churn (nodes and links creation or destruction) and various privacy and security attacks that cannot be easily encapsulated in the existing distributed models. Therefore, new models and new algorithms have to be designed.

Communication services are the building blocks for any distributed system and they have received a particular attention in the lately years. Their efficiency greatly depends on the performances of the underlying routing overlay. These overlays should be optimized to reduce the network overload. Moreover, in order to avoid security and privacy attacks the number of network nodes that are used only for the overlay connectivity have to be minimized. Additionally, the overlays have to offer some quality of services while nodes or links fail.

The work in designing optimized communication overlays for the new emergent networks has been conducted in both structured (DHT-based) and un-structured networks. Communication primitives using DHT-based schemes such as Pastry, CAN or Chord [CDHR03] build upon a global naming scheme based on hashing nodes identifiers. These schemes are optimized to efficiently route in the virtual name space however they have weak energy performances in MANETs or

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sensor networks where the maintenance of long links reduces the network perennial. Therefore, alternative strategies [KvS07], mostly based on gossip techniques, have been recently considered. These schemes, highly efficient when nodes have no information on the content and the topology of the system, offer only probabilistic guarantees on the message delivery.

In this paper we are interested in the study of overlays targeted to efficiently connect a group of nodes that are not necessarily located in the same geographical area (e.g. sensors that should communicate their sensed data to servers located outside the deployment area, P2P nodes that share the same interest and are located in different countries, robots that should participate to the same task but need to remotely coordinate). Steiner trees are good candidates to implement the above mentioned requirements since the problem have been designed for efficiently connect a subset of the network nodes, referred as Steiner members.

**The Steiner tree problem.** The Steiner tree problem can be informally expressed as follows: given a weighted graph in which a subset  $S$  of nodes is identified, find a minimum-weight tree spanning  $S$ . The Steiner tree problem is one of the most important combinatorial optimization problems and finding a Steiner tree is NP-hard.

A survey on different heuristics for constructing Steiner trees with different competitiveness levels can be found in [Bau96, Win87]. In our work we are interested in dynamic variants of Steiner trees first addressed in [IW91] in a centralized online setting. They propose a  $\log |S|$ -approximation algorithm for this problem that copes only with Steiner member arrivals. This algorithm can be implemented in a decentralized environment (see [GRG05]).

Our work considers the fully dynamic version of the problem where both Steiner members and ordinary nodes can join or leave the system. Additionally, our work aims at providing a superstabilizing approximation of a Steiner tree. The property of self-stabilization [Dij74, Dol00] enables a distributed algorithm to recover from a transient fault regardless of its initial state. The superstabilization [DH95] is an extension of the self-stabilization property for dynamic settings. The idea is to provide some minimal guarantees while the system repairs after a topology change.

To our knowledge there are only two self-stabilizing approximations of Steiner trees [KK02a, KK02b]. Both works assume the shared memory model and an unfair centralized scheduler. In [KK02a] the authors propose a self-stabilizing algorithm based on a pruned minimum spanning tree. The computed solution has a competitiveness of  $|V| - |S| + 1$  where  $V$  is the set of nodes in the network. In [KK02b], the authors proposed a four-layered algorithm that builds upon the techniques proposed in [WWW86] in order to obtain a 2 approximation.

The above cited algorithms work only for static networks.

**Our results.** We describe a super-stabilizing algorithm for the Steiner tree problem. This algorithm has the following properties:

- First, it is distributed, i.e., completely decentralized. That is, nodes locally self-organize in a Steiner tree. The cost of the constructed Steiner tree is at most  $\log |S|$  times the cost of an optimal solution, where  $S$  is the Steiner group.
- Second, our algorithm is specially designed to cope with user dynamism. In other words, our solution withstand when nodes (or links) join and leave the system.
- Third, our algorithm includes self-stabilization policies. Starting from an arbitrary state (nodes local memory corruption, counter program corruption, or erroneous messages in the

network buffers), our algorithm is guaranteed to converge to a tree spanning the Steiner members.

- Fourth, our algorithm is *superstabilizing*. That is, while a topology change occurs, i.e., during the restabilization period, the algorithm offers the guarantee that only the subtree connected through the crashed node/edge is reconstructed.

	Approximation ratio	Self-Stabilizing	Superstabilizing
Chen et al. [CHK93]	2	No	No
Kamei and Kakugawa [KK02a]	$ V  +  S  - 1$	Yes	No
Kamei and Kakugawa [KK02b]	2	Yes	No
This paper	$O(\log  S )$	Yes	Yes

Table 1: Distributed (deterministic) algorithms for the Steiner tree problem.

Table 1 summarizes our contribution compared to previous works. Hence, our algorithm is the first superstabilizing algorithm for the Steiner tree problem. Its approximation ratio is logarithmic, which is not as good as the 2-approximation algorithm by Kamei and Kakugawa in [KK02b]. However, this latter algorithm is not superstabilizing. Designing a superstabilizing 2-approximation algorithm for the Steiner tree problem is a challenge. Indeed, all known 2-approximation distributed algorithms (self-stabilizing or not) for the Steiner tree problem use a minimum spanning tree (MST), and the design of a superstabilizing algorithm for MST is a challenge by itself.

## 2 Model and notations

We consider an undirected weighted connected network  $G = (V, E, w)$  where  $V$  is the set of nodes,  $E$  is the set of edges and  $w : E \rightarrow \mathbb{R}$  is a cost function. Nodes represent processors and edges represent bidirectional communication links. Each node in the network has a unique identifier.  $S \subseteq V$  defines the set of members we have to connect. For any pair of nodes  $u, v \in V$ , we note  $d(u, v)$  the distance of the shortest path  $P(u, v)$  between  $u$  and  $v$  in  $G$  (i.e.  $d(u, v) = \sum_{e \in P(u, v)} w(e)$ ). For a node  $v \in V$ , we denote the set of its neighbors  $\mathcal{N}(v) = \{u, (u, v) \in E\}$ . A Steiner tree,  $T$  in  $G$  is a connected acyclic sub-graph of  $G$  such that  $T = (V_T, E_T)$ ,  $S \subseteq V_T \subseteq V$  and  $E_T \subset E$ . We denote by  $W(T)$  the cost of a tree  $T$ , i.e.  $W(T) = \sum_{e \in T} w(e)$ .

We consider an asynchronous communication message passing model with FIFO channels (on each link messages are delivered in the same order as they have been sent).

A *local state* of a node is the value of the local variables of the node and the state of its program counter. We consider a fine-grained communication atomicity model [BK07, Dol00]. That is, each node maintains a local copy of the variables of its neighbors. These variables are refreshed via special messages (denoted in the sequel **InfoMsg**) exchanged periodically by neighboring nodes. A *configuration* of the system is the cross product of the local states of all nodes in the system plus the content of the communication links. The transition from a configuration to the next one is produced by the execution of an atomic step at a node. An *atomic step* at node  $p$  is an internal computation based on the current value of  $p$ 's local variables and a single communication operation (send/receive) at  $p$ . An *execution* of the system is an infinite sequence of configurations,

$e = (c_0, c_1, \dots, c_i, \dots)$ , where each configuration  $c_{i+1}$  follows from  $c_i$  by the execution of a single atomic step.

In the sequel we consider the system can start in any configuration. That is, the local state of a node can be corrupted. Note that we don't make any assumption on the bound of corrupted nodes. In the worst case all nodes in the system may start in a corrupted configuration. In order to tackle these faults we use *self-stabilization* techniques.

Given  $\mathcal{L}_A$  a non-empty *legitimacy predicate*<sup>1</sup> an algorithm  $\mathcal{A}$  is *self-stabilizing* iff the following two conditions hold: (i) Every computation of  $\mathcal{A}$  starting from a configuration satisfying  $\mathcal{L}_A$  preserves  $\mathcal{L}_A$  (*closure*). (ii) Every computation of  $\mathcal{A}$  starting from an arbitrary configuration contains a configuration that satisfies  $\mathcal{L}_A$  (*convergence*).

A *legitimate configuration* for the Steiner Tree is a configuration that provides an instance of a tree  $T$  spanning  $S$ . Additionally, we expect a competitiveness of  $\log(z)$ , i.e.  $\frac{W(T)}{W(T^*)} \leq \log(z)$ , with  $|S| = z$  and  $T^*$  an optimal Steiner tree.

In the following we propose a self-stabilizing Steiner tree algorithm. We expect our algorithm to be also *superstabilizing* [DH95]. That is, given a class of topology changes  $\Lambda$  and a passage predicate, an algorithm is superstabilizing with respect to  $\Lambda$  iff it is self-stabilizing, and for every execution<sup>2</sup>  $e$  beginning at a legitimate state and containing a single topology change event of type  $\Lambda$ , the passage predicate holds for every configuration in  $e$ .

In the following we propose a self-stabilizing Steiner tree algorithm and extend it to a superstabilizing Steiner tree algorithm that copes with the Steiner members and tree edges removal. During the tree restabilization the algorithm verifies a passage predicate detailed below. Then, we discuss the extension of the algorithm to fully dynamic settings (the add/removal of members, nodes or links join/leave). This second extension offers no guarantees during the restabilization period.

### 3 The Superstabilizing Algorithm S3T

The section describes a superstabilizing algorithm for the Steiner tree problem, called S3T. It implements the technique proposed by Imase and Waxman [IW91], in a stabilizing manner. That is, each Steiner member is connected to the existing Steiner tree via a shortest path. Note that in a stabilizing setting the initial configuration may be arbitrary hence nodes have to perpetually verify the coherency of their state: a Steiner member has to be connected to the Steiner tree via a shortest path while a not Steiner node which does not serve for the tree connectivity has to be recognized as disconnected. In our implementation we assume a special node that acts as the root of the Steiner tree. To this end, we assume an underlying overlay that elects a leader within the Steiner group. That is, we assume a leader oracle that returns to every node in the system its status: leader or follower. The leader of the system is a node in the Steiner group. Note that the implementation of a leader oracle is beyond the scope of the current work. Several implementations fault-tolerant and self-stabilizing can be found in [DGDF07]. Recently, algorithms that implement leader oracles in dynamic settings are proposed in [PB08] for example.

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<sup>1</sup>A legitimacy predicate is defined over the configurations of a system and is an indicator of its correct behavior.

<sup>2</sup>[DH95] use the notion of trajectory which is the execution of a system enriched with dynamic actions.

$\text{CRoot}(v) \equiv \text{dist}_v = 0 \wedge \text{parent}_v = \text{ID}_v \wedge \text{need}_v \wedge \text{connected}_v \wedge \text{connect\_pt}_v \wedge \text{level}_v = 0$ $\text{CParent}(v) \equiv (\exists u \in N(v), \text{parent}_v = \text{ID}_u) \wedge (\text{level}_v = \text{level}_{\text{parent}_v} + 1) \wedge (\nexists u \in N(v), \text{parent}_u = \text{ID}_v \wedge \text{level}_u \neq \text{level}_v + 1)$ $\text{Asked\_Connection}(v) \equiv (\exists u \in N(v), \text{parent}_u = \text{ID}_v \wedge \text{need}_u)$ $\text{Better\_Path}(v) \equiv (\neg \text{connected}_v \wedge \text{dist}_v \neq \text{distNotConnect}(v)) \vee (\text{connected}_v \wedge \text{dist}_v \neq \text{distConnect}(v))$ $\text{Connect\_Pt\_Stab}(v) \equiv (\text{member}_v \wedge \text{connect\_pt}_v) \vee (\neg \text{member}_v \wedge  \{u : u \in N(v) \wedge \text{parent}_u = \text{ID}_v \wedge \text{connected}_u\}  > 1)$ $\text{Connect\_Stab}(v) \equiv \text{need}_v \wedge \text{connected}_{\text{parent}_v} \wedge [\text{member}_v \vee (\neg \text{member}_v \wedge \text{Asked\_Connection}(v))]$ $\text{distNotConnect}(v) \equiv \min(\min\{w(u, v) : u \in N(v) \wedge \text{connected}_u\}, \text{dist}_u + w(u, v) : u \in N(v) \wedge \neg \text{connected}_u\})$ $\text{parentNotConnect}(v) \equiv \arg(\text{distNotConnect}(v))$ $\text{distConnect}(v) \equiv \min(\min\{w(u, v) : u \in N(v) \wedge \text{connected}_u \wedge \text{connect\_pt}_u\}, \min\{\text{dist}_u + w(u, v) : u \in N(v) \wedge [\neg \text{connected}_u \vee (\text{connected}_u \wedge \neg \text{connect\_pt}_u)]\})$ $\text{parentConnect}(v) \equiv \arg(\text{distConnect}(v))$
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Figure 1: Predicates used by the algorithm.

### 3.1 Detailed description

#### 3.1.1 Variables and Predicates

For any node  $v \in V(G)$ ,  $N(v)$  is the neighbors set of  $v$  in the network  $G$  (our algorithm is built upon an underlying self-stabilizing protocol that regularly updates the neighbor set of every node). We denote by  $\text{ID}_v \in \mathbb{N}$  the unique network identifier of  $v$ . Every node  $v$  maintains seven variables for constructing and maintaining a Steiner tree. Three of them are integers, and the others are booleans.

- $\text{parent}_v$ : ID of the parent of node  $v$  in the current tree;
- $\text{level}_v$ : number of nodes on the path between the root and  $v$  in spanning tree;
- $\text{dist}_v$ : the shortest distance to a node already connected to the current tree;
- $\text{member}_v$ : *true* if  $v \in S \subseteq V$ , *false* otherwise (this is not a variable wrote by the algorithm but only read);
- $\text{need}_v$ : *true* if  $v \in S \subseteq V$  or  $v$  has a descendant which is a member, *false* otherwise;
- $\text{connected}_v$ : *true* if  $v$  is in the current tree, *false* otherwise;
- $\text{connect\_pt}_v$ : *true* if  $v$  is a member or  $v$  has more than one children in the current tree, *false* otherwise.

#### 3.1.2 Description of the algorithm

Every node  $v \in V$  sends periodically its local variables to each of its neighbors using `InfoMsg` messages. Upon the reception of this message a neighbor updates the local copy of its neighbor variables. The description of a `InfoMsg` message is as follows:

$\text{InfoMsg}_v[u] = \langle \text{InfoMsg}, \text{parent}_v, \text{level}_v, \text{dist}_v, \text{need}_v, \text{connected}_v, \text{connect\_pt}_v \rangle$ .

Our algorithm is a four phase computation: (1) first nodes update their distance to the existing Steiner tree, then (2) nodes request connection (if they are members or they received a connection demand), then (3) they establish the connection, and finally (4) they update the state of the current Steiner tree. These phases have to be performed in the given order. That is, a node cannot initiate a request for connection for example if it has not yet updated its distance.

Note that if a node detects a distance modification in its neighborhood, it can change its connection to the current tree. Therefore a node before computing any other action must update its distance to the current tree.

Every node in the network, maintains a parent link. The parent of a node is one of its neighbors having the shortest distance to the current tree. Note that erroneous initial configurations may create cycles in the parent link. To break these cycles, we use the notion of tree level, defined by the variable `level`: the root has the level zero and each node has the level equal to its parent level plus one.

When a member tries to connect to the tree, it sets its variable `need` to `true`. When a node in the current tree receives a demand for connection, an acknowledgment is sent back along the requesting path enabling every node along this path to set a variable `connected` to `true`. Nodes with `connected` set `true` are called “connected nodes”.

Whenever a node detects an incoherency in its neighborhood it disconnects from the current tree.

In order to give a  $\log(z)$ -approximate Steiner tree, we introduce a variable `connect_pt`. This variable signals if a node is a connection point or not. A connection point is a connected node which is a member or has more than one connected child.

**Algorithm:** Upon the reception of a `InfoMsg` nodes correct their local state via the rules explained below then broadcast their new local state in their local neighborhood.

**Root:** In a coherent state the root has a distance and a level equal to zero, variables `need` and `connected` are `true` since the root is always connected (it always belongs to the Steiner tree). Variable `connect_pt` is `true` because the root is a member so a connection point. Whenever the state of the root is incoherent the Rule  $\mathcal{RR}$  below is enabled.

$\mathcal{RR}$ : (**Root reinitialization**)

**If** `Is_Root(v)  $\wedge$   $\neg$ CRoot(v)` **then**  
`distv := 0; parentv := IDv; needv := true; connectedv := true;`  
`connect_ptv := true; levelv := 0;`

**Distance update:** Rule  $\mathcal{DR}_1$  enables to a not connected node to compute its shortest path distance to the Steiner tree as follows: Take the minimum between the edge weights with connected neighbors and the distances with not connected neighbors. If a not connected node detects it has a better shortest path (see Predicate `Better_Path`) then it updates its distance (using Predicates `distNotConnect` and `distConnect`) and changes its other variables accordingly.

The same rule is used to reinitiate the state of a node if it observes that its parent is no more in its neighborhood.

Similarly, Rule  $\mathcal{DR}_2$  enables to a connected node to compute its shortest path distance. In order to execute this rule a connected node must have a stabilized connection. The distance is computed as for a not connected node but a connected node compares this distance with its local distance towards its connection point and takes the minimum.

**DR<sub>1</sub>: (Distance stabilization for not connected nodes)**

**If**  $\neg \text{Is\_Root}(v) \wedge [(\neg \text{connected}_v \wedge \text{Better\_Path}(v)) \vee \neg \text{CParent}(v)]$  **then**  
 $\text{dist}_v := \text{distNotConnect}(v)$ ;  $\text{parent}_v := \text{parentNotConnect}(v)$ ;  
 $\text{connected}_v := \text{false}$ ;  $\text{connect\_pt}_v := \text{false}$ ;  $\text{level}_v := \text{Level}_{\text{parent}_v} + 1$ ;

**DR<sub>2</sub>: (Distance stabilization for connected nodes)**

**If**  $\neg \text{Is\_Root}(v) \wedge \text{connected}_v \wedge \text{Connect\_Stab}(v) \wedge \text{Better\_Path}(v) \wedge \text{CParent}(v) \wedge \text{Connect\_Pt\_Stab}(v)$  **then**  
 $\text{dist}_v := \text{distConnect}(v)$ ;  $\text{parent}_v := \text{parentConnect}(v)$ ;  
 $\text{level}_v := \text{Level}_{\text{parent}_v} + 1$ ;

**Request to join the tree:** Variable *need* is used by a not connected node to ask to its parent a connection to the current Steiner tree. Since a member must be connected to the Steiner tree, each member sets this variable to *true* using Rule  $\mathcal{NR}_1$ . A not member and not connected node which detects that a child wants to be connected (see Predicate *Asked\_Connection*) changes its variable *need* to *true*. This connection request is forwarded in the spanning tree until a not connected node neighbor of a connected node is reached.

A not connected node sets its variable *need* to *false* using Rule  $\mathcal{NR}_2$  if it is not a member and it has no child requesting a connection.

**$\mathcal{NR}_1$ : (Nodes which need to be connected)**

**If**  $\neg \text{Is\_Root}(v) \wedge \neg \text{need}_v \wedge \neg \text{connected}_v \wedge \neg \text{Better\_Path}(v) \wedge \text{CParent}(v) \wedge [\text{member}_v \vee (\neg \text{member}_v \wedge \text{Asked\_Connection}(v))]$   
**then**  $\text{need}_v := \text{true}$ ;

**$\mathcal{NR}_2$ : (Nodes which need not to be connected)**

**If**  $\neg \text{Is\_Root}(v) \wedge \neg \text{connected}_v \wedge \text{need}_v \wedge \neg \text{member}_v \wedge \neg \text{Asked\_Connection}(v) \wedge \neg \text{Better\_Path}(v) \wedge \text{CParent}(v)$   
**then**  $\text{need}_v := \text{false}$ ;

**Member connection:** When a not connected node neighbor of a connected node (i.e. which belongs to the Steiner tree) detects a connection request from a child (i.e. Predicate *Asked\_Connection* is *true*), an acknowledgment is sent backward using variable *connected* along the request path. Therefore every not connected node on this path uses Rule  $\mathcal{CR}_1$  and sets *connected* to *true* until the member that asked the connection is connected. Only a node that has (1) no better path, (2) its variable *need* = *true* and (3) a connected parent can use Rule  $\mathcal{CR}_1$ .

A connected node becomes not connected if its connection path is no more stabilized (i.e. Predicate *Connect\_Stab* is false). Therefore, it sets *connected* to *false* using Rule  $\mathcal{CR}_2$ .

The parent distance is used for the disconnection of a subtree whenever a fault occurs in the network. If a fault occurs (parent distance is infinity), a connected node in the subtree below a faulty node or edge in the spanning tree must be disconnected using Rule  $\mathcal{CR}_3$ . So the node sets *connected* to false and *dist* to infinity and waits until all its subtree is disconnected (i.e. it has no connected child).

**$\mathcal{CR}_1$ : (Nodes which must be connected)**

**If**  $\neg \text{Is\_Root}(v) \wedge \neg \text{connected}_v \wedge \text{Connect\_Stab}(v) \wedge \neg \text{Better\_Path}(v) \wedge \text{CParent}(v)$   
**then**  $\text{connected}_v := \text{true}$ ;

**$\mathcal{CR}_2$ : (Nodes which must not be connected)**

**If**  $\neg \text{Is\_Root}(v) \wedge \text{connected}_v \wedge \neg \text{Connect\_Stab}(v) \wedge \text{CParent}(v) \wedge \text{dist}_{\text{parent}_v} \neq \infty$  **then**  $\text{connected}_v := \text{false}$ ;



$\mathcal{CR}_3$ : (Consequence of a deletion)

If  $\neg \text{ls\_Root}(v) \wedge \text{connected}_v \wedge \neg \text{Connect\_Stab}(v) \wedge \text{CParent}(v) \wedge \text{dist}_{\text{parent}_v} = \infty$  then  $\text{connected}_v := \text{false}; \text{dist}_v := \infty; \text{connect\_pt}_v := \text{false};$   
 send  $\text{InfoMsg}_v$  to all  $u \in N(v)$  and wait until  $(\nexists u \in N(v), \text{parent}_u = \text{ID}_v \wedge \text{connected}_u)$

**Update the Steiner tree:** Since we use shortest paths to connect members to the existing Steiner tree, we must maintain distances from members to connection points. A connection point is a connected member or a connected node with more than one connected children, i.e. the root of the branch connecting a member. Every connected node updates its distance if it has a better path. So thanks to connection points and distance computation, we maintain a shortest path between a member and the Steiner tree in order to respect the construction in [IW91]. Rule  $\mathcal{TR}$  is used by a connected node to change its variable  $\text{connect\_pt}$  and to become or not a connection point. This rule is executed only if the connected node has a stabilized connection path (i.e. Predicate  $\text{Connect\_Stab}$  is *true*).

$\mathcal{TR}$ : (Connected path stabilization)

If  $\neg \text{ls\_Root}(v) \wedge \text{connected}_v \wedge \text{Connect\_Stab}(v) \wedge \text{CParent}(v) \wedge \neg \text{Connect\_Pt\_Stab}(v)$   
 then If  $\text{member}_v$  then  $\text{connect\_pt}_v := \text{true};$   
 Else  $\text{connect\_pt}_v := |\{u : u \in N(v) \wedge \text{parent}_u = \text{ID}_v \wedge \text{connected}_u\}| > 1;$

## 4 Correctness and proof in Static setting

**Definition 1 (Legitimate state of DST)** A configuration of algorithm is legitimate iff each process  $v \in V$  satisfies the following conditions:

1. a Steiner tree  $T$  spanning the set of members  $S$  is constructed;
2. a shortest path connects each member  $v \in S$  to the existing tree.

**Lemma 1** Eventually the node's parent relation constructs a rooted spanning tree in the network.

**Proof.** Function  $\text{ls\_Root}(v)$  is a perfect oracle which returns true if  $v$  is the root of the tree and false otherwise. So we assume that there is a time after which only one root exists in the network. Moreover Rule  $\mathcal{RR}$  is only used by the root to correct its corrupted variables.

Since there is only one root in the network, to have a spanning tree we must show that each node has one parent and there is no cycle. First note that each node  $v$  could have at each time only one parent in its neighborhood (see predicate  $\text{CParent}(v)$ ) designed by variable  $\text{parent}_v$ , only root has its parent equal to itself. Each node maintains its level stored in variable  $\text{level}_v$  which is updated by Rules  $\mathcal{RR}$ ,  $\mathcal{DR}_1$  and  $\mathcal{DR}_2$ . The level of each node is equal to the level of its parent plus one, except for the root which has a level at zero (see Rule  $\mathcal{RR}$ ). Suppose there is a cycle in the node's parent relation. This implies that there is a time after which we have a sequence of nodes with a growing sequence of levels. But there is at least one node  $x$  with a smaller level than its parent  $y$  in the cycle. That is, for  $x$  we have  $\text{level}_x \neq \text{level}_y + 1$  and for  $y$  we have  $\text{parent}_x = \text{ID}_y \wedge \text{level}_x \neq \text{level}_y + 1$ . So predicate  $\text{CParent}$  is false for  $x$  and  $y$ , thus  $x$  and  $y$  can execute Rule  $\mathcal{DR}_1$  to reset their variables and break the cycle. Therefore, there is a time after which no cycle exists in the structure described by the node's parent relation. Since there is only

one root in the network (i.e.  $\text{level}_v = 0$  and  $\text{parent}_v = ID_v$ ) and there is no cycle, thus the node's parent relation describe one tree spanning the network.  $\square$

**Lemma 2** *Eventually each non-connected node knows its distance to the current tree.*

**Proof.** A node  $v$  is connected iff  $\text{connected}_v = \text{true}$ . There is at least one connected node because the root is always connected (see Rule  $\mathcal{RR}$ ), otherwise there is a time where the root corrects its variables using Rule  $\mathcal{RR}$ . According to Lemma 1, a tree spanning the network is constructed. Let  $x$  be a non-connected node,  $d_x$  the distance of the shortest path from  $x$  to any connected node and  $y$  the neighbor on this shortest path. Suppose  $\text{dist}_x > d_x$ , thus it exists a time after which a neighbor offers a better path and  $x$  can execute Rule  $\mathcal{DR}_1$  because predicate  $\text{Better\_Path}(x)$  is true. So  $x$  corrects  $\text{dist}_x$  as the minimum distance in its neighborhood (see function  $\text{distNotConnect}(x)$ ). Therefore there is a time after which  $\text{dist}_x = d_x$ . Moreover, at each time  $x$  executes Rule  $\mathcal{DR}_1$  the variable  $\text{parent}_x$  is modified respectively to variable  $\text{dist}_x$  (see function  $\text{parentNotConnect}(x)$ ) and thus  $\text{parent}_x$  stores the neighbor of  $x$  which offers to  $x$  the shortest path to any connected node. Therefore, there is a time after which when we have  $\text{dist}_x = d_x$  then  $\text{parent}_x = y$ .  $\square$

**Lemma 3** *Eventually each Steiner member is linked to root via a connected path.*

**Proof.** A node  $v$  is connected iff  $\text{connected}_v = \text{true}$ . There is at least one connected node because the root is always connected (see Rule  $\mathcal{RR}$ ), otherwise there is a time where the root corrects its variables using Rule  $\mathcal{RR}$ . Moreover, according to lemma 1, there is only one root and a rooted tree spanning the network is constructed. Thus it exists a path between each member and the root.

To prove the lemma, we first show that for each node  $v$  on the path connecting a member we have  $\text{need}_v = \text{true}$ .

Each node  $v$  (except the root) can change the value of its variable  $\text{need}_v$  or  $\text{connected}_v$  to true respectively with Rule  $\mathcal{NR}_1$  and  $\mathcal{NR}_2$  only when  $v$  has no neighbor with a lower distance than its parent (i.e.  $v$  has no better path so  $\mathcal{DR}_1$  and  $\mathcal{DR}_2$  are not executable). Otherwise  $\text{Better\_Path}(v)$  returns true and Rules  $\mathcal{DR}_1$  or  $\mathcal{DR}_2$  are uppermost used to correct  $\text{dist}_v$  and  $\text{parent}_v$ . So we suppose that  $\text{Better\_Path}(v)$  returns false.

Note that for any member  $v$  we must have  $\text{need}_v = \text{true}$  otherwise  $v$  executes Rule  $\mathcal{NR}_1$  to correct  $\text{need}_v$ . Since there is a path from each member  $v$  to the root, the parent  $u$  of a member will execute Rule  $\mathcal{NR}_1$  because according to procedure  $\text{Asked\_Connection}(u)$ ,  $u$  has at least a child  $v$  s.t.  $\text{need}_v = \text{true}$ . Thus  $u$  changes the value of its variable  $\text{need}_v$  if necessary. Therefore one can show by induction using the same scheme that for each node  $v$  on the path between a member and the root we have  $\text{need}_v = \text{true}$ .

Each node  $v$  (except the root) with  $\text{connected}_v = \text{false}$  can correct its variable  $\text{connected}_v$  only when Rule  $\mathcal{NR}_1$  is not executable (i.e.  $\text{need}_v = \text{true}$ ) because predicate  $\text{Connect\_Stab}(v) = \text{false}$  and Rule  $\mathcal{CR}_1$  can not be executed. Since the root  $u$  is always connected (i.e.  $\text{connected}_u = \text{true}$ ), each child  $v$  of the root with  $\text{need}_v = \text{true}$  and  $\text{connected}_v = \text{false}$  can execute Rule  $\mathcal{CR}_1$  to change the value of its variable  $\text{connected}_v$  if necessary because predicate  $\text{Connect\_Stab}(v)$  is satisfied. Thus one can show by induction that for any node on the path between a member and the root we have  $\text{connected}_v = \text{true}$ .  $\square$

**Lemma 4** *Eventually  $\text{Connect\_Pt\_Stab}(v)$  is true for every connected node  $v$  on the path between each member and the root in the network.*

**Proof.** According to Lemma 3, there is a time after which we have paths of connected nodes between members and the root. Note that in this case predicate  $\text{Connect\_Stab}(v)$  is true.

Suppose that  $\text{Connect\_Pt\_Stab}(v)$  for a connected node  $v$  is false. If  $v$  is a member then this implies that  $\text{connect\_pt}_v = \text{false}$  (see predicate  $\text{Connect\_Pt\_Stab}(v)$ ), so  $v$  can execute Rule  $\mathcal{TR}$  to change the value of  $\text{connect\_pt}_v$  to true and we have  $\text{Connect\_Stab}(v) = \text{true}$ . Otherwise, let  $v$  be the parent of a member  $u$  on the path of connected nodes connecting  $u$  to the root. This implies that  $\text{connect\_pt}_v \neq |\{u : u \in N(v) \wedge \text{parent}_u = \text{ID}_v \wedge \text{connected}_u\}| > 1$  (see predicate  $\text{Connect\_Pt\_Stab}(v)$ ), so  $v$  can execute Rule  $\mathcal{TR}$  to update  $\text{connect\_pt}_v$  and we have  $\text{Connect\_Stab}(v) = \text{true}$ . Thus one can show by induction on the height of the tree that it exists a time where  $\text{Connect\_Stab}(v)$  is true for every connected node  $v$  on the path between each member and the root.  $\square$

**Lemma 5** *Eventually each member is connected by a shortest path to the current tree.*

**Proof.** Let  $T_{i-1}$  be the tree constructed by the algorithm before the connection of the member  $v_i$ . To prove the lemma, we must show that for any member  $v_i$  we have a shortest path from  $v_i$  to  $T_{i-1}$  when  $\text{Connect\_Pt\_Stab}(v_i) = \text{true}$  and  $\text{Better\_Path}(v_i) = \text{false}$  (i.e. Rule  $\mathcal{DR}_2$  can not be executed by a member and so there is no better path to connect the member).

Initially, according to Rule  $\mathcal{RR}$  the root  $v_0$  is always connected and we have  $\text{Connect\_Pt\_Stab}(v_0) = \text{true}$  and  $\text{Better\_Path}(v_0) = \text{false}$  (because  $\text{dist}_{v_0} = 0$ ). We show by induction on the number of members that the property is satisfied for each member. At iteration 1, let  $v_1$  be a not connected member then according to Lemma 2 the path  $P_1$  from  $v_1$  to  $v_0$  in the spanning tree is a shortest path, so there is a time s.t.  $\text{Connect\_Pt\_Stab}(v_1) = \text{true}$  (see Lemma 4) since  $P_1$  is a shortest path between  $v_1$  and  $v_0$  (i.e.  $T_0$ ), we have  $\text{Better\_Path}(v_1) = \text{false}$ , thus the property is satisfied for  $v_1$ . We suppose that the tree  $T_i$  satisfies the desired property for every member  $v_j, j \leq i$ . At iteration  $i+1$ , when member  $v_{i+1}$  is not connected, according to Lemma 2 the path  $P_{i+1}$  from  $v_{i+1}$  to  $T_i$  is a shortest path, so there is a time s.t.  $\text{Connect\_Pt\_Stab}(v_{i+1}) = \text{true}$  (see Lemma 4). Since  $P_{i+1}$  is a shortest path between  $v_{i+1}$  and  $T_i$ , we have  $\text{Better\_Path}(v_{i+1}) = \text{false}$  and the property is satisfied for  $v_{i+1}$ .

Note that a member  $v_{i+1}$  can create a connection point  $u$  (i.e.  $\text{connect\_pt}_u = \text{true}$ ) on the path  $P_j$  connecting a member  $v_j, j \leq i$ . In this case, the property is still satisfied for  $v_j$  because the path between  $u$  and  $v_j$  is part of  $P_j$  so it is a shortest path since a subpath of a shortest path is a shortest path. Moreover, when we have  $\text{connect\_pt}_u = \text{true}$  for  $u$  then all nodes on the path between  $u$  and  $v_j$  update their distance with Rule  $\mathcal{DR}_2$  (see predicate  $\text{Better\_Path}$ ).  $\square$

**Lemma 6** *Eventually a Steiner tree is constructed.*

**Proof.** According respectively to Lemmas 1 and 3 a spanning tree is constructed (i.e.  $S$  is also spanned) and there is a path of connected nodes between each member and the root. To prove the lemma we must show that every leaf of  $T$  is a member.

Consider the connected node  $v$  (i.e.  $\text{need}_v = \text{true}$  and  $\text{connected}_v = \text{true}$ ), such that  $v$  is a leaf of  $T$ . Since  $v$  is a leaf, this implies that  $v$  has no connected child in  $T$ , so predicate  $\text{Asked\_Connection}(v)$  is false.

Suppose that  $v$  is not a member. Thus  $v$  can execute Rule  $\mathcal{NR}_2$  and change the value of  $\text{need}_v$  to false. As a consequence predicate  $\text{Connect\_Stab}(v)$  is false and  $v$  can then execute Rule  $\mathcal{CR}_2$  which changes the value of  $\text{connected}_v$  to false. Therefore  $v$  is not connected and is no more a leaf of  $T$ .

By using the same scheme we can show by induction on the height of  $T$  that every node on a path of connected nodes which contains no member nodes can not belong to  $T$  after a finite bounded of time.

Now suppose that  $v$  is a member, the guard of Rule  $\mathcal{NR}_2$  is not satisfied so  $\text{need}_v$  remains true. Since  $\text{need}_v = \text{true}$ , predicate  $\text{Connect\_Stab}(v)$  remains true too and  $v$  is maintained by the algorithm as a leaf of  $T$ .  $\square$

**Lemma 7 (Convergence)** *Starting from an illegitimate configuration eventually the algorithm reaches in a finite time a legitimate configuration.*

**Proof.** Let  $C$  be an illegitimate configuration, i.e.  $C \notin \mathcal{L}$ . According to Lemmas 1, 5 and 6, in a finite time a legitimate state is reached for any process  $v \in V$ . Therefore in a finite time a legitimate configuration is reached in the network.  $\square$

**Lemma 8 (Correction)** *The set of legitimate configurations is closed.*

**Proof.** According to the model, **InfoMsg** messages are exchanged periodically with the neighborhood by all nodes in the network, so **InfoMsg** messages maintain up to date copies of neighbor states. Thus starting in a legitimate configuration the algorithm maintains a legitimate configuration.  $\square$

## 5 Correctness and proof in Dynamic setting

In this section, we consider dynamic networks and we prove that topology changes can be correctly treated by extending our algorithm, given in Figure 2. Moreover, we show that a passage predicate is satisfied during restabilizing execution of given algorithm.

In the following, we define the topology change events, noted  $\varepsilon$ , that we must consider:

- an add (resp. a removal) of a member  $v$  ( $v$  remains in the network) noted **add<sub>v</sub>** (resp. **del<sub>v</sub>**);
- an add (resp. a removal) of edge  $(u, v)$  in the network noted **recov<sub>uv</sub>** (resp. **crash<sub>uv</sub>**);
- an add (resp. a removal) of a neighbor node  $u$  of  $v$  in the network noted **recov<sub>u</sub>** (resp. **crash<sub>u</sub>**).

Algorithm given in Figure 2 completes the self-stabilizing algorithm described in precedent sections and allows to a node  $v$  to take into account topology change events. In the sequel we suppose that after every topology change the network remains connected. We prove in the next subsection that algorithm of Figure 2 has a superstabilizing property.

### 5.1 Correctness under restricted dynamism

We provide below definitions of the topology change events class  $\Lambda$  and passage predicate for protocol given in Figure 2.

**Definition 2 (Class  $\Lambda$  of topology change events)** ***del<sub>v</sub>**, **crash<sub>uv</sub>** and **crash<sub>v</sub>** compose the class  $\Lambda$  of topology change events.*

<p><b>Do forever:</b> send <math>\text{InfoMsg}_v</math> to all <math>u \in N(v)</math></p> <p><b>Upon receipt of <math>\text{InfoMsg}_u</math> from <math>u</math>:</b></p> <hr style="width: 80%; margin-left: 0;"/> <p style="margin-left: 20px;">use all the rules to correct the local state of <math>v</math></p> <p style="margin-left: 20px;">send <math>\text{InfoMsg}_v</math> to all <math>u \in N(v)</math></p> <p><b>Interrupt Section:</b></p> <hr style="width: 80%; margin-left: 0;"/> <p style="margin-left: 20px;"><b>If</b> <math>\varepsilon</math> is a <math>\text{del}_v</math> event or (<math>\varepsilon</math> is a <math>\text{crash}_{uv}</math> or <math>\text{crash}_u</math> event and <math>\text{parent}_v = ID_u</math>)</p> <p style="margin-left: 20px;"><b>then</b> <math>\text{connected}_v := \text{false}</math>; <math>\text{dist}_v := \infty</math>; <math>\text{connect\_pt}_v := \text{false}</math>;</p> <p style="margin-left: 40px;">send <math>\text{InfoMsg}_v</math> to all <math>u \in N(v)</math></p> <p style="margin-left: 20px;"><b>wait until</b> (<math>\nexists u \in N(v), \text{parent}_u = ID_v \wedge \text{connected}_u</math>);</p>
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Figure 2: Algorithm describing message exchanges and treatment of topology change events.

**Definition 3 (Passage predicate)** *Parent relations can be modified for nodes in the subtree connected by the removed member, edge or node, and parent relations are not changed for any other node in the tree.*

**Lemma 9** *Starting from a legitimate configuration, if a member  $x$  leaves the set of members  $S$  or node  $x$  or edge  $(y, x)$  is removed from the network then each connected node  $v$  in the subtree of  $x$  is disconnected from the tree and a legitimate configuration is reached by the system.*

**Proof.** According to the description of the complete algorithm, when a member  $x$  leaves the set of members  $S$  then  $x$  changes first its variables as following:  $\text{connected}_x = \text{false}$  and  $\text{dist}_x = \infty$ , then  $x$  sends its state to its neighborhood and finally  $x$  waits until it has no connected child. In the same way, if a node  $x$  (resp. edge  $(y, x)$  (assume  $\text{parent}_x = ID_y$ )) is removed from the network then each child  $v$  of  $x$  (resp.  $x$ ) changes first its variables as following:  $\text{connected}_v = \text{false}$  and  $\text{dist}_v = \infty$  (resp.  $\text{connected}_x = \text{false}$  and  $\text{dist}_x = \infty$ ), then  $v$  (resp.  $x$ ) sends its state to its neighborhood and finally  $v$  (resp.  $x$ ) waits until it has no connected child.

When a connected child  $u$  of  $v$  (resp. of  $x$ ) receives message  $\text{InfoMsg}_v$  from  $v$  (resp.  $\text{InfoMsg}_x$  from  $x$ ), since predicate  $\text{Connect\_Stab}(u)$  is false (because  $\text{connected}_{\text{parent}_u} = \text{false}$ ) and  $\text{dist}_{\text{parent}_u} = \infty$  the node  $u$  executes Rule  $\mathcal{CR}_3$  changing the variables of  $u$  like  $v$ 's or  $x$ 's variables, sends its state to its neighborhood and waits until it has no connected child. According to Lemma 11, no node in the subtree of  $x$  executing Rule  $\mathcal{CR}_3$  perpetually waits it has no connected child. As a consequence, after a finite time every connected node  $v$  in the subtree of  $x$  is no more connected.

Since each node in the subtree of  $x$  is not connected, there is at least one of those nodes  $v$  such that predicate  $\text{Better\_Path}(v)$  is true. Thus  $v$  can execute Rule  $\mathcal{DR}_1$ . According to Lemmas 1 and 2, there is a time after which each node in the subtree of  $x$  knows its correct shortest path distance to a connected node. Moreover, by Lemmas 3 and 5 each not connected member will be connected by a shortest path to a connected node in the existing Steiner tree. Therefore, in a finite number of steps the system reaches a legitimate configuration  $C' \in \mathcal{L}$ .  $\square$

**Lemma 10** *The proposed protocol is superstabilizing for the class  $\Lambda$  of topology change events, and the passage predicate (Definition 3) continues to be satisfied while a legitimate configuration is reached.*

**Proof.** Consider a configuration  $\Delta \vdash \mathcal{L}$ . Suppose  $\varepsilon$  is a removal of edge  $(u, v)$  from the network. If  $(u, v)$  is not a tree edge then the distances of  $u$  and  $v$  are not modified neither  $u$  nor  $v$  changes its parent, thus no parent relation is modified. Otherwise let  $\text{parent}_v = u$ ,  $u$ 's distance and  $u$ 's parent are not modified, it is true for any other node not contained in the subtree of  $v$  since the distances are not modified (i.e. predicate `Better_Path` is not satisfied). However,  $u$  is no more a neighbor of  $v$  so according to the handling of an edge removal by the algorithm  $v$ 's variables are reseted. Then  $v$  sends its state to its neighborhood and waits until it has no connected child. According to Lemma 9, all its children will become not connected and eventually change their parent by executing Rule  $\mathcal{DR}_1$  because there is a better path (i.e. predicate `Better_Path` is satisfied). Therefore, only any node in the subtree connected by the edge  $(u, v)$  may change its parent relation.

Suppose  $\varepsilon$  is a removal of node  $u$  from the network. Any node not contained in the subtree of  $u$  do not change its parent relation because the distances are not modified (i.e. predicate `Better_Path` is not satisfied). Consider each edge  $(u, v)$  between  $u$  and its child  $v$ , we can apply the same argument described above for an edge removal. Therefore, only any node contained in the subtree connected by  $u$  may change its parent relation.  $\square$

A fault which occurs in the network is detected using a distance with an infinity value. To handle a fault, we introduce Rule  $\mathcal{CR}_3$  to bootstrap connected nodes in the subtree below a faulty node/edge. We show in Lemma 11 that even Rule  $\mathcal{CR}_3$  is executed when no fault occurs in the network then no node perpetually waits (no deadlock) because of Rule  $\mathcal{CR}_3$ .

**Lemma 11** *Starting from an arbitrary configuration, Rule  $\mathcal{CR}_3$  introduces no deadlock in the network.*

**Proof.** Consider a configuration which simulates the presence of a fault in the network (but there is not really a fault) and allows the execution of Rule  $\mathcal{CR}_3$  by a node  $v$ , i.e.  $v$  is a connected node and has a not connected parent  $u$  with  $\text{dist}_{\text{parent}_v} = \infty$ . According to Rule  $\mathcal{CR}_3$ ,  $v$  becomes a not connected node and sets its distance to infinity (i.e. `connectedv = false` and `distv = ∞`), then it sends its state to its neighbors and waits until it has no connected child. There are two cases: (1)  $v$  has no connected child or (2)  $v$  has at least one connected child. In case (1),  $v$  is a leaf of the connected subtree and does not wait. Otherwise, in case (2) the subtree of connected nodes rooted in  $v$  has a finite height so we can show by induction that in a finite time every node in the subtree executes Rule  $\mathcal{CR}_3$ . According to case (1), there is no deadlock for the leaves of the connected subtree. Therefore, we can show by induction on the height of the subtree rooted in  $v$  that after a finite time there is no connected node and  $v$  wakes up.  $\square$

## Correctness under fully dynamism assumptions

In the precedent subsection guarantees are given on the conservation of the tree structure, only for removal topology events. Here, we consider all the different topology change events presented in Section 4 (i.e. add/removal of members, nodes or edges). We must maintain a quality of service on the weight of the structure reserved to interconnect all members. Therefore, legitimate configurations take into account a global constraint on the Steiner tree weight. As a consequence, we can not give any guarantees on the tree structure during the stabilization of protocol defined by the presented rules and algorithm of Figure 2 (i.e. no passage predicate is satisfied) if an add of a member, node or edge arises in the network. However to maintain a quality of service on the

structure weight, we show here that the protocol is able to restabilize when one of the previous mentioned topology change events arises in the network.

Lemma 9 proves that a legitimate configuration is reached starting from an arbitrary configuration if removal topology change events arises in the network. The following lemma considers add topology change events and shows that a legitimate configuration is reached too.

**Lemma 12** *Starting from a legitimate configuration, after a member add to  $S$  or a node or edge add in the network, eventually the algorithm leads in a finite number of steps to a legitimate configuration.*

**Proof.** We must consider three cases: an edge add, a node add and the add of a path in the network.

Consider the add of an edge between two existing nodes  $u$  and  $v$  with a weight  $w(u, v)$ . If predicate `Better_Path` is false for  $u$  and  $v$  (i.e.  $\text{dist}_u \leq \text{dist}_v + w(u, v)$  and  $\text{dist}_v \leq \text{dist}_u + w(u, v)$ ) then the system is still in a legitimate configuration  $C' \in \mathcal{L}$ . Otherwise `Better_Path` is true and Rule  $\mathcal{DR}_1$  (resp.  $\mathcal{DR}_2$ ) can be executed if  $u$  or  $v$  is not connected (resp. connected) to correct its distance. In the same way, other tree nodes  $u$  or  $v$  correct their distances, thus after a finite number of steps the system reaches a legitimate configuration  $C' \in \mathcal{L}$ .

Consider the add of a node  $v$  to an existing node  $u$  by an edge  $(u, v)$ .  $v$  corrects its variables by executing Rule  $\mathcal{DR}_1$ . If  $v$  is not a member, variable `needv` is corrected if necessary with Rule  $\mathcal{NR}_2$  otherwise according to Lemmas 3, 4 and 5  $v$  is connected by a shortest path to the existing tree, which leads the system to a legitimate configuration  $C' \in \mathcal{L}$ .

Consider the add of a path  $P$ . If  $P$  is a path between an existing node  $u$  and a new node  $v$  then all nodes of  $P$  behave like the case of a node add  $v$  to an existing node  $u$ . Otherwise  $P$  is a path between two existing nodes  $u$  and  $v$ , all nodes of  $P$  behave like the case of a node add to an existing node and  $u$  and  $v$  behave like the case of an edge add if  $P$  offers a better path. Thus, in a finite number of steps the system reaches a legitimate configuration  $C' \in \mathcal{L}$ .  $\square$

## Complexity and Cost Issues

**Theorem 1** *Using the notation of Theorem 2, Algorithm S3T performs in  $O(D \cdot |S|)$  rounds where  $D$  is the current diameter of the network. It uses  $O(\Delta \log n)$  bits of memory in the send/receive model<sup>3</sup>, where  $\Delta$  is the current maximal degree of the network.*

**Proof.** We consider the worst case in which all the tree must be reconstructed because of topological or member set modifications. Let  $T_i = (V_{T_i}, E_{T_i})$  be a tree constructed at some step  $i$  of the algorithm. Our algorithm can be viewed as a special case of a shortest path tree construction in which all nodes  $v \in V_{T_i}$  are considered as a single virtual root and all nodes  $v \notin V_{T_i}$  computes the shortest distance from this virtual root. So we can show by induction that the algorithm connects in at most  $O(D)$  rounds the nearest member to the tree  $T_i$ . Initially when the root  $r$  is stabilized and connected to  $T_0$ ,  $r$  initiates a classic shortest path computation. So after  $3D$  rounds the algorithm connects the nearest member to the root (we need at most  $D$  rounds to compute the shortest path to the root and at most  $2D$  rounds for the nodes on the path to change their states from not connected to connected). We assume that following the first  $3iD$  rounds  $i$  members are

<sup>3</sup>In the classical message passing model the memory complexity is  $O(\log |S|)$

connected to the tree  $T_i$ . We prove that after  $3D$  additional rounds  $i + 1$  members are connected. In at most  $D$  rounds all nodes  $v \notin T_i$  compute their shortest path to  $T_i$ , in additional  $2D$  rounds all nodes on the path from the nearest member  $v \notin T_i$  to  $T_i$  change their state to connected. So after  $3(i + 1)D$  rounds  $i + 1$  members are connected in tree  $T_{i+1}$ . Thus as  $0 \leq i \leq z$  the algorithm connects all members in at most  $O(zD)$  rounds.

In the following we analyze the memory complexity of our solution. Each node maintains a constant number of local variables of size  $O(\log n)$  bits. However, due to specificity of our model (the send/receive model) the memory complexity including the copies of the local neighborhood is  $O(\delta \log n)$  where  $\delta$  is the maximal degree of the network.  $\square$

Since we use the shortest distance metric between nodes in the network, any network can be represented by a complete graph so the following Lemma can be applied.

**Lemma 13 (Imase and Waxman [IW91])** *Let  $G = (V, E)$  be a complete graph with a cost function  $C : E \rightarrow \mathbb{R}^+$  satisfying the triangle inequality, and let  $S$  be any nonempty subset of  $V$  with  $|S| = z$ . If  $2P$  is the cost of an optimal tour for  $S$  and  $l : V \rightarrow \mathbb{R}^+$  satisfying the following conditions:*

1.  $d(u, v) \geq \min(l(u), l(v))$  for all nodes  $u, v \in S$ , and
2.  $l(v) \leq P$  for all nodes  $v \in S$ ,

then  $(\sum_{v \in S} l(v)) - \max_{v \in S} l(v) \leq (\lceil \log z \rceil)P$ .

**Theorem 2** *Let  $G = (V, E, w)$  be a dynamic network, and let  $S$  be a set of members. Algorithm s3T is a superstabilizing algorithm that returns a steiner tree  $T$  for  $S$  satisfying  $\frac{W(T)}{W(T^*)} \leq \lceil \log |S| \rceil$ , where  $T^*$  is an optimal Steiner tree for  $S$ .*

**Proof.** Let a set  $S$  of members, and  $z = |S|$ . According to Lemmas 5 and 6, when our algorithm completes each member  $v \in S$  is connected in  $T$  by a shortest path to a node  $u$ , such that  $u$  has been connected in  $T$  before  $v$ . Let  $T_{i-1}$  the tree constructed by our algorithm before the connection of a member  $v_i \in S$ . As in [IW91] (proof of theorem 2), if we let  $l(v_i) = \min_{0 \leq j < i} d(v_i, v_j)$  for  $1 \leq i \leq z$ , then the cost of the path selected by the algorithm to connect  $v_i$  to  $T_{i-1}$  is less than or equal to  $l(v_i)$ . Let  $l(v_0) = \max_{1 \leq j \leq z} d(v_0, v_j)$ , so  $l(v_0) \geq \max_{0 \leq j < i} l(v_j)$ . Thus we have  $W(T) \leq (\sum_{j=0}^z l(v_j)) - l(v_0)$ . Moreover for any pair of nodes  $v_j, v_k$ , according to definition of function  $l$  we have  $l(v_k) \leq d(v_j, v_k)$  so (1) of lemma 13 holds. Note that a tour of set  $S$  can be constructed from a Steiner tree for  $S$  of cost of  $P$  such that the cost of the tour is no more than twice the cost of the Steiner tree. Since  $l(v_j) \leq P$  for all  $j$ ,  $0 \leq j \leq z$ , (2) of lemma 13 holds and according to lemma 13 the theorem follows.

Since  $S$  is a dynamic set of member, we must consider two cases: the add of a member and the removal of a member. Consider the add of a new member  $v$  to  $S$ . By Lemma 12, the system reaches a legitimate configuration. Thus,  $v$  is connected by a shortest path to the existing Steiner tree and  $W(T) \leq (\sum_{j=0}^z l(v_j)) - l(v_0)$  is still satisfied. The same argument is true for the add of a node or an edge of the network. Consider the removal of a member  $v$  from  $S$ . By Lemma 9, the system reaches a legitimate configuration. Thus, each member  $v$  of  $S$  is connected by a shortest path to a connected member in the Steiner tree and  $W(T) \leq (\sum_{j=0}^z l(v_j)) - l(v_0)$  is satisfied again. The same argument is true for the removal of a node or an edge of the network. Therefore, considering a dynamic network  $G$  and a dynamic set of members the theorem is always satisfied.  $\square$



## 6 Conclusion

We propose a self-stabilizing algorithm for the Steiner tree problem, based on the heuristic proposed in [IW91], and achieves starting from any configuration a competitiveness of  $\log(z)$  in  $O(zD)$  rounds with  $z$  the number of members and  $D$  the diameter of the network. Additionally, we show that our algorithm works for dynamic networks in which a fault may occur on a node or edge. Moreover, we prove that if a fault occurs in a legitimate configuration our algorithm is superstabilizing and is able to satisfy a "passage predicate" about the tree structure.

For future works, it will be interesting to design a self-stabilizing algorithm in dynamic networks for the Steiner tree problem, which achieves a constant competitiveness of 2. For example, by using the self-stabilizing algorithm proposed in [KK02b] and extending it for dynamic networks or by using another heuristic.

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