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Fluctuational magnetization and conductivity in anisotropic Fulde-Ferrell-Larkin-Ovchinnikov superconductors

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We derive the magnetization and the paraconductivity of Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superconductors in their normal state. The FFLO superconducting fluctuations induce oscillations of the magnetization between diamagnetism and unusual paramagnetism which originates in the competition between paramagnetic and orbital effects. We also predict a strong anisotropy of the paraconductivity when the FFLO transition is approached in contrast with the case of a uniform BCS state. Finally building a Ginzburg-Levanyuk argument, we demonstrate that these fluctuation effects can be safely treated within the Gaussian approximation since the critical fluctuations are prominent only within an experimentally inaccessible temperature interval.

I. INTRODUCTION.

Forty years ago, Fulde and Ferrell¹, and Larkin and Ovchinnikov² predicted that the paramagnetism of the electron gas might induce a novel superconducting state wherein the order parameter is modulated in real-space. In their original proposal, these authors considered a singlet *s*-wave superconductor perturbed by the Zeeman effect only, and neglected completely the orbital coupling and the disorder. For most type-II superconductors, the superconductivity is destroyed by the orbital pair-breaking effect which leads to a more familiar inhomogeneous superconducting state: the Abrikosov vortex lattice. In order to observe the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, the paramagnetic effect must break Cooper pairs more efficiently than the orbital one. Such a situation may be realized in tridimensional (3D) superconductors with large internal exchange fields, like the rare-earth magnetic superconductor ErRh₄B₄, see³ for a review. Another possibility corresponds to a quasi two-dimensional (2D) layered superconductor wherein the weakness of the interplane hopping suppresses the orbital effect for in-plane magnetic field. Being the ratio of the critical fields in the pure orbital $H_c^{orb}(T=0)$ and pure paramagnetic limit $H_p(T=0)$, the Maki parameter $\alpha_m = \sqrt{2}H_c^{orb}(T=0)/H_p(T=0)$ is a measure of the relative strength of those pair breaking mechanisms. Besides demanding a large Maki parameter ($\alpha_m > 1.8$), the occurrence of the FFLO state also requires very clean samples since it is far less robust against disorder than the usual vortex lattice, see⁴ and⁵ for recent reviews.

Recently, there have been mounting evidence that the heavy fermion superconductor CeCoIn₅ under magnetic field might fulfill those stringent conditions^{6,7,8}. The magnetism of this system is still under debate, but mysterious superconducting phases have been reported at large magnetic field and low temperature, both for parallel and perpendicular orientations of the field relatively to the basal plane of the tetragonal CeCoIn₅ lattice⁵. In the field-induced organic superconductor⁹ λ -(BETS)₂FeCl₄, and in the layered organic superconductor¹⁰ κ -(BEDT-TTF)₂Cu(NCS)₂ the FFLO state have been reported

when a strong magnetic field (20 T for latter one) is applied along the superconducting planes.

However in practice, the identification of the FFLO state is hindered by the interplay between orbital and paramagnetic effects. The first available experimental clue is the shape of the transition line $H_c(T)$ separating the normal state from the inhomogeneous superconducting state. A lot of theoretical works have been devoted to the description of this $H_c(T)$ line. For moderate Maki parameters, $\alpha_m < 9$, the structure of the FFLO modulation involves a zero Landau level (index $n = 0$) function (Gaussian with no additional modulation)¹¹. For higher Maki parameter, $\alpha_m > 9$, the Cooper pair wave function of a 3D superconductor consists in a cascade of more exotic solutions, the so-called multi-quanta states, which are described by a higher (index $n > 0$) Landau level¹². Such values of Maki parameters are rather high for 3D compounds (for instance CeCoIn₅ has $\alpha_m = 4.6 - 5$) but they can be achieved in layered quasi-2D superconductors (or superconducting thin films) under in-plane magnetic fields¹³. All these studies were performed so far in the framework of isotropic models, namely for the idealistic case of a spherical Fermi surface in the normal state. Moreover it has been shown that an elliptic Fermi surface leads to the same phenomenology at cost of introducing an angle-dependent Maki parameter¹⁴.

In real compounds, the crystal lattice (or the pairing symmetry) induces a non trivial anisotropy which matters a lot for the modulated state^{15,16} since it essentially lifts the degeneracy between various orientations of the FFLO modulation. Recently, the interplay of paramagnetic and orbital effects was reconsidered in the presence of such a non trivial anisotropy, namely for a Fermi surface which slightly differs from the spherical or elliptical shape¹⁷. Using a perturbative approach, it was found that even a small anisotropy stabilizes the exotic multi-quanta states which can therefore exist at lower Maki parameter (any $\alpha_m > 1.8$) than predicted by the idealized isotropic models. According to this prediction such states are therefore likely to occur in any real anisotropic Pauli limited superconductor. More specifically in the tetragonal symmetry, 3 scenarios are possible for the FFLO

state: a) Maximal FFLO modulation along the field with zero Landau level state, b) Highest Landau level modulation in the plane perpendicular to the field and no FFLO modulation, and c) Both Landau level and FFLO modulations. This three scenario picture corresponds to the tetragonal symmetry and was derived within a single Landau level approximation, which is valid at large field. It may thus be relevant to explain the observation of two mysterious high-field and low temperature phases of CeCoIn₅, existing for distinct magnetic field orientations (field inside or perpendicular to the CeIn₃ planes) with contrasted behaviors. Nevertheless the shape of the $H_c(T)$ transition line is far from sufficient to establish a clear correspondance between one phase and a particular class of solutions among the three a-c possibilities. It is thus necessary to gain complementary informations about the transition in order to determine which scenario among a-c is actually realized. As natural precursors of the transition, the fluctuations provide informations about the symmetry breaking superconducting state and we shall show here that they enable to detect the presence of a FFLO state and to discriminate between the various a-c scenarios in the tetragonal 3D case.

In this paper, we evaluate the fluctuation induced conductivity (paraconductivity) and magnetization near the FFLO transition in both 2D and 3D anisotropic superconductors. We use the modified Ginzburg-Landau functional^{18,19,20} which is valid at the vicinity of the tricritical point. Previously we calculated the fluctuational specific heat and conductance near the pure FFLO transition in the absence of orbital effect²¹. Our motivation was to establish a relation between the topology of the lowest energy fluctuation modes and the divergencies of the physical properties at the pure FFLO transition. In the isotropic model, those divergencies are very different than the standard BCS ones since the topologies of the degenerate FFLO and BCS modes differ fundamentally. Unfortunately, in the anisotropic models, this degeneracy is lifted and the topologies of FFLO and BCS modes become quite similar, thereby leading to less contrasted behaviors.

Here we show that the anisotropy ratio σ_{xx}/σ_{yy} of the fluctuational conductivity near the FFLO transition differs drastically than the one near a standard BCS transition. Moreover we demonstrate that the fluctuation induced magnetization of thin films may oscillate between positive (paramagnetism) and negative (diamagnetism) values. These oscillations originate from the competition between orbital and paramagnetic effects which tend to promote respectively Landau level modulation and FFLO modulation. Being precursors of the Meissner or Abrikosov lattice state, the superconducting fluctuations are usually diamagnetic. Therefore the paramagnetism predicted here is a hallmark of the unconventional FFLO state. This result can be easily extended to the case of layered 2D compounds like λ -(BETS)₂FeCl₄ or κ -(BEDT-TTF)₂Cu(NCS)₂. In 3D superconductors under high magnetic field, these oscilla-

tions are blurred out when scenario a) is realized whereas they pertain when scenario b) takes place, thereby providing an experimental test to distinguish among the various possible structures of the order parameter described in Ref.¹⁷. Experimentally, the superconducting fluctuations in CeCoIn₅ have been investigated far above T_c and under low fields^{22,23}. Here we suggest similar measurements near T_c under strong magnetic field and near the FFLO critical temperature.

The paper is organized as follows. In Sec II, we present the modified Landau-Ginzburg formalism and the low energy effective functionals used in the sequel of the paper. In Sec III, we analyse the case of thin superconducting films under in-plane magnetic field and predict a strong dependence of the conductance upon the mutual orientation of the current flow and magnetic field. We also derive the fluctuation magnetization induced by a tilted magnetic field pointing out of the film plane. In Sec IV, we discuss the 3D anisotropic compounds with emphasis on the fluctuation magnetization. Finally, we provide a detailed derivation of the Ginzburg-Levanyuk criterion for the FFLO transition in order to discuss the role of the critical fluctuations.

II. FORMALISM

At the vicinity of the tricritical point, the FFLO transition can be described by a modified Ginzburg-Landau functional derived from the microscopic BCS theory including the Zeeman effect. This modified Ginzburg-Landau functional was first derived in the case of clean isotropic s -wave superconductors¹⁸ and has been extended to clean/dirty (s -wave and d -wave) singlet superconductors with various crystalline symmetry^{19,20}. The isotropic functional consists in the following expansion with respect to the superconducting order parameter Ψ and its spatial derivatives:

$$\begin{aligned}
 H[\Psi(\mathbf{r})] = & \int d\mathbf{r} \left[\alpha |\Psi|^2 - g |\nabla\Psi|^2 + \gamma |\nabla^2\Psi|^2 \right] \\
 & + \int d\mathbf{r} \left[\beta_1 |\Psi|^4 + \mu |\Psi|^2 |\nabla\Psi|^2 + \beta_2 |\Psi|^6 \right] \\
 & + \int d\mathbf{r} \left[\eta \left((\Psi^*)^2 (\nabla\Psi)^2 + (\Psi)^2 (\nabla\Psi^*)^2 \right) \right] \quad (1)
 \end{aligned}$$

Here $\alpha = a(T - \tilde{T}_c)$ changes sign at the critical temperature $\tilde{T}_c(h)$ of the uniform superconducting second order phase transition. The other coefficients also depend on the temperature and Zeeman splitting h . In particular, the coefficient g is positive when $h/2\pi T > 0.3$ and negative when $h/2\pi T < 0.3$. This induces an inhomogeneous superconducting phase ($g > 0$) below the FFLO critical temperature $T_c(h)$. This FFLO critical temperature $T_c(h)$ is larger than $\tilde{T}_c(h)$ when $h/2\pi T > 0.3$. Moreover, γ is positive in the vicinity of the tricritical point thereby ensuring a finite wave vector modulation.

The first line of the MGL functional Eq.(1) is sufficient to study the Gaussian fluctuations. In section V we evaluate the effect of the two other lines and demonstrate that they can be safely neglected by establishing a Ginzburg-Levanyuk criterion for the FFLO transition. In the Fourier representation, the Gaussian functional is given by

$$H[\Psi] = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} |\Psi_{\mathbf{k}}|^2. \quad (2)$$

where the fluctuation spectrum is $\varepsilon_{\mathbf{k}} = \alpha - gk^2 + \gamma k^4$ in the isotropic case corresponding to Eq.(1). In the sequel of this paper, we will consider the effects associated with an anisotropic spectrum which can be written generically as:

$$\varepsilon_{\mathbf{k}} = \alpha - g_i k_i^2 + \gamma_{ij} k_i^2 k_j^2. \quad (3)$$

The terms $\gamma_{ij} k_i^2 k_j^2$ generically produce a deviation from an elliptic Fermi surface, excepting when the coefficients g_i and γ_{ij} are related in a very special manner (such that $\varepsilon_{\mathbf{k}}$ is a function of \mathbf{k}^2 and \mathbf{k}^4 only).

Finally the density of free energy F , the paraconductivity tensor $\sigma_{\alpha\beta}$ and the magnetization M are given by the expressions^{24,25,26}

$$F = \frac{k_B T}{L^d} \sum_{\mathbf{k}} \ln \frac{\varepsilon_{\mathbf{k}}}{\pi k_B T} \quad (4)$$

$$\sigma_{\alpha\beta} = \frac{\pi e^2 a k_B T}{4\hbar} \frac{1}{L^d} \sum_{\mathbf{k}} \frac{v_{\mathbf{k}\alpha} v_{\mathbf{k}\beta}}{\varepsilon_{\mathbf{k}}^3} \quad (5)$$

$$M = -\frac{\partial F}{\partial H} \quad (6)$$

where $v_{\mathbf{k}\alpha} = \partial \varepsilon_{\mathbf{k}} / \partial k_{\alpha}$ is the velocity component along the α axis.

III. SUPERCONDUCTING THIN FILMS

Here we consider thin superconducting films under in-plane magnetic field wherein the FFLO state is expected to occur due to the smallness of the orbital effect. We investigate the anisotropy of the paraconductivity when measured along or perpendicular to the applied magnetic field $\mathbf{H} = H\mathbf{e}_x$. We demonstrate that the conductivity becomes extremely anisotropic when approaching the FFLO transition, whereas $\sigma_{xx}/\sigma_{yy} \simeq 1$ near the BCS transition, thereby providing an experimental tool to identify the FFLO state. Finally, we also discuss the effect of a tilted magnetic field. Under strong perpendicular field, the magnetization exhibits oscillations between diamagnetic and paramagnetic behavior in sharp contrast with the usual fluctuation induced diamagnetism predicted and observed close to the BCS transition^{25,26}.

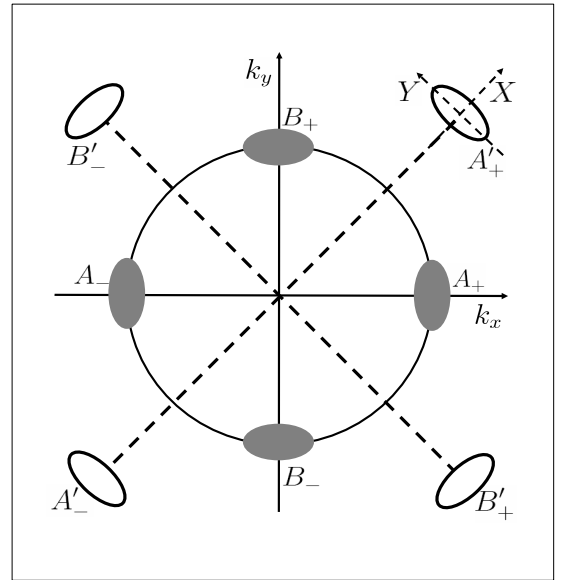


FIG. 1: Minima of the fluctuation spectrum Eq. (7) for the two-dimensional model with square symmetry. The open (filled) ellipses indicate the locations of the lowest energy fluctuations for $\eta < 1$ ($\eta > 1$). Note that in the isotropic model ($\eta = 1$), there is an infinity of degenerate minima located on the circle (solid line).

A. Square symmetry

We first consider films which are sufficiently thin to neglect the orbital effect induced by the in-plane magnetic field $\mathbf{H} = H\mathbf{e}_x$. This approximation is correct if $H\xi d \ll \Phi_0$, where ξ is the superconducting coherence length and $\Phi_0 = h/2e$ the superconducting quantum of flux. Owing to the sizeable Zeeman effect, the parameter g may change sign, thereby indicating an instability towards the inhomogeneous FFLO state when g is negative. Then in the normal state, fluctuations are described by the spectrum

$$\varepsilon_{\mathbf{k}} = \alpha - gk^2 + \gamma k^4 + 2\gamma(\eta - 1)k_x^2 k_y^2, \quad (7)$$

where $\gamma > 0$. Although the x and y directions are equivalent, the quartic terms $k_x^2 k_y^2$ introduce a non trivial anisotropy controlled by the parameter η . Isotropy is restored only for the case $\eta = 1$ which corresponds to a circular Fermi surface and has been treated elsewhere²¹.

Here we investigate the effect of the anisotropy near the FFLO transition ($g > 0$), starting with the case $\eta > 1$. The spectrum $\varepsilon_{\mathbf{k}}$ has four minima located along the x and y axis, namely at points $A_{\pm}(\pm q_0, 0)$ or $B_{\pm}(0, \pm q_0)$ of the reciprocal space, where $q_0^2 = g/2\gamma$ (Fig.1). The spectrum can be expanded around each of those minima. For instance near the minima A_{\pm} located along the x axis, the spectrum is approximated by

$$\varepsilon_{\mathbf{k}}^{(A_{\pm})} \approx \tau + 2g(k_x \mp q_0)^2 + g(\eta - 1)k_y^2 \quad (8)$$

with $\tau = \alpha - g^2/4\gamma$. Since the effective masses along x and y directions are different, the fluctuations around

each of those two minima break the symmetry between x and y axis: $\sigma_{xx}^{(A\pm)}/\sigma_{yy}^{(A\pm)} \neq 1$. Of course this symmetry must be restored if one takes into account the minima $B_{\pm}(0, \pm q_0)$ located along the y axis. Indeed the spectrum around B_{\pm} is given by

$$\varepsilon_{\mathbf{k}}^{(B\pm)} \approx \tau + g(\eta - 1)k_x^2 + 2g(k_y \mp q_0)^2, \quad (9)$$

where the effective masses along x and y axis are exchanged in comparison with (8).

Finally, we evaluate the integral Eq.(5) over the whole reciprocal space by summing the contributions from the four minima. Accordingly the total paraconductivity $\sigma_{xx} = 2(\sigma_{xx}^{(A+)} + \sigma_{xx}^{(B+)})$ diverges at the FFLO transition as:

$$\sigma_{xx} = \sigma_{yy} = \frac{e^2 a k_B}{4\sqrt{2}\hbar} \frac{1 + \eta}{\sqrt{\eta - 1}} \left(\frac{T_c}{T - T_c} \right). \quad (10)$$

The second case $\eta < 1$ can be treated along the same line of reasoning, albeit the four degenerate minima (q_{0x}, q_{0y}) are now located on the diagonals with $q_{0x}^2 = q_{0y}^2 = q_0^2/2$. The spectrum can be expanded as

$$\varepsilon_{\mathbf{k}} \approx \tau - g_{ij}(k_i - q_{0i})(k_j - q_{0j}), \quad (11)$$

around any of those four minima. Here we have $\tau = \alpha - g^2/2\gamma(1 + \eta)$ and Einstein summation over index $i, j = x, y$ is used. For instance, we find

$$g_{xx} = g_{yy} = \frac{2g}{1 + \eta} \quad \text{and} \quad g_{xy} = g_{yx} = \frac{2g\eta}{1 + \eta} \quad (12)$$

around $A'_+(\frac{q_0}{\sqrt{2}}, \frac{q_0}{\sqrt{2}})$. Diagonalization of this tensor leads to the eigenvalues $\lambda_X = 2g$ and $\lambda_Y = 2g(1 - \eta)/(1 + \eta)$ along the principal axis X and Y . Finally summation over the four minima (Fig.1) restores the equality between σ_{xx} and σ_{yy}

$$\sigma_{xx} = \sigma_{yy} = \frac{e^2 a k_B}{2\hbar} \frac{g^2 \eta^2}{(1 + \eta)^3} \sqrt{\frac{1 + \eta}{1 - \eta}} \left(\frac{T_c}{T - T_c} \right) \quad (13)$$

Note that above expressions Eqs.(10,13) both diverge for $\eta \rightarrow 1$, which indicates stronger Gaussian fluctuations in the isotropic model²¹.

B. Rectangular anisotropy

Here we consider superconducting films having non equivalent x and y directions. In order to describe the physics of the FFLO fluctuations in the absence of orbital effect (limit $H\xi d \ll \Phi_0$), we consider the spectrum

$$\varepsilon_{\mathbf{k}} = \alpha - gk^2 + \gamma_1 k_x^4 + \gamma_2 k_y^4 + 2\gamma\eta k_x^2 k_y^2 \quad (14)$$

close to the FFLO transition ($g > 0$).

The positions of the $\varepsilon_{\mathbf{k}}$ minima depend on the parameters γ , γ_1 and γ_2 (Fig.2). There are three different possibilities depending on the range of parameters:

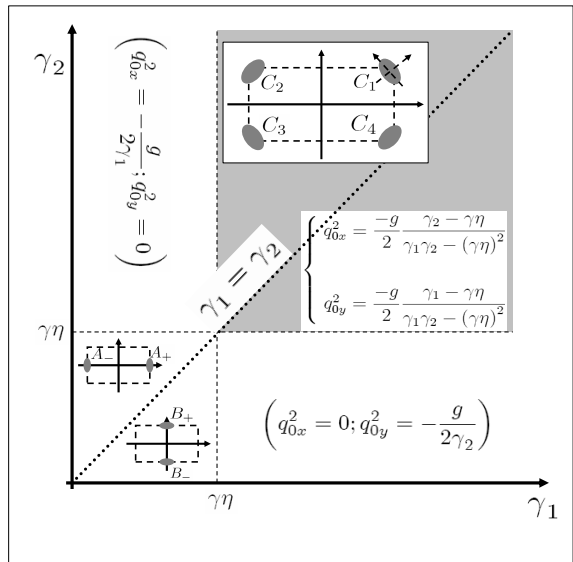


FIG. 2: Lowest energy fluctuations in the two-dimensional model with rectangular symmetry. In the gray area, the four minima of the fluctuation spectrum are located on the diagonals when both γ_1 and γ_2 exceeds a threshold value $\gamma\eta$. Otherwise, the spectrum has only two minima located on the axis x (or y). Note that the line $\gamma_1 = \gamma_2$ corresponds to the square model illustrated in Fig. 1.

two minima $A_{\pm}(\pm\sqrt{g/2\gamma_1}, 0)$ along x axis, or two minima $B_{\pm}(0, \pm\sqrt{g/2\gamma_2})$ along y axis, or four minima C_i ($i = 1, \dots, 4$). As in the preceding subsection, the paraconductivity is obtained by summing the contributions from the fluctuations around the relevant energy minima. We now provide the expressions of the conductivity ratio σ_{xx}/σ_{yy} for the different configurations of these minima.

Two minima along x axis ($\gamma_2 > \gamma_1$):

$$\frac{\sigma_{xx}}{\sigma_{yy}} = \frac{\sigma_{xx}^{(A+)} + \sigma_{xx}^{(A-)}}{\sigma_{yy}^{(A+)} + \sigma_{yy}^{(A-)}} = \frac{\gamma\eta - \gamma_2}{2\gamma_2}. \quad (15)$$

Two minima along y axis ($\gamma_2 < \gamma_1$):

$$\frac{\sigma_{xx}}{\sigma_{yy}} = \frac{\sigma_{xx}^{(B+)} + \sigma_{xx}^{(B-)}}{\sigma_{yy}^{(B+)} + \sigma_{yy}^{(B-)}} = \frac{2\gamma_1}{\gamma\eta - \gamma_1}. \quad (16)$$

Four minima shifted from the x and y axis ($\gamma_1, \gamma_2 > \gamma\eta$):

$$\frac{\sigma_{xx}}{\sigma_{yy}} = \frac{\sum_i \sigma_{xx}^{(C_i)}}{\sum_i \sigma_{yy}^{(C_i)}} = \frac{\gamma_1 \gamma\eta - \gamma_2}{\gamma_2 \gamma\eta - \gamma_1}. \quad (17)$$

In the latter case, one recovers the square symmetry ($\sigma_{xx} = \sigma_{yy}$) when $\gamma_1 = \gamma_2$. Otherwise (when $\min(\gamma_1, \gamma_2) < \gamma\eta$) the square case is retrieved provided that all the contributions from the A_{\pm} and B_{\pm} minima are collected.

C. Anisotropic films with in-plane magnetic field.

In the models presented above, the orbital motion of the electrons was neglected, which corresponds to the limit of vanishing film thickness ($H\xi d \ll \Phi_0$). Here we treat perturbatively the orbital effect of the in-plane magnetic field which fixes the direction of the FFLO modulation. Even in the square symmetric case, the spectrum $\varepsilon_{\mathbf{k}}$ has only two-degenerate true minima instead of four, and the paraconductivity σ_{xx} may differ from σ_{yy} . In presence of the orbital effect, the modified Ginzburg-Landau functional is obtained as

$$\begin{aligned} H = & \alpha |\Psi|^2 - g \left[|\Pi_x \Psi|^2 + |\Pi_y \Psi|^2 \right] + \\ & + \gamma \left[|\Pi_x^2 \Psi|^2 + |\Pi_y^2 \Psi|^2 + |\Pi_x \Pi_y \Psi|^2 + |\Pi_y \Pi_x \Psi|^2 \right] + \\ & + \gamma \eta \left[\Pi_x^2 \Psi (\Pi_y^2 \Psi)^* + (\Pi_x^2 \Psi)^* \Pi_y^2 \Psi \right] \end{aligned} \quad (18)$$

from Eq.(1) after the usual Peierls substitution $\Pi_x = \partial_x$ and $\Pi_y = \partial_y + 2iezH/\hbar$ within the Landau gauge. In the case of thin films with a strong confinement in the z direction, the dispersion can be written as

$$\begin{aligned} \varepsilon_{\mathbf{k}} = & \alpha - g \left(k^2 + \frac{(Hd)^2}{12\Phi_0^2} \right) + \gamma \left(k^4 + \frac{(Hd)^4}{80\Phi_0^4} \right) + \\ & + 2\gamma(\eta - 1) k_x^2 k_y^2 + \frac{(Hd)^2}{6\Phi_0^2} (\gamma \eta k_x^2 + 3\gamma k_y^2) \end{aligned} \quad (19)$$

where $k^2 = k_x^2 + k_y^2$, and d is the width of the film along the z -axis²⁷. If $d = 0$, one recovers the model of section III A, while for $d \neq 0$, the minima are displaced as follows ($\eta > 1$):

$$A_{\pm} \rightarrow A_{\pm} \left(\sqrt{\frac{g}{2\gamma} - \frac{\eta(Hd)^2}{12\Phi_0^2}}, 0 \right), \quad (20)$$

$$B_{\pm} \rightarrow B_{\pm} \left(0, \sqrt{\frac{g}{2\gamma} - \frac{(Hd)^2}{4\Phi_0^2}} \right). \quad (21)$$

The critical temperature associated with modulation A_{\pm} differs from the one for B_{\pm} . It turns out that the FFLO modulation occurs along the field (points A_{\pm}) for $\eta < 3$. Strikingly for $\eta > 3$ the modulation occurs along the y -axis (points B_{\pm}) which is perpendicular to the applied field²⁷. Owing to the smallness of the parameter $H\xi d/\Phi_0$, we can expand the "effective mass" tensors g_{ij} around their values for the model without orbital effect ($H\xi d/\Phi_0 = 0$). Finally summing over the A_+ and A_- (or the B_+ and B_-) contributions the ratio between the paraconductivities measured along or perpendicular to the field is

$$\frac{\sigma_{xx}}{\sigma_{yy}} = 1 + \frac{\gamma}{6g} \left(\frac{Hd}{\Phi_0} \right)^2 \left[\frac{\eta - 3}{\eta - 1} + \frac{g^2}{2\gamma\tau} \frac{(\eta - 3)^2}{\eta + 1} \right] \quad (22)$$

for $1 < \eta < 3$ and with $\tau = \alpha - g^2/4\gamma$.

As a main result of this section, the ratio σ_{xx}/σ_{yy} contains a contribution which diverges at the FFLO transition. This is in striking contrast with the regular behavior of this ratio

$$\left(\frac{\sigma_{xx}}{\sigma_{yy}} \right)_{BCS} = 1 - \frac{\gamma(\eta - 3)}{6g} \left(\frac{Hd}{\Phi_0} \right)^2 \quad (23)$$

near the BCS transition. Such an enhancement of σ_{xx}/σ_{yy} may serve as an experimental signature of the FFLO state. This property is reminiscent of the recent finding that critical current oscillates as a function of magnetic orientation in anisotropic 2D films²⁷.

D. Effect of a tilted magnetic field

Finally, we discuss the effect of a tilted magnetic field $\mathbf{H} = H_{\parallel} \mathbf{e}_x + H_{\perp} \mathbf{e}_z$ pointing out of the film plane. The perpendicular component $H_{\perp} \mathbf{e}_z$ quantizes the in-plane motion of the fluctuating Cooper pairs, and induces a finite magnetization. In this subsection, we neglect the orbital motion associated with the in-plane part of the field, which is correct for $H_{\parallel} \xi d \ll \Phi_0$. Thus, the Ginzburg-Landau functional is obtained by the Peierls substitution $D_x = \partial_x$ and $D_y = \partial_y + 2ie_x H_{\perp} / \hbar$. In the anisotropic case, the eigenmodes of the operator

$$\begin{aligned} H = & \alpha |\Psi|^2 - g \left[|D_x \Psi|^2 + |D_y \Psi|^2 \right] + \\ & + \gamma \left[|D_x^2 \Psi|^2 + |D_y^2 \Psi|^2 + |D_x D_y \Psi|^2 + |D_y D_x \Psi|^2 \right] + \\ & + \gamma \eta \left[D_x^2 \Psi (D_y^2 \Psi)^* + (D_x^2 \Psi)^* D_y^2 \Psi \right] \end{aligned} \quad (24)$$

are not known exactly precluding an analytical evaluation of the magnetization. In the following, we derive a simple formula for the film magnetization within the isotropic model. Recently this isotropic model was used in order to compute the fluctuational magnetization of small rings made of a FFLO superconductor (persistent currents)²⁸. Here we evaluate the magnetization in the simpler planar geometry.

The isotropic version of the operator H in Eq.(24) is simply $H = \alpha - g\mathbf{D}^2 + \gamma\mathbf{D}^4$ which can be diagonalized exactly²⁹. The eigenmodes are the standard Landau wavefunctions whose energies are modified as

$$E_n = \tau + \gamma(Q_n^2 - q_0^2)^2, \quad (25)$$

where we introduced

$$Q_n^2 \equiv \frac{4eH_{\perp}}{\hbar} \left(n + \frac{1}{2} \right) \quad n = 0, 1, 2, \dots \quad (26)$$

and $q_0^2 = g/2\gamma$. This particular form of the eigenenergies enables both degeneracies between the Landau levels ($E_n = E_{n+1}$) and commensurability effects between the wavevectors Q_n and q_0 .

The Gaussian integration over the fluctuating order parameter leads to the free energy per surface unit

$$F = \frac{H_{\perp}}{\Phi_0} k_B T \sum_{n=0}^{\infty} \ln \frac{E_n}{\pi k_B T}, \quad (27)$$

where $\Phi_0 = h/2e$ is the superconducting flux quantum and H_{\perp}/Φ_0 is the degeneracy of the n -th Landau level per unit surface of the film³⁰.

For large perpendicular field, namely $H_{\perp}/\Phi_0 \gg \sqrt{\tau/\gamma}$, the Landau levels are well separated from each others and the main contribution to the free energy (27) comes either from the single level with minimal energy E_n , or from two levels when a degeneracy ($E_n = E_{n+1}$) occurs.

Let us first consider the nondegenerate case. Then the free energy is simply given by the single level contribution

$$F = \frac{H_{\perp}}{\Phi_0} k_B T \ln \frac{E_n}{\pi k_B T} \quad (28)$$

and the corresponding orbital magnetization (per unit surface)

$$M_n = -\frac{(8n+4)k_B T}{\Phi_0^2} \left(\frac{\gamma(Q_n^2 - q_0^2)}{\tau + \gamma(Q_n^2 - q_0^2)^2} \right) H_{\perp} \quad (29)$$

is highly nonlinear since the prefactor of H_{\perp} depends strongly on the field and on temperature. Importantly the magnetization may change sign due to the presence of the factor $Q_n^2 - q_0^2$ in the numerator. In order to make more transparent the formula Eq.(29), one may introduce the field-dependent temperature $T_{cn}(H)$ where the denominator vanishes:

$$a(T - T_{cn}) = \tau + \gamma(Q_n^2 - q_0^2)^2 \quad (30)$$

This relation defines the second-order transition line $T_c^{(n)}(H)$ between the normal and the modulated superconducting state described by the n -th Landau level. We also define the points like A,C,E (Fig.3) along this transition line where the numerator vanishes since $Q_n^2 = q_0^2$. Those points are also located on the second-order transition line $T_{cP}(H)$ between the normal and the FFLO superconducting state calculated in the pure paramagnetic limit. In the normal state, the orbital magnetization can be therefore reexpressed as

$$M_n = -\frac{(8n+4)k_B T}{\Phi_0^2} \frac{\gamma}{a} \left(\frac{Q_n^2 - q_0^2}{T - T_{cn}} \right) H_{\perp} \quad (31)$$

This 2D magnetization is diamagnetic when $Q_n^2 > q_0^2$ and paramagnetic when $Q_n^2 < q_0^2$ (Fig.3). In contrast, the fluctuation magnetization is always diamagnetic in the BCS case. However M_n follows a similar power law $(T - T_c^{(n)})^{-1}$ and is on the same order of magnitude than the BCS magnetization²⁶. Consequently we expect that the oscillations between diamagnetism and paramagnetism should be measurable in thin films of FFLO superconductors. This single mode approximation breaks

down when the n -th and $(n+1)$ -th Landau levels are degenerate, namely when $E_n = E_{n+1}$. Then the two levels must be included together in the free energy, whereas the other Landau levels are still far in energy and can be neglected safely. The resulting magnetization $M_n + M_{n+1}$ is slightly diamagnetic at degeneracy.

The single mode approximation also breaks down in the weak field limit ($H_{\perp}/\Phi_0 \ll \sqrt{\tau/\gamma}$) where the Landau level separation becomes so small that all the levels have to be taken into account. This situation corresponds to a magnetic field which is slightly tilted out of the film plane. Then the discrete sum (27) can be replaced by an integral since the level spacing is also small. Using the formula

$$\sum_{n=0}^{\infty} f\left(n + \frac{1}{2}\right) \approx \int_0^{\infty} f(x) dx + \frac{1}{24} f'(0), \quad (32)$$

the fluctuational free energy

$$F = F_0 - \frac{\pi k_B T}{3} \frac{\gamma q_0^2}{\Phi_0^2} \frac{H_{\perp}^2}{\tau + \gamma q_0^4} \quad (33)$$

is obtained. The integral in the right side of Eq.(32) corresponds to the free energy $F_0 = F(H_{\perp} = 0)$ in the absence of perpendicular field. Here $\tau = \alpha - g^2/4\gamma$ and $\gamma q_0^2 = g/2$, thus the denominator is simply $\tau + \gamma q_0^4 = \alpha = a(T - T_{c0})$. Finally the orbital magnetization

$$M = \frac{\pi k_B T}{3} \frac{g}{\Phi_0^2} \frac{H_{\perp}}{a(T - T_{c0})} \quad (34)$$

corresponds to an unusual paramagnetic response for superconducting fluctuations. We have followed closely the derivation of the fluctuation magnetization at the BCS superconducting transition³¹. We find a very similar result than the BCS one. The important difference is that but here the coefficient g becomes positive at the tricritical point (Fig.3), turning the usual diamagnetism of superconducting fluctuations into paramagnetism.

As a main result of this section, we have shown that under strong perpendicular field the FFLO fluctuations induce an oscillatory behavior of the magnetization which switches between diamagnetism and paramagnetism. Under a small perpendicular field, the magnetization is always paramagnetic. This situation is in strong contrast with the standard BCS case where the magnetization is always diamagnetic. Nevertheless the power law $(T - T_{c0})^{-1}$ describing the divergency of the magnetization is the same than in the BCS case for 2D superconductors²⁶.

IV. ANISOTROPIC 3D SUPERCONDUCTORS

It is commonly believed that the FFLO state in CeCoIn₅ corresponds to a modulation along the applied magnetic field. Nevertheless it was argued recently that

this situation is unlikely to happen for arbitrary field orientations when the tetragonal anisotropy of CeCoIn₅ is properly taken into account. Apparently if the order parameter modulation is along the field for $\mathbf{H} \perp \mathbf{c}$ (resp. $\mathbf{H} \parallel \mathbf{c}$), then the modulation is likely to be perpendicular to the field for $\mathbf{H} \parallel \mathbf{c}$ (resp. $\mathbf{H} \perp \mathbf{c}$)¹⁷. Here we investigate the FFLO fluctuations in anisotropic 3D compounds, building upon the various mean-field scenarios reported in Ref¹⁷. We evaluate the paraconductivity σ and the fluctuational magnetization M along the magnetic field $\mathbf{H} = H\mathbf{e}_z$. In particular, we demonstrate below that the magnetization oscillates between sizeable diamagnetism and paramagnetism when the modulation is perpendicular to the field (Landau level like). Those oscillations are the 3D counterparts of the ones predicted in the previous section for superconducting films. In contrast the magnetization is shown to be strongly suppressed when the modulation occurs along the field (FFLO like modulation). In the 3D case, magnetization measurements therefore provide an experimental tool to discriminate between the two possible order parameter structures uncovered in Ref¹⁷.

A. Mean field

We start by a short reviewing of the mean-field properties of the functional

$$\begin{aligned}
H = & \alpha |\Psi|^2 - \sum_{i=x,y,z} g |D_i \Psi|^2 + \gamma \left| \sum_{i=x,y,z} D_i^2 \right|^2 \\
& + \varepsilon_z |D_z^2 \Psi|^2 + \varepsilon_x (|D_x D_y \Psi|^2 + |D_y D_x \Psi|^2) \\
& + \varepsilon (|D_x D_z \Psi|^2 + |D_z D_x \Psi|^2) \\
& + \varepsilon (|D_y D_z \Psi|^2 + |D_z D_y \Psi|^2) \quad (35)
\end{aligned}$$

consistent with the tetragonal symmetry of CeCoIn₅. The terms ε_z , ε_x and ε describe nontrivial (namely different from a simple elliptical) anisotropy¹⁷. Note that the cubic symmetry corresponds to $\varepsilon_x = \varepsilon$ and $\varepsilon_z = 0$. It was shown that two kinds of modulated superconducting states (scenarios a) and b) mentioned above in the general introduction) are the most likely to occur when anisotropies are properly taken into account. The first class of solutions corresponds to order parameters modulated along the field with characteristic FFLO wave-vector q_0 and in the $n = 0$ Landau level. The corresponding fluctuations have typical spectrum

$$E(k_z) = \tau + \gamma \left(\frac{2eH}{\hbar} + k_z^2 - q_0^2 \right)^2 \quad (36)$$

which indicates an instability towards finite modulation along the z axis (magnetic field). Here the lowest Landau level is favorable, $n = 0$ and γ is a renormalized parameter which depends on ε_z , ε_x and ε . In the second class of solutions, the modulation occurs in the plane perpendicular to the field and is described by a higher ($n > 0$)

Landau level. Then the dispersion of the fluctuations can be approximated as follows

$$E(k_z) = \tau + \gamma(Q_n^2 - q_0^2)^2 + gk_z^2 \quad (37)$$

where

$$Q_n^2 \equiv \frac{4eH}{\hbar} \left(n + \frac{1}{2} \right) \quad n = 0, 1, 2, \dots \quad (38)$$

This spectrum differs from Eq.(25) by the kinetic energy gk_z^2 along the field. Note that here γ and g are also renormalized parameters which depends on ε_z , ε_x and ε in a complicated manner¹⁷.

B. Fluctuation magnetization

Here we evaluate the magnetization induced by the FFLO fluctuations taking into account the intrinsic anisotropy present in 3D compounds. The FFLO transition might happen under low or strong field, depending on the underlying microscopic mechanism. For instance, in the rare earth magnetic superconductor ErRh₄B₄ a small field is sufficient to polarize the internal moments, and the FFLO transition is thus expected at low applied magnetic field³. Here we treat the case of the FFLO transition occurring under strong magnetic field which is relevant for the case of the heavy fermion superconductor CeCoIn₅. Using a single Landau level approximation, we demonstrate that the magnetization exhibits qualitatively distinct behaviors depending on the class of solutions.

FFLO-like modulation along the field, characterized by a finite wave-vector q_0 and Landau index $n = 0$ (scenario a) discussed in the introduction). The 3D density of free energy is given by the integral

$$F = k_B T \frac{H}{\Phi_0} \int \frac{dk_z}{2\pi} \ln \frac{E(k_z)}{\pi k_B T} \quad (39)$$

where the energy depends on the magnetic field as follows:

$$E(k_z) = \tau + \gamma \left(\frac{2eH}{\hbar} + k_z^2 - q_0^2 \right)^2. \quad (40)$$

Hence the most divergent part of the orbital magnetization (per unit volume) $M = -\partial F / \partial H$ is given by

$$\begin{aligned}
M = & -\frac{2k_B T}{\Phi_0^2} \times \\
& \times \left(\int_{-\infty}^{\infty} dk_z \frac{\gamma (2eH/\hbar + k_z^2 - q_0^2)}{\tau + \gamma (2eH/\hbar + k_z^2 - q_0^2)^2} \right) H, \quad (41)
\end{aligned}$$

where $\Phi_0 = h/(2e)$. Since the numerator of the integrand cancels and changes sign as a function of k_z , one expects a strong suppression of the fluctuation magnetization compared to the uniform BCS case wherein such

a cancellation does not occur. Indeed the magnetization M diverges logarithmically at the FFLO transition which is less divergent than the $\tau^{-1/2}$ law predicted in the standard BCS case. Therefore the presence of a genuine FFLO state should be detected as a suppression of the fluctuation diamagnetism observed near the BCS transition. In comparison with the 2D case, the oscillations between paramagnetism and diamagnetism predicted in the previous section are blurred out by the dispersion over the momentum k_z along the field.

Landau level modulation perpendicular to the field (scenario b) discussed in the introduction). The dispersion of the fluctuations

$$E(k_z) = \tau + \gamma(Q_n^2 - q_0^2)^2 + gk_z^2 \quad (42)$$

now favors the absence of modulation along the z axis in contrast to the spectrum Eq.(40). Upon increasing the parameter q_0^2 , the lowest energy Landau level is successively $n = 0$, then $n = 1$ etc... Near the BCS transition ($g > 0$), the fluctuations induce diamagnetism and a lowering of the critical field $H_{c2}(T)$ below the purely paramagnetic critical field $H_P(T)$ at the same temperature (Fig.3). When the n -th Landau level is realized and when all the other Landau levels are distant in energy, one can single out the contribution of this main level to the density of free energy

$$F = k_B T \frac{H}{\Phi_0} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \ln \frac{E_n(k_z)}{\pi k_B T}. \quad (43)$$

Writing the orbital magnetization as

$$M_n = -\frac{(4n+2)k_B T}{\Phi_0^2} \times \left(\int_{-\infty}^{\infty} dk_z \frac{\gamma(Q_n^2 - q_0^2)}{\tau + \gamma(Q_n^2 - q_0^2)^2 + gk_z^2} \right) H_{\perp}, \quad (44)$$

shows that the 2D oscillations are no longer suppressed by the integration over k_z since the numerator is independent of k_z . Calculating the integral shows that the magnetization diverges as

$$M_n = -\frac{(4n+2)k_B T}{\Phi_0^2} \frac{\gamma}{(ag)^{1/2}} \frac{Q_n^2 - q_0^2}{(T - T_{cn})^{1/2}} H_{\perp} \quad (45)$$

with the same power law than in the BCS transition of 3D superconductors²⁶. Unlike the BCS case, this fluctuation magnetization changes sign being diamagnetic when $Q_n^2 > q_0^2$ (arcs BC , DE on Fig.3) and paramagnetic when $Q_n^2 < q_0^2$ (arcs AB , CD).

In brief, the magnetization is sizeable and oscillates between para- and diamagnetism when the superconducting order parameter is modulated perpendicularly to the field, whereas it is strongly suppressed when the order parameter is modulated along the field. Therefore magnetization measurements may serve as a test to discriminate between FFLO and Landau level like modulations in 3D anisotropic superconductors.

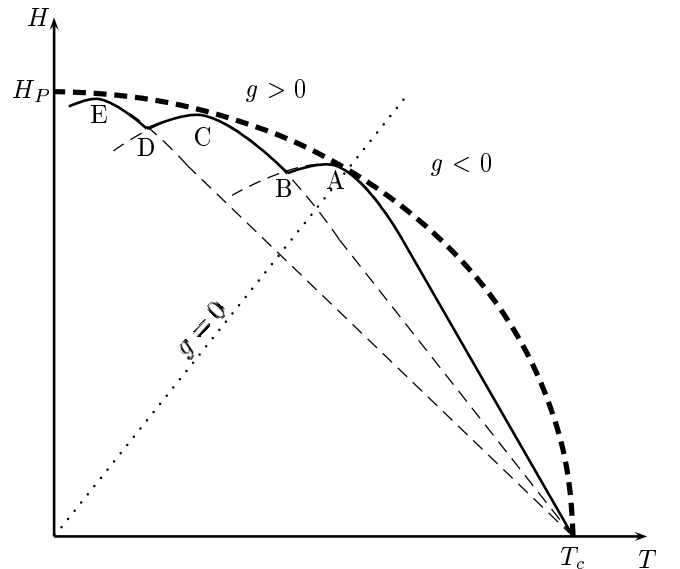


FIG. 3: Schematic field-temperature (H, T) phase diagram showing the cascade of Landau levels. The thick dashed curve represents the critical field in the absence of orbital effect. In presence of orbital effect, the critical field is reduced and described by the thin dashed curves which corresponds respectively to the $n = 0$, $n = 1$ and $n = 2$ Landau levels. The solid line represents schematically the expected transition line between the normal and the superconducting states. The fluctuations are diamagnetic between zero field ($H = 0, T_c$) and B , paramagnetic near the arc AB , then again diamagnetic near the arc BC , etc... The Landau levels are degenerate at points B, D , etc... This schematic picture is relevant for both 2D FFLO superconductors and for 3D ones where the Landau level modulation is realized (scenario b) evoked in the introduction).

V. GINZBURG-LEVANYUK CRITERION FOR FFLO PHASES.

Now, we address the validity of the Gaussian analysis used in the preceding sections. Using a Ginzburg-Levanyuk type of argument^{32,33}, we obtain that the size of the critical region in FFLO superconductors is more extended than in usual uniform superconductor case. Nevertheless the main conclusion of this section is that the critical region remains extremely small in FFLO superconductors thereby supporting our Gaussian analysis for small ratio T_c/E_F between the critical temperature and the Fermi energy E_F . This fact makes very difficult the observation of the phenomena (first-order transition) predicted by renormalization group studies^{34,35}.

A. Isotropic model

When the temperature is sufficiently close to the critical one, interactions between the fluctuation modes be-

come so strong that the Gaussian approximation breaks down. In order to quantify the range of temperature where this breakdown occurs, we shall use the isotropic MGL functional^{18,19,20}

$$H[\Psi] = N_d(0) \int d\mathbf{r} \left[\alpha |\Psi|^2 - g \xi^2 |\partial\Psi|^2 + \xi^4 |\partial^2\Psi|^2 \right] + N_d(0) \int d\mathbf{r} \left[\frac{g}{T_c^2} |\Psi|^4 + \frac{1}{T_c^4} |\Psi|^6 \right]. \quad (46)$$

Here $N_d(0)$ is the d -dimensional electronic density of states and $\xi = v_F/T_c$ the superconducting coherence length. In this section, we will use the notations $k_B = \hbar = 1$, $\alpha = (T - \bar{T}_c)/\bar{T}_c$ and $\tau = \alpha - g^2/4 = (T - T_c)/T_c$. In the spirit of the original Ginzburg-Levanyuk criterion, we evaluate the various free and interaction terms within the Gaussian approximation

$$\langle H \rangle_0 = N_d(0) \sum_{\mathbf{k}} \left[\tau + (k^2 - q_0^2)^2 \xi^4 + \tilde{g} \frac{2}{T_c^2} \frac{1}{L^d} \sum_{\mathbf{k}'} \left\langle |\Psi_{\mathbf{k}'}|^2 \right\rangle_0 + \left(\frac{2}{T_c^2} \frac{1}{L^d} \sum_{\mathbf{k}'} \left\langle |\Psi_{\mathbf{k}'}|^2 \right\rangle_0 \right)^2 \right] \left\langle |\Psi_{\mathbf{k}}|^2 \right\rangle_0 \quad (47)$$

and look for the conditions under which the interaction terms can be safely neglected in comparison to the Gaussian ones. Note that in this problem the form of the free field correlator

$$\left\langle |\Psi_{\mathbf{k}}|^2 \right\rangle_0 = \frac{\pi T_c / N_d(0)}{\tau + \xi^4 (k^2 - q_0^2)^2}. \quad (48)$$

is rather special due to the proximity of the FFLO transition. Here the wavevector q_0 is defined by $g = 2q_0^2 \xi^2$. The dimensionless function g cancels at the tricritical point, and increases when one follows the FFLO/normal transition line towards the low temperatures (and thus high fields). We must distinguish two regimes depending on the relative strength of the $|\Psi|^4$ and $|\Psi|^6$ interaction terms, and on the quadratic or quartic dispersion of the fluctuations.

Far from the tricritical point, namely when $g > (T_c/E_F)^{2(d-1)/(6-d)}$, the leading correction to the Gaussian behavior originates from the $|\Psi|^4$ interaction term. Moreover the fluctuations propagate with a quadratic dispersion, and the correlator Eq.(48) can be approximated as

$$\left\langle |\Psi_{\mathbf{k}}|^2 \right\rangle_0 = \frac{\pi T_c / N_d(0)}{\tau + 4q_0^2 \xi^4 (k - q_0)^2} \quad (49)$$

when evaluating the sum

$$\frac{g}{T_c^2 L^d} \sum_{\mathbf{k}'} \left\langle |\Psi_{\mathbf{k}'}|^2 \right\rangle_0 = \frac{g^{d/2} \tau^{-1/2}}{T_c N_d(0) \xi^d}. \quad (50)$$

Using $T_c N_d(0) \xi^d = (E_F/T_c)^{d-1}$, we find that the $|\Psi|^4$ interaction terms are negligible in comparison to the Gaussian ones when the condition (Ginzburg-Levanyuk criterion)

$$\tau \gg g^{d/3} \left(\frac{T_c}{E_F} \right)^{2(d-1)/3} \quad (51)$$

is fulfilled. The critical region width is larger than in the standard BCS case ($\tau \gg (T_c/E_F)^4$ for $d = 3$ and $\tau \gg T_c/E_F$ for $d = 2$) but it remains extremely thin.

Near the tricritical point, when $g < (T_c/E_F)^{2(d-1)/(6-d)}$, the $|\Psi|^6$ interaction becomes stronger than the $|\Psi|^4$ one since this latter contribution is suppressed by the extremely small prefactor g . In particular, along the $g = 0$ line in the (H, T) diagram, the $|\nabla\Psi|^2$ and $|\Psi|^4$ terms are totally absent from the functional¹⁸. Therefore one should compute the mean value $\left\langle |\Psi_{\mathbf{k}}|^2 \right\rangle_0$ with a purely quartic momentum dependence. Since

$$\frac{1}{T_c^2} \frac{1}{N_d(0) L^d} \sum_{\mathbf{k}'} \left\langle |\Psi_{\mathbf{k}'}|^2 \right\rangle_0 = \frac{\tau^{(d-4)/4}}{T_c N_d(0) \xi^d}, \quad (52)$$

the condition to neglect the $|\Psi|^6$ interaction between the fluctuation modes is thus

$$\tau \gg \left(\frac{T_c}{E_F} \right)^{4(d-1)/(6-d)} \quad (53)$$

The critical fluctuations are present in a larger region of the phase diagram than for BCS superconductivity²⁶. During the completion of this work, we became aware of Ref.²⁸ where the Ginzburg-Levanyuk criterion is derived by evaluating exclusively the $|\Psi|^4$ interaction term. We therefore obtain the same Ginzburg-Levanyuk criterion as in Ref.²⁸ for the large g regime whereas our criteria differ when approaching the tricritical point. In spite of this discrepancy, both procedures lead to the same practical conclusion that the critical region remains extremely thin and inaccessible for experimental observations, because of the smallness of the ratio $T_c/E_F \sim 10^{-2} - 10^{-3}$.

B. Anisotropic model.

We now derive the Ginzburg-Levanyuk criterion in the case of anisotropic FFLO superconductors. The large g

regime is modified in comparison to the isotropic case, since there the low energy fluctuations are located around few isolated points instead being spread over a large shell of radius q_0 .

Far from the tricritical point, namely when $g > (T_c/E_F)^{2(d-1)/(6-d)}$, the leading correction to the Gaussian behavior originates from the $|\Psi|^4$ interaction term. Moreover the fluctuations propagate with a quadratic dispersion, and the correlator Eq.(48) can be approximated as

$$\langle |\Psi_{\mathbf{k}}|^2 \rangle_0 = \frac{\pi T_c / N_d(0)}{\tau + g k^2 \xi^2}. \quad (54)$$

Evaluating the sum

$$\frac{g}{T_c^2 L^d} \sum_{\mathbf{k}'} \langle |\Psi_{\mathbf{k}'}|^2 \rangle_0 = \left(\frac{\tau}{g} \right)^{(d-2)/2} \frac{1}{T_c N_d(0) \xi^d} \quad (55)$$

and using $T_c N_d(0) \xi^d = (E_F/T_c)^{d-1}$, we find that the $|\Psi|^4$ interaction terms are negligible in comparison to the Gaussian ones when the condition (Ginzburg-Levanyuk criterion)

$$\tau \gg g^{(2-d)/(4-d)} \left(\frac{T_c}{E_F} \right)^{2(d-1)/(4-d)} \quad (56)$$

is fulfilled. This Ginzburg-Levanyuk criterion is similar (same power of T_c/E_F) than the one encountered in the standard BCS case.

Near the tricritical point, when $g < (T_c/E_F)^{2(d-1)/(6-d)}$, the lowest energy fluctuations are located around the origin of the reciprocal space and have a quartic dispersion like in the isotropic model studied above. The Ginzburg-Levanyuk criterion is thus again

$$\tau \gg \left(\frac{T_c}{E_F} \right)^{4(d-1)/(6-d)} \quad (57)$$

near the tricritical point.

C. Concluding remarks

The FFLO critical fluctuations are present in a larger region of the phase diagram than for BCS

superconductivity²⁶. Nevertheless, for superconducting compounds, the ratio $T_c/E_F \sim 10^{-3}$ is small and thus the critical region remains hardly accessible for experimental observations.

VI. CONCLUSION.

We investigated the conductivity and the orbital magnetization associated with superconducting fluctuations above the FFLO critical temperature or field. Both in 2D and 3D models, we shown that these properties differ considerably than their counterparts at the vicinity of a standard BCS transition towards an homogeneous superconducting state, thereby providing an experimental tool to detect the inhomogeneous state. First, the paraconductivity of thin superconducting films exhibits a strong anisotropy when measured parallel or perpendicular to the applied magnetic field. Second, the orbital magnetization oscillates between diamagnetic and paramagnetic behaviors whereas the uniform BCS state always induces diamagnetic fluctuations above T_c . We suggest performing magnetization and conductance measurements along the FFLO transition line in compounds where the FFLO state has been recently reported. In 2D organic superconductors^{9,10}, the magnetization oscillations should be even more pronounced than in the 3D magnetic superconductors (ErRh₄B₄, see³) or in the case of the anisotropic 3D heavy fermion superconductor CeCoIn₅^{6,7,8}. In the 3D case, we find that the absence of such oscillations reveals a FFLO state modulated along the field whereas presence of oscillations should be associated with a multiquanta Landau modulation perpendicular to the field.

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¹ P. Fulde, and R. A. Ferrell. Phys. Rev. **135**, A550 (1964).

² A. Larkin, and Y. Ovchinnikov. Sov. Phys. JETP **20**, 762 (1965).

³ L.N. Bulaevskii et al., Adv. Phys. **34**, 175 (1985).

⁴ R. Casalbuoni, and G. Nardulli. Rev. Mod. Phys. **76**, 263

(2004).

⁵ Y. Mastuda, and H. Shimahara J. Phys. Soc. Jap. **76**, 051005 (2007).

⁶ H. Radovan *et al.* Nature **425**, 51 (2003).

⁷ A. Bianchi *et al.* Phys. Rev. Lett. **89**, 137002 (2002).

⁸ A. Bianchi *et al.* Phys. Rev. Lett. **91**, 187004 (2003).

⁹ S. Uji *et al.* Phys. Rev. Lett. **96**, 157001 (2006).

- ¹⁰ R. Lortz *et al.* Phys. Rev. Lett. **99**, 187002 (2007).
- ¹¹ L.W. Gruenberg and L. Gunther, Phys. Rev. Lett. **16**, 996 (1966).
- ¹² A.I. Buzdin and J.P. Brison, Phys. Lett. A **218**, 359 (1996).
- ¹³ A.I. Buzdin and J.P. Brison, Europhys. Lett. **35**, 707 (1996).
- ¹⁴ J.P. Brison *et al.*, Physica C **250**, 128 (1995).
- ¹⁵ K. Yang, and S. Sondhi. Phys. Rev. B **57**, 8566 (1998).
- ¹⁶ A. Vorontsov, J. Sauls, and M. Graf. Phys. Rev. B **72**, 184501 (2005).
- ¹⁷ D. Denisov, A. Buzdin, and H. Shimahara, Phys. Rev. B **79**, 064506 (2009).
- ¹⁸ A. I. Buzdin, and H. Kachkachi. Phys. Lett. A **225**, 341 (1997).
- ¹⁹ D. Agterberg, and K. Yang. J. Phys.: Condens. Matter **13**, 9259 (2001).
- ²⁰ M. Houzet and V. P. Mineev, Phys. Rev. B **74**, 144522 (2006).
- ²¹ F. Korschelle, J. Cayssol, and A. I. Buzdin, Europhys. Lett. **79**, 67001 (2007).
- ²² R. Bel *et al.*, Phys. Rev. Lett. **92**, 217002 (2004).
- ²³ Y. Onose, Lu Li, C. Petrovic, N. P. Ong, Europhys. Lett. **79**, 17006 (2007).
- ²⁴ L. D. Landau, and E. M. Lifshitz. *Course of theoretical physics, Volume 5 : Statistical Physics, 3rd edition* (Butterworth-Heinemann, 1980).
- ²⁵ W.J. Skocpol, and M. Tinkham, Rep. on Prog. Phys. **38**, 1049 (1975).
- ²⁶ A. Larkin, and A. Varlamov. *The physics of superconductors, Vol. I : Conventional and high- T_c superconductors*, chapter 3, Fluctuation phenomena in superconductors (K.H. Bennemann and J.B. Ketterson, Springer-Verlag, 2003).
- ²⁷ A.I. Buzdin, Y. Matsuda, and T. Shibauchi. Europhys. Lett. **80**, 67004 (2007).
- ²⁸ A.A. Zyuzin, A.Yu. Zyuzin, arXiv:0901.1562; A.A. Zyuzin, A.Yu. Zyuzin, JETP Lett. **88**, 147 (2008).
- ²⁹ This useful property relies on the isotropy of the model treated in Sec. II. In the anisotropic case (Sec. III) $\gamma\mathbf{D}^4$ is replaced by a more complicated operator $\gamma_{ij}\mathbf{D}_i^2\mathbf{D}_j^2$ whose eigenmodes differ from the ones of \mathbf{D}^2 and are not known exactly.
- ³⁰ L. D. Landau, and E. M. Lifshitz. *Course of theoretical physics, Volume 3 : Quantum Mechanics, 3rd edition* (Butterworth-Heinemann, 2001).
- ³¹ A. Schmid. Phys. Rev. **180**, 527 (1969).
- ³² A. Levanyuk. Sov. Phys. JETP **36**, 571 (1959).
- ³³ V. L. Ginzburg. Sov. Solid State **2**, 61 (1960).
- ³⁴ S. A. Brazovskii. Sov. Phys. JETP **41**, 85 (1975).
- ³⁵ D. Dalidovich, and K. Yang. Phys. Rev. Lett. **93**, 247002 (2004).