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## Research Article

# Quality Assessment of Stereoscopic Images

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Several metrics have been proposed in literature to assess the perceptual quality of two-dimensional images. However, no similar effort has been devoted to quality assessment of stereoscopic images. Therefore, in this paper, we review the different issues related to 3D visualization, and we propose a quality metric for the assessment of stereopairs using the fusion of 2D quality metrics and of the depth information. The proposed metric is evaluated using the SAMVIQ methodology for subjective assessment. Specifically, distortions deriving from coding are taken into account and the quality degradation of the stereopair is estimated by means of subjective tests.

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## 1. INTRODUCTION

3D imaging is a wide research area driven both by the entertainment industry and by scientific applications. Some of the most recently advances have been recently published in [1]. From John Logie Baird who introduced the first version of stereo TV, many techniques have been developed [2]: stereoscopic vision with polarizing glasses, autostereoscopic displays for free viewpoint TV, or sophisticated holographic systems. In parallel, methods for 3D scene representation [3] and data content broadcasting [4] have been widely studied.

Applications are numerous. They range from entertainment (videos, games) to more specialized applications such as the educational ones [5] and medical applications like body exploration [6, 7], therapeutic purposes [8], and so forth.

Several signal processing operations [9, 10] have been specifically designed for stereoscopic images. Therefore, the necessity to define standardized protocols to assess the perceived quality of the processed stereo images is evident.

Quality assessment of multimedia content is achievable either through subjective tests or through objective metrics. The best way to assess image and video quality would surely be to run subjective tests according to standardized protocols, which are defined in order to obtain correct,

universal, and reliable quality evaluations. However, the use of subjective tests is a time consuming approach. Furthermore, the analysis of the obtained results is not straightforward. Therefore, the definition of objective metrics reliably predicting the perceived quality of images would be a great improvement in the quality assessment field.

A great effort has been devoted by both the academic and the industrial communities to develop objective metrics able to quantitatively evaluate the amount of degradation undergone by a signal, an image, or a video sequence. In fact objective metrics can be used to accomplish different tasks. Among the multitude of possible applications, it is worth pointing out that they can be used for benchmarking purposes to choose among several processing systems which can be used for the same purpose on a digital media; the system providing the best metric value will be used. Moreover, when image and video delivery takes place in an error prone scenario, objective quality metrics can be used as side information for the image and video server to take the necessary actions to improve the quality of the received data, like prefiltering, optimal bit assignment algorithms, error concealment methods, and so on.

However, although several subjective and objective quality assessment methods have been proposed in literature for

images and videos, no comparable effort has been devoted to the quality assessment of stereoscopic images. With the widespread of 3D technology applied to different fields such as entertainment, CAD, medical applications, to cite only a few, 3D images and videos need to be processed. Therefore, the necessity to define both subjective procedures and objective metrics to assess the quality of the processed stereo images is becoming an issue of paramount importance. From a visual point of view, 3D perception involves new critical points which have to be taken into account. First, subjective experiments [11–13] have to be performed in order to identify the main new issues. Indeed, compared to 2D images, perception of stereo content involves several peculiar elements which cannot be considered when dealing with the fruition of 2D content. Previous research tried to identify these new factors, including the notion of “presence” [14] which is related to the sensation of immersion in the 3D visual scene. Moreover, the different technologies on which 3D displays are based on and are so different that two issues must be considered: what is the impact of each technology on the observer viewing experience and in a more general way, independently of the technology, which factors have to be taken into account to quantify 3D image quality and how do they impact on visual perception? Subjective experiments must be conducted to understand these two problems and the related models have to be designed.

Taking into account these considerations, we first propose to review quality issues for 3D images and recent works on this purpose. Regarding the wide open area, we then proposed to limit our study to stereopair images. Both subjective and objective assessments are addressed within this context taking care of the heritage of 2D image quality assessment. The first attempt to build objective quality metrics specifically tailored to stereo images was proposed in [15] where a metric making use of reliable 2D metrics applied to both the left and the right views has been proposed. However, the depth information is not taken into account

In this paper, we take a different perspective by using also the depth information to design an objective metric for 3D quality assessment.

The paper is organized as follows. In Section 2, quality issues for 3D content display are briefly summarized. Section 3 presents an overview of 3D subjective test used in this work. Sections 4 and 5 present, respectively, the objective quality metric we propose and the related results. In Section 6, the obtained results are analyzed and conclusions are drawn.

## 2. QUALITY ISSUES IN 3D

Because of the different physiological mechanisms on which the fruition of stereo images is based with respect to those involved when 2D content is analyzed, several new issues have to be taken into account.

Generally speaking, 3D perception is based on various depth cues such as illumination, relative size, motion, occlusion, texture gradient, geometric perspective, disparity, and many others. However, a very effective depth perception

sensation is obtained by viewing a scene from slightly different viewing positions. From a physiological point of view, given a scene in the real world, 2D slightly different scenes are projected on the retina of each eye. This implies that the 3D depth information is lost at this stage. Then, the primary visual cortex in the brain fuses the corresponding points of the stereopair by means of the stereopsis mechanism and a prior knowledge on the 3D world. Therefore, humans can perceive the depth starting from the bidimensional images on the retina of each eye. When 3D imaging systems try to mimic the behaviour of the human visual system, the role of the eyes is taken over by stereo cameras which capture a scene from slightly different positions. The depth information can be obtained using stereo vision techniques by means of the disparity, the relative displacement of the stereo camera as well as its geometry.

### 2.1. 3D perception and 3D displays

In the literature, 3D content visualization criteria generally include image quality, naturalness, viewing experience, and depth perception. These criteria are linked to the specific display technology and also to the used data format. Since several display technologies have been developed, it is of paramount importance to study their impact on image quality, depth perception, naturalness, and so forth.

Roughly speaking, the systems used to display stereo images present alternatively to the left and the right eyes two slightly different images in such a way that the human visual system gets a perception of depth. More in detail, the 3D rendering systems can be classified as either *autostereoscopic* or *stereoscopic* displays. Autostereoscopic displays do not need any special viewing glasses, but the viewing angle is not very wide. On the other side, stereoscopic displays require viewing glasses such as anaglyphic lenses, polarized glasses for passive systems, or liquid crystal shutter glasses for active systems. These systems allow the left and right images to be projected onto a screen with different polarization or colours. They are more affordable than autostereoscopic displays and they can be used in commercial theatre as well as in a home environment.

Then, considering one 3D imaging system, effects such as crosstalk between views, key-stone distortion, depth-plane curvature, puppet theater effect, cardboard effect, shear distortion, picket-fence effect, and image flipping can appear [12]. Also, compared to bidimensional data, raw stereo data representation requires higher storage capacity and higher bandwidth for transmission. Therefore, in order to make these technologies deployable in real-life applications, coding schemes have to be developed and their effect on visual perception must be carefully analyzed. Previous studies already report distortion effects such as blocking, blurring, jerkiness, and ghosting. As a general rule, perception and quality constancy regarding field of view have to be investigated and the impact of depth representation, data formats, and compressions have to be clearly identified. Both the technological and the psychovisual factors influencing stereopairs fruition are summarized in Table 1.

TABLE 1: Issues in 3D from a technological and visual perception point of view.

Technology factors	Impact on perception
Data formats	Impact of depth
Compression	Quality constancy
Depth representation	Field of view
Crosstalk	Viewing experience
Distorsions	Presence

## 2.2. Subjective studies

In [16], a wide variety of subjective tests to identify how depth information retrieval, crosstalk, depth representation, and asymmetrical compression impact on image quality, naturalness, viewing experience presence, and visual strain are described. These studies are related to specific 3D display, but general considerations can be drawn for a much larger audience. Some experiments on asymmetric JPEG coding on stereopairs have highlighted that observers give a global score depending on the image of the stereopair having the lowest quality. The same experiments were performed with asymmetric blur in [17]. However, in this case, the final score depends on the image of the stereopair having the highest quality. Therefore, the perceived quality of a stereopair, whose images have been asymmetrically distorted, strictly depends on the applied distortions, which is related to the level of the human visual system masking effects. Following the impact of asymmetric stereo images coding, tests were carried out in order to identify the impact of eye dominance. In [16, 18, 19], no effect of eye dominance was noticed for image quality evaluation. Nevertheless, in [20], it was observed that eye dominance improves the performance of visual search task by aiding visual perception in binocular vision, and the eye dominance effect in 3D perception and asymmetric view coding was also analyzed. To clarify this contradiction, other experiments should be designed in order to clearly identify the role to eye dominance.

In [21] a depth perception threshold model is designed and a 3D display benchmark is performed in order to identify the most suitable technology for depth representation. Nevertheless, the mechanisms related to depth perceptions have still to be fully understood.

The impact of the depth information on the perceived 3D image quality is one of the main issues that has to be investigated and it is still controversial. Recent studies [16, 22] hypothesize that, from a psychovisual point of view, depth is not related to the perceived three-dimensional effect. Nevertheless, other studies point out the importance of depth for quality perception. For example in [23] a blurring filter, whose blur intensity depends on the depth of the area where it is applied, is used to enhance the viewing experience. This work is validated by the study reported in [24] which shows that blurring 3D images reduce discrepancy between responses of accommodation and convergence, so that blur increases viewers' experience. Also, methods which aim at enhancing the local depth information on objects are

proposed, as in [25], where the algorithm directly impacts on the image quality by taking into account depth information.

This overview, although incomplete, shows that the role of depth in the perception mechanism of stereo images is still not clearly identified. Nevertheless, depth information is required to design objective quality metrics in order to take into account viewers' experience as well as signal processing operations affecting depth information.

## 2.3. Discussion

### 2.3.1. Human perception and visual comfort

Since 3D displays design requires the knowledge of the mechanisms driving 3D perception, human perception investigations must be conducted, several factors have to be taken into account such as accommodation issues, and intereye masking effect can appear. Also physiological differences between people (interpupillary distance [26, 27], age [28, 29], etc.) impact on individual perception. One of the most well known effects is related to visual fatigue and visual discomfort [11, 30, 31]. Indeed, as 3D displays allow the synthesis of objects at different distances from the screen, artificial 3D content visualization can introduce an accommodation and convergence discrepancy [32]. Indeed, when viewing real 3D objects, both eyes converge on the object and accommodation is naturally performed at the object depth position. Nevertheless, when viewing an object by means of a 3D screen, the eyes still converge at the virtual object position but the accommodation has to be performed at the screen depth level. This discrepancy is one of the causes of visual fatigue and may also impact on visual functions performance.

### 2.3.2. Safety and health issues

In addition to human factors related only to 3D perceptions, it is important to identify all the cues related to human vision performance degradation prevention for such display technologies. Indeed, some recent studies [33, 34] enlighten some possible problems created by 3D display like decline of visual functions after experiments requiring vergence adaptation on 3D content. Also, asymmetrical image distortions can cause vision degradation such as myopia increase [32]. Some ophthalmologists remain concerned that viewing stereoscopic images may cause strabismus, an abnormality in binocular alignment in young children. However, there is no evidence that the fruition of stereoscopic images causes strabismus except for what is reported in [35]. An extensive survey on the potential health problems related to 3D technologies is given in [32].

### 2.3.3. Further development

This brief overview shows that the design of a 3D quality metric is a very challenging goal that involves many factors interacting each other in a way that still needs to be clearly modeled. At a first level of approximation, a preliminary analysis can be done by focusing on a specific technology

and by studying the influence of a limited set of parameters on the perceived quality of 3D images. In [15], in the process of defining an objective quality metric specifically designed for stereoscopic images, we evaluate whether 2D image quality objective metrics are also suited for quality assessment of stereo images. This method showed interesting results when considering image distortions such as burr, JPEG, and JPEG2000 compression applied symmetrically to the stereopair images. Nevertheless, since depth information is not exploited, particular aspects of the 3D perception such as viewing experience and visual comfort are not taken into account. Therefore, in this paper, we enhance the preliminary study made in [15] by including also the depth information in order to design an objective quality metric for stereo images which takes into account the basic mechanisms of the human visual system involved in the fruition of stereo images.

### 3. SUBJECTIVE STEREO IMAGE QUALITY ASSESSMENT

In general the design of objective quality assessment metrics needs to be validated by subjective quality assessment. Then the definition of specific test setups for subjective test experiments is required. Methods have been proposed for 2D quality such as double stimulus continuous quality scale (DSCQS) [36] and SAMVIQ [37]. We choose to follow the SAMVIQ protocol which stability allows to conduct the experiments in a more reliable way. More precisely, the test was performed in a controlled environment as recommended in ITU BT 500-11 [36], by using displays with active liquid crystal shutter glasses. SAMVIQ is a methodology for subjective test of multimedia applications using computer displays, whose application can be extended to embrace the full format television environment as well. The method proposed by SAMVIQ specification makes it possible to combine quality evaluation capabilities and ability to discriminate similar levels of quality, using an implicit comparison process. The proposed approach is based on a random access process to play sequence files. Observers can start and stop the evaluation process as they wish and can follow their own paces in rating, modifying grades, repeating play out when needed. Therefore, SAMVIQ can be defined as a multistimuli continuous quality scale method using explicit and hidden references. It provides an absolute measure of the subjective quality of distorted sequences which can be compared directly with the reference. As the assessors can directly compare the impaired sequences among themselves and against the reference, they can grade them accordingly. This feature permits a high degree of resolution in the grades given to the systems. Moreover, there is no continuous sequential presentation of items as in DSCQS method, which reduces possible errors due to lack of concentration, thus offering higher reliability. Nevertheless, since each sequence can be played and assessed as many times as the observer wants, the SAMVIQ protocol is time consuming and a limited number of tests can be done.

At the end of the test sessions, the difference mean opinion score ( $DMOS$ ) for the  $i$ th image is computed as



FIGURE 1: Experimental setup: the user is facing the screen with crystal shutter glasses.

the difference between the  $MOS$  for the hidden reference, namely,  $MOS_{hr}$ , and the one relative to the image  $i$ ,  $MOS_i$ ,

$$DMOS = MOS_{hr} - MOS_i. \quad (1)$$

#### 3.1. Test setup

Figure 1 shows the experimental setup we have used and which is detailed hereafter.

In this paper, we perform subjective tests using six stereo images shown in Figure 2. We consider for each image five degradation levels per image distortion (JPEG and JPEG2000) which leads to sixty degraded images plus the six original images. More in detail, the image mean size is  $512 \times 448$  pixels viewed at standard resolution (no upscaling, centred on the display) on a  $1024 \times 768$  frame resolution, 21" Samsung SyncMaster 1100 MB display. JPEG2000 compressions used bit rates ranging from 0.16 bits per pixel (bpp) to 0.71 bpp while JPEG compression involved bit rates ranging from 0.24 bpp to 1.3 bpp.

#### 3.2. Human subjects

Seventeen observers, mostly males familiar with subjective quality tests, with an average age of 28.2 years and a standard deviation of 6.7 took part in the test. The observers had a visual acuity, evaluated at a three-meter distance, at least 9 out of 10. Three observers have discarded because the correlations between their individual scores and the mean opinion score were lower than a fixed threshold that has been set to 0.85. Each subject was individually briefed about the goal of the experiment, and a demonstration of the experimental procedure was given.

Each observer participated in two 30-minute sessions. For each image evaluation step, observers were asked to score the quality of the original stereo image (reference image), the hidden reference, and seven degraded versions on a continuous scale ranging from 0 to 100. Each distorted image was picked up in a random order. Each observer scored the sixty-six images available in the test. Subjective experiments lead to ninety  $DMOS$  values.

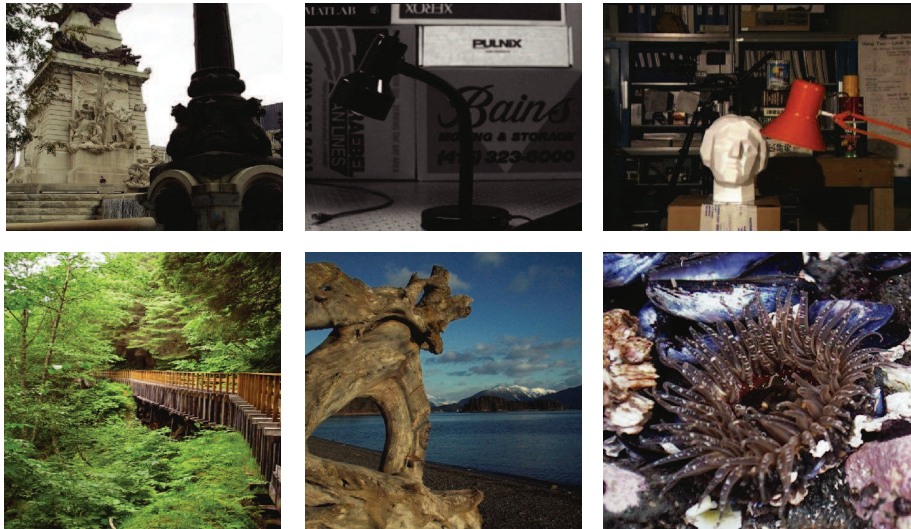


FIGURE 2: Left views of the tested stereopairs.

## 4. OBJECTIVE STEREO QUALITY ASSESSMENT

### 4.1. Overview of the proposed approach

In [15], we have introduced a metric for stereo images quality assessment which relies on the use of some well-known 2D quality metrics. Among the ones we have used in [15], it is worth to briefly summarize the Structural SIMilarity (SSIM) [38] and  $C4$  [39] which have been used also in the proposed approach as follows:

- (i) *Structural SIMilarity (SSIM)* is an objective metric for assessing perceptual image quality, working under the assumption that human visual perception is highly adapted for extracting structural information from a scene. Quality evaluation is thus based on the degradation of this structural information assuming that error visibility should not be equated with loss of quality as some distortions may be clearly visible but not so annoying. Finally SSIM directly evaluates the structural changes between two complex-structured signals.
- (ii)  $C4$  is a metric based on the comparison between the structural information extracted from the distorted and the original images. This method exploits an implementation of an elaborated model of the human visual system. The full process can be decomposed into two phases. During the first step, perceptual representation is built for the original and the distorted images, then, during the second stage, representations are compared in order to compute a quality score.

In [15], all the employed 2D metrics have been applied separately on each image (left and right eyes) and fusion methods, to obtain one overall score for the given stereopair, have been investigated. The correlation between  $DMOS$  and each of the objective metrics for each of considered

distortions has been calculated after a “mapping” operation in order to evaluate the performances of the metrics. More in detail “mapping” refers to the application of nonlinear function as recommended by VQEG [40] in order to map metrics scores into subjective score space. For each condition, parameters of the mapping function have been optimized. As a preliminary result the average of both left and right eyes measures gave the best result among the employed fusion methods.

However, in the metric design in [15] no information about the depth perception was taken into account. As outlined in Section 2, the lack of depth information can lead to discrepancy between 2D and 3D quality measures. Indeed, for example, in some cases, the degradation of the single images of a stereopair by using a blurring filter can help to get better stereo viewing experience, whereas the measure of the 2D degradation does not correlate with the enhanced quality of stereo perception [24]. Therefore, in this paper, we take this fact into account and, starting from the metric designed in [15], we investigate the amount of information added, if any, into the quality assessment process using depth information. To this purpose, we propose to enhance the original model by considering information strictly related to the nature of the stereo images. Specifically, we choose to focus on the disparity information. Indeed, as well known [1, 41], the sense of stereo vision is related to the difference in the viewpoint between eyes. Given two corresponding points in the left and the right images of a stereopair, the vector between the two points is called disparity. In general, disparity can be used to reproduce one of the two images of the stereopairs having the other one. More in detail, two different disparity computation algorithms have been selected for our purposes: the one described in [42], namely, “bpVision” and the one presented in [43], namely, “kz1”. These two algorithms model the disparity by means of Markov random field (MRF). Nevertheless, bpVision algorithm uses belief propagation for inference,

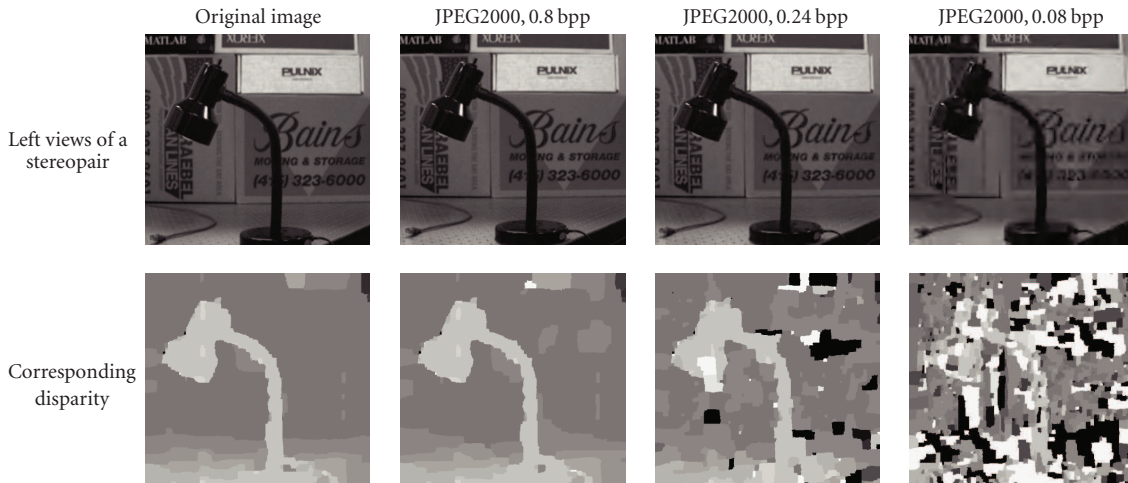


FIGURE 3: Original disparity map (left) and disparity maps computed after different JPG2000 compressions using bpVision algorithm.

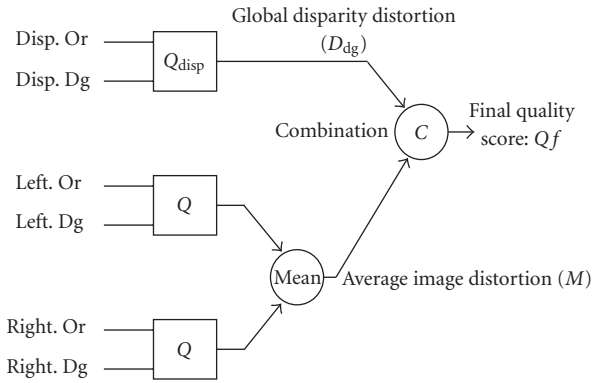


FIGURE 4: Quality estimation of stereopairs using original left and right views (Left.Or, Right.Or) compared with the degraded versions (Left.Dg, Right.Dg) and the related original disparity map compared to the degraded disparity map (Disp.Or and Disp.Dg) using a global approach.

while kz1 algorithm uses graph cuts and their formulations of the MRF are different. The comparative study presented in [44] shows that performances of the two methods are close to each other and superior to those of other algorithms proposed in literature. Graph cuts based methods give smoother results because they are able to find a lower-energy solution. On the other hand, belief propagation can maintain some structures which are lost in the graph cuts solution. From a computational cost point of view, accelerated belief propagation methods such as bpVision are faster than graph cuts based methods.

When distortions occur because of transmission on error prone channels or signal processing operations, the disparity map of the given stereopair is altered; see for example Figure 3 where the original disparity map together with the disparity map of a JPEG2000 coded stereopair is displayed. These considerations suggest us to employ also this information to assess the perceived quality of the

stereopair. However, only after validation of the quality model by means of subjective experiments we can infer that the depth information is relevant to the stereo image quality evaluation process.

For the proposed metric, we measure the quality of the distorted stereopair by measuring the following.

- (i) The difference between original (left or right) images and the corresponding (left or right) distorted version. For this purpose, one can use usual 2D perceptual quality metric such as SSIM or C4. As in [15], the two measures per pair are averaged in order to get the global 2D image distortion measure  $M$ .
- (ii) The difference between the disparity map of the original stereopair and the disparity map of the distorted stereopair. It is worth pointing out that since disparity maps are not natural images, perceptual-based distortion metrics cannot be applied.

The combination of this information is made in two different manners.

The first approach (sketched in Figure 4) is to measure a global disparity distortion and to combine this information with the one coming from the evaluation of the stereopair as a couple of two 2D images [15]. In this way, we investigate the impact of the quality estimation in a global approach. We evaluate individually the left and right views using either SSIM or C4 2D metrics and mean the results. The so obtained 2D quality score is fused with the score related to the disparity distortion measure.

In the second approach (sketched in Figure 5), the disparity distortion is measured locally and then it is fused with the quality measures coming from 2D quality assessment performed independently to the left and right images of the stereopair. The final score is the mean score of left and right distortions measures. SSIM is appropriate for this approach since SSIM measures are available for each pixel of the images by using the SSIM map (that we call  $M_{\text{map}}$ ). On the other hand, C4 cannot be used since its algorithm focuses on

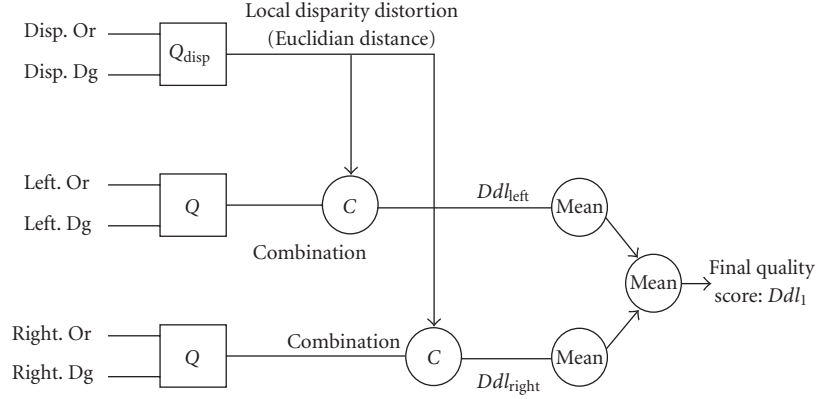


FIGURE 5: Quality estimation of stereopairs using original left and right views (Left.Or, Right.Or) compared with the degraded versions (Left.Dg, Right.Dg) and the related original disparity map compared to the degraded disparity map (Disp.Or and Disp.Dg) using a local approach.

discrete areas on the image. The two different proposed approaches are detailed in the two following subsections.

#### 4.2. Image quality and global disparity distortion measure

In this first approach, the impact of the global disparity distortion measure  $D_{dg}$  is computed using the correlation coefficient between the original disparity maps and the corresponding disparity maps processed after image degradation.

The final quality measure  $d$  is obtained after the fusion of the disparity distortion measure  $D_{dg}$ , and the averaged left and right image distortion measures  $M$ . These two measures both rank from 0 (maximum error measure) to 1 (no error measured). Two different fusion rules, shown in (2), have been tested. Moreover, the disparity distortion measure  $D_{dg}$  has been considered by itself for comparison purposes, note that other combinations can be considered but we focused on these ones in order to limit the over training with subjective data due to many possible combination. The main objective is not to determine the best possible combination but to find out a tradeoff tendency. Main differences between chosen combinations are related to the weight assigned to disparity distortions compared to intrainage (left or right) distortions:  $d_3$  only considers the disparity distortion while  $d_1$  and  $d_2$  combine both disparity and intrainage distortions (actually,  $d_1$  gives more weight to the disparity distortion;  $d_2$  first focuses on the 2D distortion measures and adds a cross factor related to the disparity distortion measure):

$$\begin{aligned} d_1 &= M \cdot \sqrt{D_{dg}}, \\ d_2 &= M \cdot (1 + D_{dg}), \\ d_3 &= D_{dg}. \end{aligned} \quad (2)$$

By using C4 and SSIM metrics, we obtain seven different global metrics to perform quality assessment: SSIM (no disparity), C4 (no disparity),  $d_3$  (disparity only), SSIM using  $d_1$ , C4 using  $d_1$ , SSIM using  $d_2$ , and C4 using  $d_2$ . The metric  $d_1$  limits the influence of the disparity distortion measure while  $d_2$  gives more weight to this measure.

Note that the correlation coefficient computation for disparity distortion measure can be replaced by other methods. For example, root mean square error (RMSE) can be used since this method is currently involved in disparity algorithm performances evaluation in [45], but in our context, global RMSE gives quality metrics with lower performances. We choose to present only correlation coefficient-based metrics in order to make the paper more readable.

#### 4.3. Image quality and local disparity distortion measure

In this second approach, we propose an enhancement of the metric proposed in the previous section by using the local SSIM metric in conjunction with the local disparity distortions measures. Indeed, SSIM estimates image quality by evaluating three factors: luminance, contrast, and structure constancy (refer to [38] for more details). Here, we add the contribution of a fourth factor related to the disparity distortion measure, this “weight” being related to disparity constancy. Following this idea, we propose to measure locally the disparity distortion using the Euclidian distance thus obtaining a weight for the local measure (no distortion gives 1, while the maximum distortion measure gives 0). The proposed metric is thus evaluated by measuring the local SSIM measure map  $M_{map}$  and by fusing it with the local disparity distortion measure using point-wise product. The evaluated disparity distortion measure for each pixel  $p$  for each view is the following (here for the left view):

$$Ddl_{left}(p) = M_{map, left}(p) \left( 1 - \frac{\sqrt{Disp.Or(p)^2 - Disp.Dg(p)^2}}{255} \right). \quad (3)$$

The final quality value  $Ddl_1$  is obtained by first computing the mean value of the  $N$  pixels of  $Ddl_{left}$  and  $Ddl_{right}$  maps and by averaging both results (see Figure 5) as follows:

$$Ddl_1 = \frac{1}{2} \left( \frac{1}{N} \sum_N Ddl_{left}(p) + \frac{1}{N} \sum_N Ddl_{right}(p) \right). \quad (4)$$



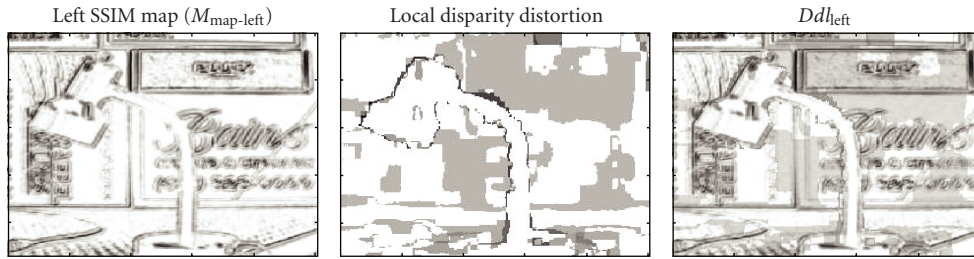


FIGURE 6: Sample of local SSIM enhancement; from left to right: original SSIM map, the local disparity distortion map, and the  $Ddl_{left}$  map.

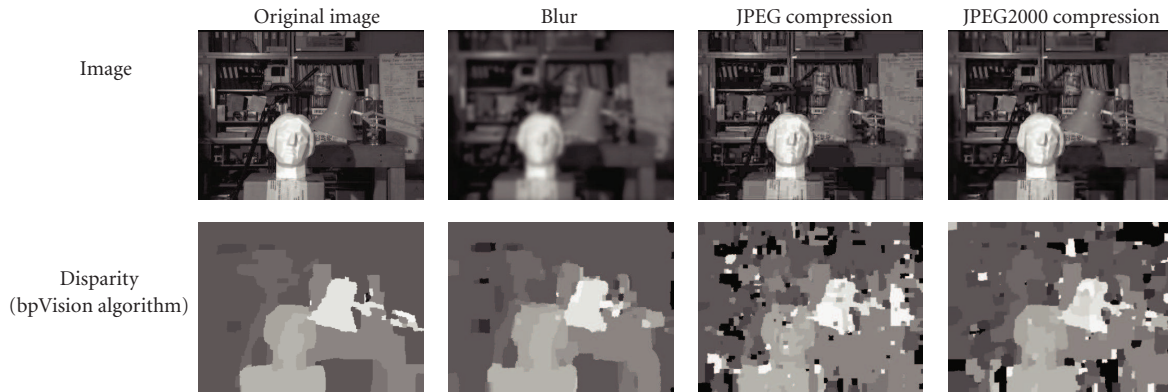


FIGURE 7: Sample of image degradations applied to the same image and the corresponding disparity maps.

Figure 6 shows examples of a 2D SSIM map result (here for the left view), the local disparity distortion map obtained with Euclidian distance measure, and the corresponding  $Ddl_{left}$  map.

## 5. RESULTS

We have computed these quality metrics on stereopairs when applying JPEG, JPEG2000 compression, and blur filtering. Figure 7 shows examples of image degradation and the corresponding disparity maps.

Contrary to [15] where the metrics were evaluated independently on each image distortion, we evaluate here the performance of the metrics on all distortions at the same time (e.g., mapping is applied on the overall database). As a consequence, we consider simultaneously a larger spectrum of possible distortions. We report the performance of all the considered metrics in the same table in order to compare them directly.

Results before mapping are presented in Table 2. We show the correlation coefficient  $CC$  between the measured subjective DMOS (ERRATA Section 3.3, (1) CORRIGE Section 3, (1)) and the scores obtained with the proposed objective metrics,  $CC$  being a reliable performance monotonicity indicator [40]. The original SSIM and C4 metrics are compared with the new approaches including disparity distortion information, using the two disparity computation algorithms bpVision and kz1.

Significant performance improvements can be observed with the SSIM-based metrics (SSIM with  $d_1$ ,  $d_2$ , and  $Ddl_1$ )

using the bpVision disparity algorithm. When comparing the correlation coefficient of both original 2D objective quality  $M$  proposed in [15] and the disparity distortion  $d_3$ , with the new proposed metrics, we can see that they are less correlated to the subjective DMOS than the proposed new metrics  $d_1$ ,  $d_2$ , and  $Ddl_1$ . Indeed, the original SSIM metric and disparity degradation give correlation coefficient equal to 0.77 and 0.67, respectively, while SSIM  $d_1$ ,  $d_2$ , and  $Ddl_1$  metrics give correlation coefficient values equal to 0.84, 0.85, and 0.88, respectively. Then, linear combinations of 2D metrics and disparity distortion measure give better results in the SSIM case. More in detail, when considering SSIM  $Ddl_1$  metric with the bpVision disparity algorithm, the resulting correlation coefficient performs even better and gives results close to C4 metric, the correlation coefficient difference being only 0.03.

In parallel, global metrics based on C4 are not enhanced by the added disparity information. Since C4 model is a perceptual metric, this fact may confirm that quality for static 3D images does not depend on the depth information as hypothesized in [16, 22]. However, when the disparity computation algorithm kz1 is used, results are more contradictory. In fact the disparity distortion  $d_3$  using kz1 is much less correlated with the subjective DMOS than when using bpVision algorithm (correlation coefficient varies from 0.59 for kz1 to 0.67 for bpVision). As a consequence, its contribution in the proposed metric is expected not to be efficient. As expected, the performance of global approaches  $d_1$  and  $d_2$ , and local metrics  $Ddl_1$  do not increase the performance of the original SSIM metric, with a correlation

TABLE 2: Metrics' performances synthesis before mapping.

	SSIM [27]	SSIM $d_1$	SSIM $d_2$	C4 [27]	C4 $d_1$	C4 $d_2$	$d_3$	SSIM $Ddl_1$
CC <sub>1</sub> bpVision	0.77	0.84	0.85	0.91	0.91	0.90	0.67	0.88
CC <sub>2</sub> kz1		0.78	0.79		0.89	0.88	0.59	0.79

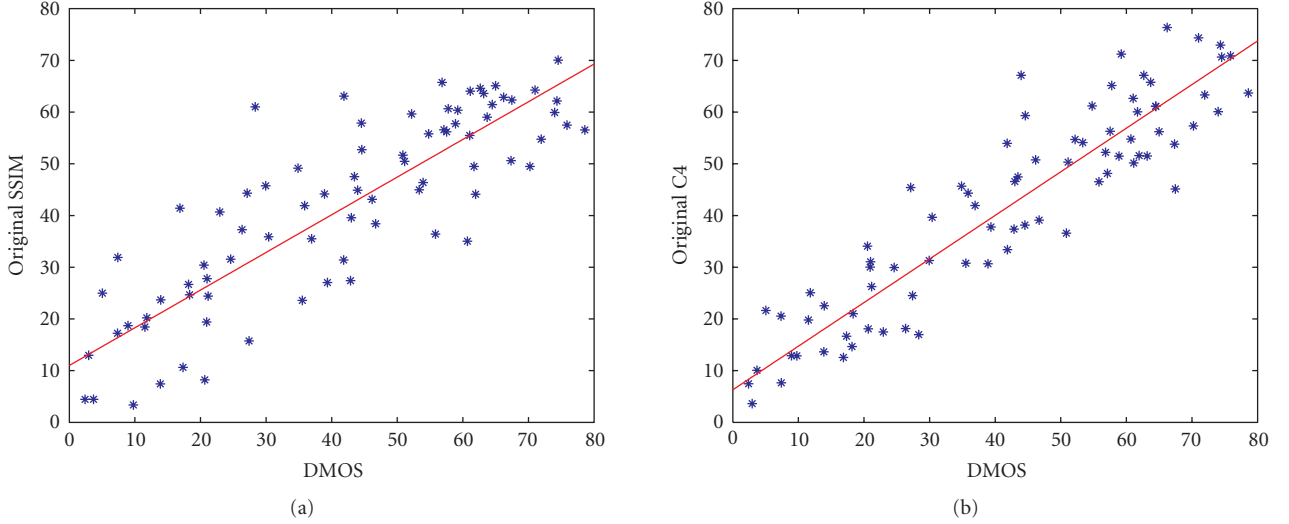


FIGURE 8: Couple of points (DMOS, mapped objective score) for original SSIM and C4 metrics.

coefficient increase of 0.02, while it decreases with C4-based metric.

Therefore, we can observe that combining the disparity distortion measure with SSIM metric enhances performances and gives results close to perceptual objective metrics such as C4. Also, the smoother disparity maps computed by the kz1 algorithm do not allow a significant performance increase while the sharper belief propagation method (bpVision) performs better.

Figure 8 shows couples of points (DMOS, mapped objective score) for SSIM and C4 original metrics. We can see from this figure that C4 correlation coefficient is higher while its RMSE is slightly lower.

Figure 9 compares plotted DMOS versus mapped objective score for SSIM using  $d_2$  metric for both disparity algorithms. We can see that kz1 disparity computation algorithm gives a more disperse plot and, as a consequence, a lower correlation coefficient. The smoother disparity maps of the kz1 algorithm are less correlated with perceived image quality than bpVision algorithm.

Figure 10 shows DMOS versus mapped objective score for SSIM  $Ddl_1$  metric with different symbols per type of image distortion. We can see that no particular distortion has a specific localization in the plot, which means that the performances of the metric do not depend on the distortion type. Also, compared to the original SSIM 2D metric, the correlation coefficient has increased and it is close to the C4 perceptual metric.

Table 3 presents more complete results obtained after data mapping, performed as detailed in Section 4.1. After this operation, more indicators metric performance becomes

available such as root mean square error ( $RMSE$  on a unitary scale) between the subjective DMOS and the objective metrics. A low  $RMSE$  value indicates a reliable accuracy of the metric with regard to the subjective DMOS. Also, the outlier ratio ( $OR$ ) is available and indicates the relative number of samples which are out of the subjective DMOS confidence interval (95%) as specified in [40]. This outlier ratio indicates the consistency of the metric with regard to the subjective measures (the lower value presents better consistency). Note that we use the confidence interval of subjective DMOS measures since such measure is based on the mean score given by a set of observers during the subjective evaluation session.

Compared to the original metrics coming from [15] which do not take into account the disparity information, the increase of the correlation coefficient due to mapping is less significant for the new metrics. We obtain a maximum correlation coefficient increase of 0.03 with the new metrics ( $d_1$ ,  $d_2$ , and  $Ddl_1$ ) while the original SSIM metric increased by 0.08 and C4 without disparity increased by 0.01. This shows that the SSIM-based metrics which include disparity distortion measures are basically more correlated with the DMOS without the help of mapping. In addition, considering metrics based on bpVision algorithm, RMSE remains stable. More precisely, SSIM-based methods ( $d_1$ ,  $d_2$ ) do not increase the original RMSE while  $Ddl_1$  allows to decrease it. In parallel, results after mapping confirm the poor results obtained when kz1 disparity algorithm is used in these new metrics. When observing the outliers ratios, we can see that with the bpVision disparity computation algorithm the ratios are lower than the ones obtained with

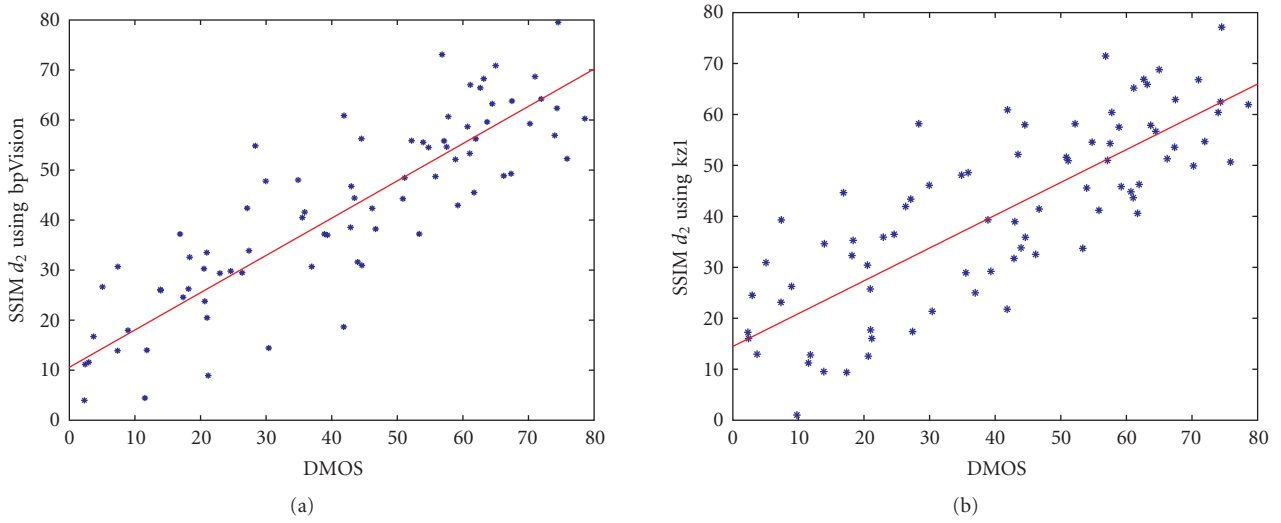


FIGURE 9: Couple of points (DMOS, mapped objective score) for SSIM  $d_2$  metric using bpVision and kz1 disparity computation algorithm.

TABLE 3: Metrics' performances synthesis after mapping.

	SSIM [27]	SSIM $d_1$	SSIM $d_2$	C4 [27]	C4 $d_1$	C4 $d_2$	$D_3$	SSIM $Ddl_1$
CC bpVision	0.85	0.86	0.86	0.92	0.92	0.90	0.70	0.90
RMSE bpVision	0.47	0.46	0.46	0.36	0.37	0.40	0.65	0.41
OR bpVision	4%	4%	4%	1%	2%	2%	15%	2%
CC kz1	0.85	0.80	0.80	0.92	0.89	0.88	0.64	0.82
RMSE kz1	0.47	0.54	0.55	0.36	0.41	0.44	0.90	0.51
OR kz1	4%	10%	11%	1%	2%	5%	27%	7%

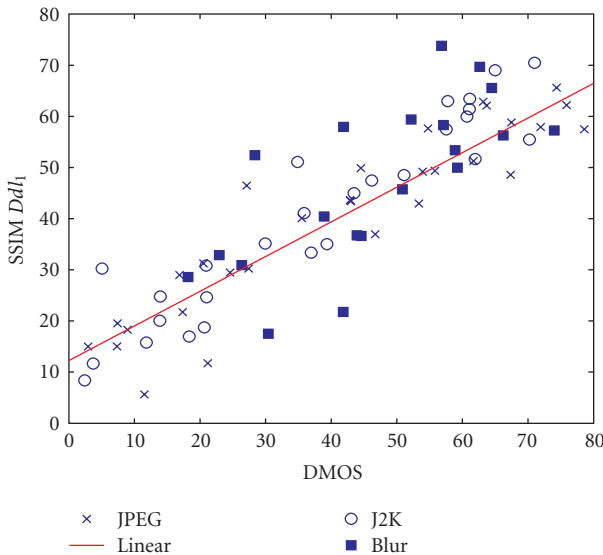


FIGURE 10: Couple of points (DMOS, mapped objective score) for SSIM  $Ddl_1$  metrics.

kz1 algorithm, and they are very close to the original SSIM and C4 2D metrics values. Belief propagation method for disparity computation is still more correlated with subjective results.

TABLE 4: Metrics' performances significance measure using Fisher test.

	M [27] versus $d_1$	M [27] versus $d_2$	M [27] versus $Ddl_1$
F statistic	0.2011	0.2530	0.2344

In order to validate such metric performance increase assumption, significance tests such as Fisher r-to Z statistical test [46] confirm that the correlation values difference between the original SSIM-based metric and the three new SSIM- and bpVision-based metrics  $d_1$ ,  $d_2$ , and  $Ddl_1$  is significantly different (see Table 4). In this table, the computed probabilities associated to the F statistic, which compare the differences among the previous and the new metrics, are reported. All these values are greater than the critical value 0.05 such that the assumption of homoscedasticity is met for each proposed new metric.

To summarize, belief propagation-based disparity (bpVision algorithm) enhances the SSIM metric and gives results close to a perceptually based metric like C4. The choice between C4 and SSIM with  $Ddl_1$  metric can be done by taking into account the computational cost of the two algorithms. In fact, C4 is a very time consuming algorithm since it integrates a global contrast sensitivity function inspired by the human visual system followed by a number of image filtering performed to determine salient areas where human beings are most likely to discriminate artifacts. In

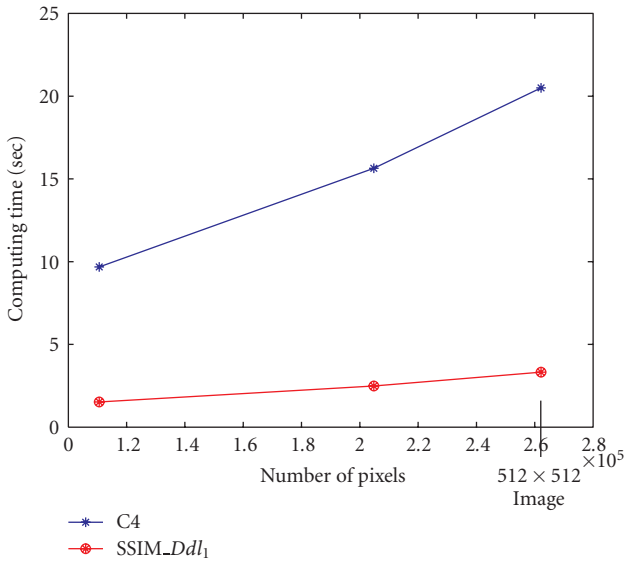


FIGURE 11: Evolution of the computation time of C4 and enhanced SSIM with  $Ddl_1$  metrics versus the image size (number of pixels).

order to compare the algorithm computational efficiency, we have evaluated the computational time of both the C4 algorithm and the SSIM local method using  $Ddl_1$  metric on an Intel Quad Core 2.4 GHz based computer equipped with 4 Gb of RAM (each algorithm using only one thread). We report on Figure 11, the evolution of the computation time versus the image resolution for the two considered stereo metrics. As shown in Figure 11, the computational time of both metrics increases with the image resolution. However, a significant difference can be observed between the 2 metrics; in fact SSIM with disparity-based method is at least 5 times faster. For example, when the metrics are computed for pictures of size  $512 \times 512$  pixels, the computational time of C4 is approximately 20 seconds while for SSIM enhanced metric with  $Ddl_1$  3 seconds are enough to perform the computation. To summarize, the considered metrics give similar quality evaluation performances but the SSIM-based metric computational time is much lower.

## 6. DISCUSSION AND CONCLUSIONS

In this paper, we have first reviewed the main quality issues in stereo data, highlighting the different aspects of both stereo technologies and stereo perception. From quality evaluation to the viewing experience, 3D involves additional factors with respect to the fruition of bidimensional content. The main goal of this paper is to introduce an objective quality metric for stereo images quality assessment. The proposed metric relies on both the use of 2D metrics and depth information. The presented results give some hints about the complex problem of stereo quality metric design. First we can notice that C4 well correlates with the subjective experiments when no disparity information is used and that it is not enhanced by the added disparity distortion information. This confirms

in a way the assumption that depth does not impact on the quality assessment given in [16, 22].

Then, it has been shown that SSIM is enhanced when adding the disparity distortion contribution. This fact may be related to the fact that the luminance, contrast, and structure criterion evaluation of the original SSIM are not sufficient to assess quality from a perceptual point of view. Then, the use of the disparity information brings to an enhancement of the original metric. As for the disparity computation algorithm, it has been shown that, within this framework, belief propagation-based algorithms are more efficient than graph cuts based methods. Indeed, the sharper disparity maps coming from belief propagation are more correlated with subjective quality metrics than smoother graph cuts maps.

Finally, we have pointed out that the 3D quality assessment method based on the use of 2D C4 metric is as efficient as the enhanced SSIM with local disparity distortion measure introduced in this paper but has a higher computational cost. In this paper, we proposed an approach involving 2D quality metrics while taking into account the stereo disparity information; this can be considered as the final limit of the conventional 2D approaches. It is worth pointing out that dealing with stereo data introduces a new perspective; in fact instead of dealing with quality assessment we should refer to quality of experience. Indeed, since 3D involves new perception factors such as the feeling of immersion, presence [14], and so forth, image quality is not anymore sufficient to represent the quality of the experience done by the observer when immersed in a stereo environment. Then, it is necessary to build a new setup which would take into account all the factors related to 3D. The first attempt has been drawn in [16] where image quality contributes with depth information to a more global “naturalness” model which contributes to a main “3D visual experience” model. But the impact of depth and visual comfort is still waiting to be investigated. New test setups have to be defined to identify all the factors related to 3D visual experience.

Our results show that the depth information can improve quality metric but the relation with image “naturalness,” “viewing experience,” and “presence” has still to be investigated in depth, depending also on the different 3D display technology used. Some of these factors have been explored from a subjective point of view in [16] but a complete analysis which could bring the definition of a universal and objective quality metric for quality of experience assessment for stereo images and video is still far to come.

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