Wind-forced modulations of gravity waves

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WIND-FORCED MODULATIONS OF GRAVITY WAVES

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Summary The effect of wind on modulational instability of gravity waves is presented. A forced nonlinear Schödinger equation that governs the evolution in deep water of weakly nonlinear packets of surface gravity waves under wind forcing is derived. Stokes waves are shown to grow exponentially following Miles' linear mechanism, while modulational instability becomes explosive. These analytical results are completed by numerical simulations and experiments showing the amplification of modulations by wind. Therefore, we suggest that wind boosts the formation of rogue waves.

INTRODUCTION

In 1957, Miles [1] discovered a linear mechanism for the amplification of infinitesimal wavy disturbances at the interface between air and water when wind blows. Ten years later, Benjamin and Feir [2] and Zakharov [3] discovered independently the modulational instability of a finite amplitude wave train. While the modulational instability has been observed both experimentally and numerically [4], the effect of wind on this latter remains a matter of controversy. Indeed, experimental results show either suppression [5] or enhancement [6] of modulations, while numerical models predict either amplification [7] or downshifting [8].

THE FORCED NLS EQUATION AND EXPLOSIVE MODULATIONS

Theoretically, some progress has been made recently for modulational instability under wind forcing [9]. Let

$$\eta(x,t) = \mathrm{Re}\{a(x,t)e^{i(kx-\omega t)}\} + O(|a|^2) \tag{1}$$

be the surface elevation of a narrow-banded gravity wave packet modulated around a carrier wave with wavenumber $|k|$ and frequency $\omega = \sqrt{g|k|}$, that propagates in deep water under the forcing of a steady wind with velocity profile $U(z)$ along the x-direction, and such that $U(0) = 0$. Let $\delta = \rho_a/\rho_w$ be the density ratio between air and water ($\delta = 1.29 \times 10^{-3}$), both assumed inviscid. The amplitude of the wave packet is governed by the forced NLS equation [7, 8, 9]

$$i(\partial_t a + c_g \partial_x a) - (\omega/8k^2)\partial_{xx} a - (\omega k^2/2)|a|^2 a = \tfrac{1}{2}\omega(\alpha + i\beta)a, \tag{2}$$

where $c_g = \partial_k \omega$ is the group velocity, and α and β are real numbers defined such that $\alpha + i\beta = \delta(\chi_0'/k + U_0'/\omega - 1)$, using the notation $f_0' = (\partial_z f)_{|z=0}$. The function $\chi(z)$ solves Rayleigh equation in the air ($z > 0$):

$$(U - c)(\chi'' - k^2\chi) - U''\chi = 0, \quad c = \omega/k, \tag{3}$$

with boundary conditions $\chi(0) = 1$ and $\chi(\infty) = 0$. Integration of (3) yields $\beta = 0$ except if there exists a critical height z_c such that $U_c = U(z_c) = \omega/k$. In that case, according to Miles [1], $\beta = -\pi\delta U_c''|\chi_c|^2/|kU_c'|$ so that $\beta > 0$ for usual wind profiles ($U''(z) < 0$). Therefore, the wave with wavenumber k grows exponentially with time in the linear regime.
In the weakly nonlinear regime, it is convenient to recast (2) as

$$2iA_T - \tfrac{1}{4}A_{XX} - |A|^2 A = i\beta A, \tag{4}$$

where $T = \omega t$, $X = k(x - c_g t)$, and $A(X,T) = ka(x,t)e^{i\omega\alpha t}$. If $\beta = 0$ (no wind forcing), the usual NLS equation is recovered and Stokes wave is described by the homogeneous solution $A(T) = R_0 e^{i(\Theta_0 - R_0^2 T/2)}$. With wind forcing ($\beta > 0$), Stokes wave becomes $A(T) = R_S(T)e^{i\Theta_S(T)}$ where: $R_S(T) = R_0 e^{\beta T/2}$ and $\Theta_S(T) = \Theta_0 - \frac{1}{2}R_S^2(T)/\beta$ so that it grows exponentially with time without saturation.
If we now disturb this solution by setting $|A(X,T)| = R_S(T)(1 + \lambda(T)\cos(KX))$, Eq. (4) yields, upon linearization:

$$\lambda''(T) + \gamma(\gamma - R_0^2 e^{\beta T})\lambda(T) = 0, \quad \gamma = \tfrac{1}{8}K^2. \tag{5}$$

If $\beta = 0$, the disturbance grows exponentially when $0 < \gamma < R_0^2$: this is the modulational instability [2, 3]. If $\beta > 0$, the solution of (5) may be expressed in terms of modified Bessel function and, for large time [9]:

$$\lambda(T) \sim 2e^{f(T)}/\sqrt{2\pi f(T)}, \quad f(T) = 2\gamma^{\frac{1}{2}} R_0 e^{\beta T/2}. \tag{6}$$

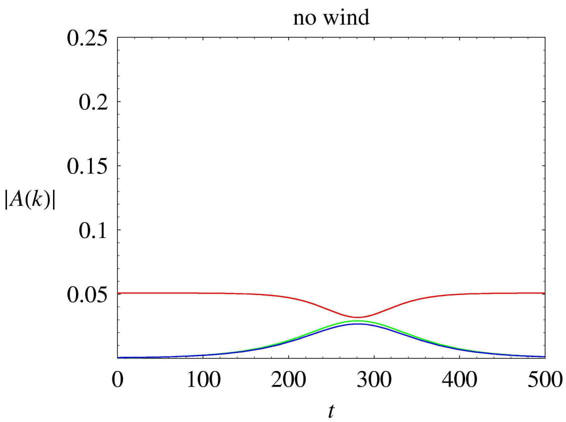
Thus, the modulations grow *superexponentially* under fair wind. The growth is however so rapid that nonlinear effects not considered here should rapidly interact. This is perhaps the reason why this surprising behavior have not been observed in experiments. Furthermore, the validity of the present approach is restricted to steepnesses $|ka|$ of order $\sqrt{\delta}$; above, higher order approximations or numerical computations are needed.

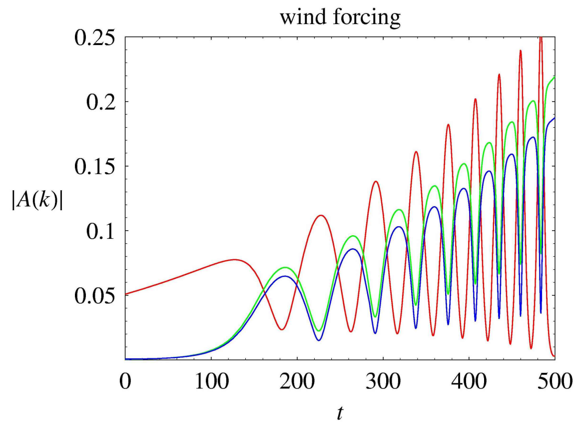
NUMERICAL COMPUTATIONS AND EXPERIMENTAL RESULTS

To test the validity of the linear results mentioned above, numerical simulations of the forced Zakharov equation [7]:

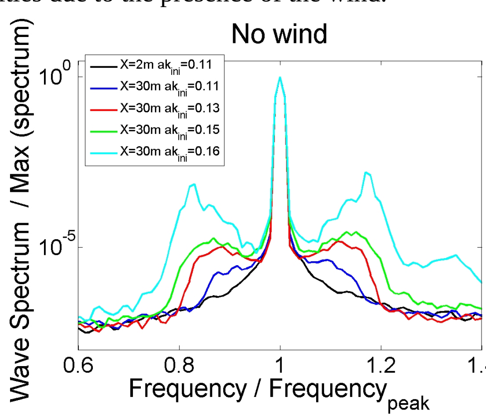
$$i\partial_t A_{\mathbf{k}} - \omega_{\mathbf{k}} A_{\mathbf{k}} - \int_{\mathbf{k}+\mathbf{p}=\mathbf{q}+\mathbf{r}} T_{\mathbf{k},\mathbf{p},\mathbf{q},\mathbf{r}} A^*_{\mathbf{p}} A_{\mathbf{q}} A_{\mathbf{r}} d\mathbf{p} d\mathbf{q} d\mathbf{r} = \tfrac{1}{2}\omega_{\mathbf{k}}(\alpha_{\mathbf{k}} + i\beta_{\mathbf{k}})A_{\mathbf{k}}, \tag{7}$$

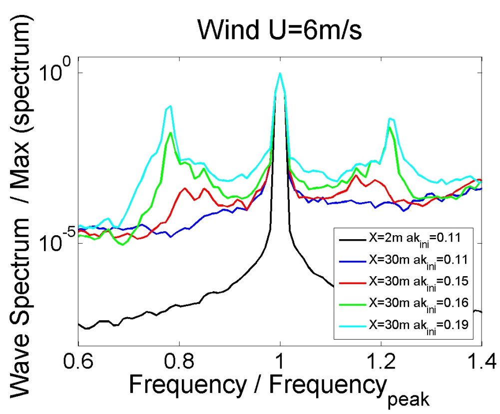
have been performed for three modes. Here, $A_{\mathbf{k}}$ is Zakharov canonical variable [3] and T is Krasitskii's nonlinear transfer term. Figure below on the left shows the evolution of a Stokes wave with steepness 0.1 and wavenumber $|\mathbf{k}| = 1.97\ \mathrm{m}^{-1}$ (in red), disturbed by two sideband disturbances (blue and green) that grow with time owing to modulational instability and then decreases in a recurrent way [4]. In the presence of wind (right), Stokes waves in red starts to grow exponentially with rate $\frac{1}{2}\omega\beta$ according to the theory exposed previously (here, $\beta \approx 7.5 \times 10^{-3}$). Then, the two disturbances (blue and green) grow suddenly much faster than Stokes wave, in fact close to the predicted superexponential growth. Then, nonlinearity makes its job, resulting in the acceleration of the cycles of modulation/demodulation. At larger times, energy must saturate either by breaking or by quenching of the wind profile [10], effects that are taken into account neither in (2) nor in (7). Such effects are actually under investigation, as far as computations based on the fully coupled Euler equations.





Experiments have been conducted in the large air-sea interaction facility of the IRPHE laboratory in Marseille. The water tank is 40 m long, 3 m wide, 1 m deep and the height of the aerodynamical flow above the water surface is 1.5 m. The wind velocity can be adjusted from 0.5 to 14 m/s. Mechanical waves in the frequency range 1 to 2 Hz with various amplitudes can be generated by a completely immersed wavemaker. We performed several runs with regular mechanical generated waves, with different steepness. Experiments were first conducted without wind, then with a $U = 6$ m/s wind blowing over the paddle waves. Waves were recorded with two capacitances wave gauges located at 2 m and 30 m away from the wavemaker. The last two figures show the normalized frequency spectrum of the water elevation for different initial wave steepness. Obviously, at 30 m away from the wavemaker, there is a large increase of the Benjamin-Feir side-band instabilities due to the presence of the wind.





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