

Spanning Trees of Bounded Degree Graphs John Michael Robson

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Appendix

The following conditions are tested to check that the multipliers for cuts with both parts having more than one vertex are subject to the same lower bounds as those where one part is a singleton. In each case the format is the same. Writing d for the degree bound, c for the cut value, i and p for the number of independent and dependent pairs:

 $(c - maxu - maxv) + f[maxu, d] + f[maxv, d] + (2 * i + p)(f[2, d] - 2) \ge f[c, d]$ followed by the numerical values of the two sides.

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Case of a cut of value 4, with maxu= 2 and maxv= 2

There are 2 dependent pairs incident at u1 or v1 (1 at u1 and 1 at v1)

For u and v greedy partition is (2, 2);

2 independent and 2 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 5: 0+f[5,2]+f[5,2]+(2*2+2)(f[5,2]-2) >= f[5,4] (7.20000>6.912000)

Degree bound 6: 0+f[6,2]+f[6,2]+(2*2+2)(f[6,2]-2) >= f[6,4] (6.666664>6.351852)

Degree bound 7: 0+f[7,2]+f[7,2]+(2*2+2)(f[7,2]-2) >= f[7,4] (6.285712>5.970845)

Degree bound 8: 0+f[8,2]+f[8,2]+(2*2+2)(f[8,2]-2) >= f[8,4] (6.00000>5.695313)

Degree bound 9: 0+f[9,2]+f[9,2]+(2*2+2)(f[9,2]-2) >= f[9,4] (5.777776>5.486968)

Degree bound 10: 0+f[10,2]+f[10,2]+(2*2+2)(f[10,2]-2) >= f[10,4] (5.60000>5.324000)

Degree bound 11: 0+f[11,2]+f[11,2]+(2*2+2)(f[11,2]-2) >= f[11,4] (5.454544>5.193088)
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Case of a cut of value 5, with maxu= 2 and maxv= 2

There are 2 dependent pairs incident at u1 or v1 (1 at u1 and 1 at v1)

For u and v greedy partition is (2, 2, 1);

6 independent and 2 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 6: 1+f[6,2]+f[6,2]+(2*6+2)(f[6,2]-2) >= f[6,5] (10.333328>9.263117)

Degree bound 7: 1+f[7,2]+f[7,2]+(2*6+2)(f[7,2]-2) >= f[7,5] (9.571424>8.529779)

Degree bound 8: 1+f[8,2]+f[8,2]+(2*6+2)(f[8,2]-2) >= f[8,5] (9.000000>8.009033)

Degree bound 9: 1+f[9,2]+f[9,2]+(2*6+2)(f[9,2]-2) >= f[9,5] (8.555552>7.620790)

Degree bound 10: 1+f[10,2]+f[10,2]+(2*6+2)(f[10,2]-2) >= f[10,5] (8.200000>7.320500)

Degree bound 11: 1+f[11,2]+f[11,2]+(2*6+2)(f[11,2]-2) >= f[11,5] (7.909088>7.081484)
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Case of a cut of value 5, with maxu= 3 and maxv= 2

There are 4 dependent pairs incident at u1 or v1 (3 at u1 and 1 at v1)

For u greedy partition is (3, 2); for v greedy partition is (2, 2, 1);

4 independent and 2 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 6: 0+f[6,3]+f[6,2]+(2*4+2)(f[6,2]-2) >= f[6,5] (9.749996>9.263117)

Degree bound 7: 0+f[7,3]+f[7,2]+(2*4+2)(f[7,2]-2) >= f[7,5] (9.061221>8.529779)

Degree bound 8: 0+f[8,3]+f[8,2]+(2*4+2)(f[8,2]-2) >= f[8,5] (8.546875>8.009033)

Degree bound 9: 0+f[9,3]+f[9,2]+(2*4+2)(f[9,2]-2) >= f[9,5] (8.148146>7.620790)

Degree bound 10: 0+f[10,3]+f[10,2]+(2*4+2)(f[10,2]-2) >= f[10,5] (7.830000>7.320500)

Degree bound 11: 0+f[11,3]+f[11,2]+(2*4+2)(f[11,2]-2) >= f[11,5] (7.570246>7.081484)
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Case of a cut of value 6, with maxu= 2 and maxv= 2
There are 2 dependent pairs incident at u1 or v1 (1 at u1 and 1 at v1)
For u and v greedy partition is (2, 2, 2);
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9 independent and 4 dependent pairs (excluding pairs incident at u1 or v1)
Degree bound 7: 2+f[7,2]+f[7,2]+(2*9+4)(f[7,2]-2) >= f[7,6] (12.857136>11.697983)
Degree bound 8: 2+f[8,2]+f[8,2]+(2*9+4)(f[8,2]-2) >= f[8,6] (12.00000>10.812195)
Degree bound 9: 2+f[9,2]+f[9,2]+(2*9+4)(f[9,2]-2) >= f[9,6] (11.333328>10.161053)
Degree bound 10: 2+f[10,2]+f[10,2]+(2*9+4)(f[10,2]-2) >= f[10,6] (10.800000>9.663060)
Degree bound 11: 2+f[11,2]+f[11,2]+(2*9+4)(f[11,2]-2) >= f[11,6] (10.363632>9.270306)
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Case of a cut of value 6, with maxu= 3 and maxv= 2

There are 4 dependent pairs incident at u1 or v1 (3 at u1 and 1 at v1)

For u greedy partition is (3, 3); for v greedy partition is (2, 2, 2);

6 independent and 5 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 7: 1+f[7,3]+f[7,2]+(2*6+5)(f[7,2]-2) >= f[7,6] (12.061219>11.697983)

Degree bound 8: 1+f[8,3]+f[8,2]+(2*6+5)(f[8,2]-2) >= f[8,6] (11.296875>10.812195)

Degree bound 9: 1+f[9,3]+f[9,2]+(2*6+5)(f[9,2]-2) >= f[9,6] (10.703700>10.161053)

Degree bound 10: 1+f[10,3]+f[10,2]+(2*6+5)(f[10,2]-2) >= f[10,6] (10.230000>9.663060)

Degree bound 11: 1+f[11,3]+f[11,2]+(2*6+5)(f[11,2]-2) >= f[11,6] (9.842972>9.270306)
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Case of a cut of value 6, with maxu= 3 and maxv= 3

There are 6 dependent pairs incident at u1 or v1 (3 at u1 and 3 at v1)

For u and v greedy partition is (3, 2, 1);

7 independent and 2 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 7: 0+f[7,3]+f[7,3]+(2*7+2)(f[7,2]-2) >= f[7,6] (12.408158>11.697983)

Degree bound 8: 0+f[8,3]+f[8,3]+(2*7+2)(f[8,2]-2) >= f[8,6] (11.593750>10.812195)

Degree bound 9: 0+f[9,3]+f[9,3]+(2*7+2)(f[9,2]-2) >= f[9,6] (10.962960>10.161053)

Degree bound 10: 0+f[10,3]+f[10,3]+(2*7+2)(f[10,2]-2) >= f[10,6] (10.460000>9.663060)

Degree bound 11: 0+f[11,3]+f[11,3]+(2*7+2)(f[11,2]-2) >= f[11,6] (10.049584>9.270306)
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Case of a cut of value 6, with maxu= 4 and maxv= 2

There are 7 dependent pairs incident at u1 or v1 (6 at u1 and 1 at v1)

For u greedy partition is (4, 2); for v greedy partition is (2, 2, 1, 1);

6 independent and 2 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 7: 0+f[7,4]+f[7,2]+(2*6+2)(f[7,2]-2) >= f[7,6] (12.256555>11.697983)

Degree bound 8: 0+f[8,4]+f[8,2]+(2*6+2)(f[8,2]-2) >= f[8,6] (11.445313>10.812195)

Degree bound 9: 0+f[9,4]+f[9,2]+(2*6+2)(f[9,2]-2) >= f[9,6] (10.820298>10.161053)

Degree bound 10: 0+f[10,4]+f[10,2]+(2*6+2)(f[10,2]-2) >= f[10,6] (10.324000>9.663060)

Degree bound 11: 0+f[11,4]+f[11,2]+(2*6+2)(f[11,2]-2) >= f[11,6] (9.920358>9.270306)
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Case of a cut of value 7, with maxu= 2 and maxv= 2

There are 2 dependent pairs incident at u1 or v1 (1 at u1 and 1 at v1)

For u and v greedy partition is (2, 2, 2, 1);

15 independent and 4 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 8: 3+f[8,2]+f[8,2]+(2*15+4)(f[8,2]-2) >= f[8,7] (16.000000>14.191006)

Degree bound 9: 3+f[9,2]+f[9,2]+(2*15+4)(f[9,2]-2) >= f[9,7] (14.999992>13.171735)

Degree bound 10: 3+f[10,2]+f[10,2]+(2*15+4)(f[10,2]-2) >= f[10,7] (14.200000>12.400927)

Degree bound 11: 3+f[11,2]+f[11,2]+(2*15+4)(f[11,2]-2) >= f[11,7] (13.545448>11.798571)
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Case of a cut of value 7, with maxu= 3 and maxv= 2

There are 4 dependent pairs incident at u1 or v1 (3 at u1 and 1 at v1)

For u greedy partition is (3, 3, 1); for v greedy partition is (2, 2, 2, 1);

12 independent and 5 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 8: 2+f[8,3]+f[8,2]+(2*12+5)(f[8,2]-2) >= f[8,7] (15.296875>14.191006)

Degree bound 9: 2+f[9,3]+f[9,2]+(2*12+5)(f[9,2]-2) >= f[9,7] (14.370364>13.171735)

Degree bound 10: 2+f[10,3]+f[10,2]+(2*12+5)(f[10,2]-2) >= f[10,7] (13.630000>12.400927)

Degree bound 11: 2+f[11,3]+f[11,2]+(2*12+5)(f[11,2]-2) >= f[11,7] (13.024788>11.798571)
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Case of a cut of value 7, with maxu= 3 and maxv= 3
There are 6 dependent pairs incident at u1 or v1 (3 at u1 and 3 at v1)
For u and v greedy partition is (3, 3, 1);
9 independent and 6 dependent pairs (excluding pairs incident at u1 or v1)
Degree bound 8: 1+f[8,3]+f[8,3]+(2*9+6)(f[8,2]-2) >= f[8,7] (14.593750>14.191006)
Degree bound 9: 1+f[9,3]+f[9,3]+(2*9+6)(f[9,2]-2) >= f[9,7] (13.740736>13.171735)
Degree bound 10: 1+f[10,3]+f[10,3]+(2*9+6)(f[10,2]-2) >= f[10,7] (13.060000>12.400927)
Degree bound 11: 1+f[11,3]+f[11,3]+(2*9+6)(f[11,2]-2) >= f[11,7] (12.504128>11.798571)
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Case of a cut of value 7, with maxu= 4 and maxv= 2

There are 7 dependent pairs incident at u1 or v1 (6 at u1 and 1 at v1)

For u greedy partition is (4, 3); for v greedy partition is (2, 2, 2, 1);

9 independent and 5 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 8: 1+f[8,4]+f[8,2]+(2*9+5)(f[8,2]-2) >= f[8,7] (14.695313>14.191006)

Degree bound 9: 1+f[9,4]+f[9,2]+(2*9+5)(f[9,2]-2) >= f[9,7] (13.820296>13.171735)

Degree bound 10: 1+f[10,4]+f[10,2]+(2*9+5)(f[10,2]-2) >= f[10,7] (13.124000>12.400927)

Degree bound 11: 1+f[11,4]+f[11,2]+(2*9+5)(f[11,2]-2) >= f[11,7] (12.556720>11.798571)
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Case of a cut of value 7, with maxu= 4 and maxv= 3 There are 9 dependent pairs incident at u1 or v1 (6 at u1 and 3 at v1) For u greedy partition is (4, 2, 1); for v greedy partition is (3, 2, 1, 1); 10 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 8: 0+f[8,4]+f[8,3]+(2*10+2)(f[8,2]-2) >= f[8,7] (14.992188>14.191006) Degree bound 9: 0+f[9,4]+f[9,3]+(2*10+2)(f[9,2]-2) >= f[9,7] (14.079556>13.171735) Degree bound 10: 0+f[10,4]+f[10,3]+(2*10+2)(f[10,2]-2) >= f[10,7] (13.354000>12.400927) Degree bound 11: 0+f[11,4]+f[11,3]+(2*10+2)(f[11,2]-2) >= f[11,7] (12.763332>11.798571)

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Case of a cut of value 7, with maxu= 5 and maxv= 2

There are 11 dependent pairs incident at u1 or v1 (10 at u1 and 1 at v1)

For u greedy partition is (5, 2); for v greedy partition is (2, 2, 1, 1, 1);

8 independent and 2 dependent pairs (excluding pairs incident at u1 or v1)

Degree bound 8: 0+f[8,5]+f[8,2]+(2*8+2)(f[8,2]-2) >= f[8,7] (14.759033>14.191006)

Degree bound 9: 0+f[9,5]+f[9,2]+(2*8+2)(f[9,2]-2) >= f[9,7] (13.843008>13.171735)

Degree bound 10: 0+f[10,5]+f[10,2]+(2*8+2)(f[10,2]-2) >= f[10,7] (13.120500>12.400927)

Degree bound 11: 0+f[11,5]+f[11,2]+(2*8+2)(f[11,2]-2) >= f[11,7] (12.536026>11.798571)
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Case of a cut of value 8, with maxu= 2 and maxv= 2

There are 2 dependent pairs incident at u1 or v1 (1 at u1 and 1 at v1) For u and v greedy partition is (2, 2, 2, 2); 20 independent and 6 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 9: 4+f[9,2]+f[9,2]+(2*20+6)(f[9,2]-2) >= f[9,8] (18.666656>16.726013) Degree bound 10: 4+f[10,2]+f[10,2]+(2*20+6)(f[10,2]-2) >= f[10,8] (17.600000>15.589737) Degree bound 11: 4+f[11,2]+f[11,2]+(2*20+6)(f[11,2]-2) >= f[11,8] (16.727264>14.709907)

Case of a cut of value 8, with maxu= 3 and maxv= 2 There are 4 dependent pairs incident at u1 or v1 (3 at u1 and 1 at v1) For u greedy partition is (3, 3, 2); for v greedy partition is (2, 2, 2, 2); 17 independent and 7 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 9: 3+f[9,3]+f[9,2]+(2*17+7)(f[9,2]-2) >= f[9,8] (18.037028>16.726013) Degree bound 10: 3+f[10,3]+f[10,2]+(2*17+7)(f[10,2]-2) >= f[10,8] (17.030000>15.589737) Degree bound 11: 3+f[11,3]+f[11,2]+(2*17+7)(f[11,2]-2) >= f[11,8] (16.206604>14.709907)

Case of a cut of value 8, with maxu= 3 and maxv= 3 There are 6 dependent pairs incident at u1 or v1 (3 at u1 and 3 at v1) For u and v greedy partition is (3, 3, 2); 14 independent and 8 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 9: 2+f[9,3]+f[9,3]+(2*14+8)(f[9,2]-2) >= f[9,8] (17.407400>16.726013) Degree bound 10: 2+f[10,3]+f[10,3]+(2*14+8)(f[10,2]-2) >= f[10,8] (16.460000>15.589737) Degree bound 11: 2+f[11,3]+f[11,3]+(2*14+8)(f[11,2]-2) >= f[11,8] (15.685944>14.709907)

Case of a cut of value 8, with maxu= 4 and maxv= 2 There are 7 dependent pairs incident at u1 or v1 (6 at u1 and 1 at v1) For u greedy partition is (4, 4); for v greedy partition is (2, 2, 2, 2); 12 independent and 9 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 9: 2+f[9,4]+f[9,2]+(2*12+9)(f[9,2]-2) >= f[9,8] (17.042516>16.726013) Degree bound 10: 2+f[10,4]+f[10,2]+(2*12+9)(f[10,2]-2) >= f[10,8] (16.124000>15.589737) Degree bound 11: 2+f[11,4]+f[11,2]+(2*12+9)(f[11,2]-2) >= f[11,8] (15.374900>14.709907)

Case of a cut of value 8, with maxu= 4 and maxv= 3 There are 9 dependent pairs incident at u1 or v1 (6 at u1 and 3 at v1) For u greedy partition is (4, 3, 1); for v greedy partition is (3, 3, 1, 1); 13 independent and 6 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 9: 1+f[9,4]+f[9,3]+(2*13+6)(f[9,2]-2) >= f[9,8] (17.301776>16.726013) Degree bound 10: 1+f[10,4]+f[10,3]+(2*13+6)(f[10,2]-2) >= f[10,8] (16.354000>15.589737) Degree bound 11: 1+f[11,4]+f[11,3]+(2*13+6)(f[11,2]-2) >= f[11,8] (15.581512>14.709907)

Case of a cut of value 8, with maxu= 4 and maxv= 4 There are 12 dependent pairs incident at u1 or v1 (6 at u1 and 6 at v1) For u and v greedy partition is (4, 2, 1, 1); 14 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 9: 0+f[9,4]+f[9,4]+(2*14+2)(f[9,2]-2) >= f[9,8] (17.640596>16.726013) Degree bound 10: 0+f[10,4]+f[10,4]+(2*14+2)(f[10,2]-2) >= f[10,8] (16.648000>15.589737) Degree bound 11: 0+f[11,4]+f[11,4]+(2*14+2)(f[11,2]-2) >= f[11,8] (15.840716>14.709907) Case of a cut of value 8, with maxu= 5 and maxv= 2 There are 11 dependent pairs incident at u1 or v1 (10 at u1 and 1 at v1) For u greedy partition is (5, 3); for v greedy partition is (2, 2, 2, 1, 1); 12 independent and 5 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 9: 1+f[9,5]+f[9,2]+(2*12+5)(f[9,2]-2) >= f[9,8] (17.287450>16.726013) Degree bound 10: 1+f[10,5]+f[10,2]+(2*12+5)(f[10,2]-2) >= f[10,8] (16.320500>15.589737) Degree bound 11: 1+f[11,5]+f[11,2]+(2*12+5)(f[11,2]-2) >= f[11,8] (15.536024>14.709907)

Case of a cut of value 8, with maxu= 5 and maxv= 3 There are 13 dependent pairs incident at u1 or v1 (10 at u1 and 3 at v1) For u greedy partition is (5, 2, 1); for v greedy partition is (3, 2, 1, 1, 1); 13 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 9: 0+f[9,5]+f[9,3]+(2*13+2)(f[9,2]-2) >= f[9,8] (17.546710>16.726013) Degree bound 10: 0+f[10,5]+f[10,3]+(2*13+2)(f[10,2]-2) >= f[10,8] (16.550500>15.589737) Degree bound 11: 0+f[11,5]+f[11,3]+(2*13+2)(f[11,2]-2) >= f[11,8] (15.742636>14.709907)

Case of a cut of value 8, with maxu= 6 and maxv= 2 There are 16 dependent pairs incident at u1 or v1 (15 at u1 and 1 at v1) For u greedy partition is (6, 2); for v greedy partition is (2, 2, 1, 1, 1, 1, 1); 10 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 9: 0+f[9,6]+f[9,2]+(2*10+2)(f[9,2]-2) >= f[9,8] (17.272159>16.726013) Degree bound 10: 0+f[10,6]+f[10,2]+(2*10+2)(f[10,2]-2) >= f[10,8] (16.263060>15.589737) Degree bound 11: 0+f[11,6]+f[11,2]+(2*10+2)(f[11,2]-2) >= f[11,8] (15.452120>14.709907)

Case of a cut of value 9, with maxu= 2 and maxv= 2 There are 2 dependent pairs incident at u1 or v1 (1 at u1 and 1 at v1) For u and v greedy partition is (2, 2, 2, 2, 1); 28 independent and 6 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 5+f[10,2]+f[10,2]+(2*28+6)(f[10,2]-2) >= f[10,9] (21.800000>19.292299) Degree bound 11: 5+f[11,2]+f[11,2]+(2*28+6)(f[11,2]-2) >= f[11,9] (20.636352>18.053067)

Case of a cut of value 9, with maxu= 3 and maxv= 2 There are 4 dependent pairs incident at u1 or v1 (3 at u1 and 1 at v1) For u greedy partition is (3, 3, 3); for v greedy partition is (2, 2, 2, 2, 1); 23 independent and 9 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 4+f[10,3]+f[10,2]+(2*23+9)(f[10,2]-2) >= f[10,9] (20.830000>19.292299) Degree bound 11: 4+f[11,3]+f[11,2]+(2*23+9)(f[11,2]-2) >= f[11,9] (19.752056>18.053067)

Case of a cut of value 9, with maxu= 3 and maxv= 3 There are 6 dependent pairs incident at u1 or v1 (3 at u1 and 3 at v1) For u and v greedy partition is (3, 3, 3); 18 independent and 12 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 3+f[10,3]+f[10,3]+(2*18+12)(f[10,2]-2) >= f[10,9] (19.860000>19.292299) Degree bound 11: 3+f[11,3]+f[11,3]+(2*18+12)(f[11,2]-2) >= f[11,9] (18.867760>18.053067) Case of a cut of value 9, with maxu= 4 and maxv= 2 There are 7 dependent pairs incident at u1 or v1 (6 at u1 and 1 at v1) For u greedy partition is (4, 4, 1); for v greedy partition is (2, 2, 2, 2, 1); 20 independent and 9 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 3+f[10,4]+f[10,2]+(2*20+9)(f[10,2]-2) >= f[10,9] (20.324000>19.292299) Degree bound 11: 3+f[11,4]+f[11,2]+(2*20+9)(f[11,2]-2) >= f[11,9] (19.283988>18.053067)

Case of a cut of value 9, with maxu= 4 and maxv= 3 There are 9 dependent pairs incident at u1 or v1 (6 at u1 and 3 at v1) For u greedy partition is (4, 4, 1); for v greedy partition is (3, 3, 2, 1); 17 independent and 10 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 2+f[10,4]+f[10,3]+(2*17+10)(f[10,2]-2) >= f[10,9] (19.754000>19.292299) Degree bound 11: 2+f[11,4]+f[11,3]+(2*17+10)(f[11,2]-2) >= f[11,9] (18.763328>18.053067)

Case of a cut of value 9, with maxu= 4 and maxv= 4 There are 12 dependent pairs incident at u1 or v1 (6 at u1 and 6 at v1) For u and v greedy partition is (4, 3, 1, 1); 18 independent and 6 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 1+f[10,4]+f[10,4]+(2*18+6)(f[10,2]-2) >= f[10,9] (20.048000>19.292299) Degree bound 11: 1+f[11,4]+f[11,4]+(2*18+6)(f[11,2]-2) >= f[11,9] (19.022532>18.053067)

Case of a cut of value 9, with maxu= 5 and maxv= 2 There are 11 dependent pairs incident at u1 or v1 (10 at u1 and 1 at v1) For u greedy partition is (5, 4); for v greedy partition is (2, 2, 2, 2, 1); 16 independent and 9 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 2+f[10,5]+f[10,2]+(2*16+9)(f[10,2]-2) >= f[10,9] (19.720500>19.292299) Degree bound 11: 2+f[11,5]+f[11,2]+(2*16+9)(f[11,2]-2) >= f[11,9] (18.717840>18.053067)

Case of a cut of value 9, with maxu= 5 and maxv= 3 There are 13 dependent pairs incident at u1 or v1 (10 at u1 and 3 at v1) For u greedy partition is (5, 3, 1); for v greedy partition is (3, 3, 1, 1, 1); 17 independent and 6 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 1+f[10,5]+f[10,3]+(2*17+6)(f[10,2]-2) >= f[10,9] (19.950500>19.292299) Degree bound 11: 1+f[11,5]+f[11,3]+(2*17+6)(f[11,2]-2) >= f[11,9] (18.924452>18.053067)

Case of a cut of value 9, with maxu= 5 and maxv= 4 There are 16 dependent pairs incident at u1 or v1 (10 at u1 and 6 at v1) For u greedy partition is (5, 2, 1, 1); for v greedy partition is (4, 2, 1, 1, 1); 18 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 0+f[10,5]+f[10,4]+(2*18+2)(f[10,2]-2) >= f[10,9] (20.244500>19.292299) Degree bound 11: 0+f[11,5]+f[11,4]+(2*18+2)(f[11,2]-2) >= f[11,9] (19.183656>18.053067)

Case of a cut of value 9, with maxu= 6 and maxv= 2 There are 16 dependent pairs incident at u1 or v1 (15 at u1 and 1 at v1) For u greedy partition is (6, 3); for v greedy partition is (2, 2, 2, 1, 1, 1); 15 independent and 5 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 1+f[10,6]+f[10,2]+(2*15+5)(f[10,2]-2) >= f[10,9] (19.863060>19.292299) Degree bound 11: 1+f[11,6]+f[11,2]+(2*15+5)(f[11,2]-2) >= f[11,9] (18.815754>18.053067)

Case of a cut of value 9, with maxu= 6 and maxv= 3 There are 18 dependent pairs incident at u1 or v1 (15 at u1 and 3 at v1) For u greedy partition is (6, 2, 1); for v greedy partition is (3, 2, 1, 1, 1, 1); 16 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 0+f[10,6]+f[10,3]+(2*16+2)(f[10,2]-2) >= f[10,9] (20.093060>19.292299) Degree bound 11: 0+f[11,6]+f[11,3]+(2*16+2)(f[11,2]-2) >= f[11,9] (19.022366>18.053067)

Case of a cut of value 9, with maxu= 7 and maxv= 2 There are 22 dependent pairs incident at u1 or v1 (21 at u1 and 1 at v1) For u greedy partition is (7, 2); for v greedy partition is (2, 2, 1, 1, 1, 1, 1); 12 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 10: 0+f[10,7]+f[10,2]+(2*12+2)(f[10,2]-2) >= f[10,9] (19.800927>19.292299) Degree bound 11: 0+f[11,7]+f[11,2]+(2*12+2)(f[11,2]-2) >= f[11,9] (18.707657>18.053067)

Case of a cut of value 10, with maxu= 2 and maxv= 2 There are 2 dependent pairs incident at u1 or v1 (1 at u1 and 1 at v1) For u and v greedy partition is (2, 2, 2, 2, 2); 35 independent and 8 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 6+f[11,2]+f[11,2]+(2*35+8)(f[11,2]-2) >= f[11,10] (24.545440>21.882506)

Case of a cut of value 10, with maxu= 3 and maxv= 2 There are 4 dependent pairs incident at u1 or v1 (3 at u1 and 1 at v1) For u greedy partition is (3, 3, 3, 1); for v greedy partition is (2, 2, 2, 2, 2); 31 independent and 10 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 5+f[11,3]+f[11,2]+(2*31+10)(f[11,2]-2) >= f[11,10] (23.842962>21.882506

Case of a cut of value 10, with maxu= 3 and maxv= 3
There are 6 dependent pairs incident at u1 or v1 (3 at u1 and 3 at v1)
For u and v greedy partition is (3, 3, 3, 1);
27 independent and 12 dependent pairs (excluding pairs incident at u1 or v1)
Degree bound 11: 4+f[11,3]+f[11,3]+(2*27+12)(f[11,2]-2) >= f[11,10] (23.140484>21.882506

Case of a cut of value 10, with maxu= 4 and maxv= 2 There are 7 dependent pairs incident at u1 or v1 (6 at u1 and 1 at v1) For u greedy partition is (4, 4, 2); for v greedy partition is (2, 2, 2, 2, 2); 27 independent and 11 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 4+f[11,4]+f[11,2]+(2*27+11)(f[11,2]-2) >= f[11,10] (23.193076>21.882506

Case of a cut of value 10, with maxu= 4 and maxv= 3 There are 9 dependent pairs incident at u1 or v1 (6 at u1 and 3 at v1) For u greedy partition is (4, 4, 2); for v greedy partition is (3, 3, 3, 1); 23 independent and 13 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 3+f[11,4]+f[11,3]+(2*23+13)(f[11,2]-2) >= f[11,10] (22.490598>21.882506) Case of a cut of value 10, with maxu= 4 and maxv= 4 There are 12 dependent pairs incident at u1 or v1 (6 at u1 and 6 at v1) For u and v greedy partition is (4, 4, 1, 1); 21 independent and 12 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 2+f[11,4]+f[11,4]+(2*21+12)(f[11,2]-2) >= f[11,10] (22.204348>21.882506)

Case of a cut of value 10, with maxu= 5 and maxv= 2 There are 11 dependent pairs incident at u1 or v1 (10 at u1 and 1 at v1) For u greedy partition is (5, 5); for v greedy partition is (2, 2, 2, 2, 2); 20 independent and 14 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 3+f[11,5]+f[11,2]+(2*20+14)(f[11,2]-2) >= f[11,10] (22.081474>21.882506)

Case of a cut of value 10, with maxu= 5 and maxv= 3 There are 13 dependent pairs incident at u1 or v1 (10 at u1 and 3 at v1) For u greedy partition is (5, 4, 1); for v greedy partition is (3, 3, 2, 1, 1); 22 independent and 10 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 2+f[11,5]+f[11,3]+(2*22+10)(f[11,2]-2) >= f[11,10] (22.469904>21.882506

Case of a cut of value 10, with maxu= 5 and maxv= 4 There are 16 dependent pairs incident at u1 or v1 (10 at u1 and 6 at v1) For u greedy partition is (5, 3, 1, 1); for v greedy partition is (4, 3, 1, 1, 1); 23 independent and 6 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 1+f[11,5]+f[11,4]+(2*23+6)(f[11,2]-2) >= f[11,10] (22.729108>21.882506)

Case of a cut of value 10, with maxu= 5 and maxv= 5 There are 20 dependent pairs incident at u1 or v1 (10 at u1 and 10 at v1) For u and v greedy partition is (5, 2, 1, 1, 1); 23 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 0+f[11,5]+f[11,5]+(2*23+2)(f[11,2]-2) >= f[11,10] (22.890232>21.882506)

Case of a cut of value 10, with maxu= 6 and maxv= 2 There are 16 dependent pairs incident at u1 or v1 (15 at u1 and 1 at v1) For u greedy partition is (6, 4); for v greedy partition is (2, 2, 2, 2, 1, 1); 20 independent and 9 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 2+f[11,6]+f[11,2]+(2*20+9)(f[11,2]-2) >= f[11,10] (22.361206>21.882506)

Case of a cut of value 10, with maxu= 6 and maxv= 3 There are 18 dependent pairs incident at u1 or v1 (15 at u1 and 3 at v1) For u greedy partition is (6, 3, 1); for v greedy partition is (3, 3, 1, 1, 1, 1); 21 independent and 6 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 1+f[11,6]+f[11,3]+(2*21+6)(f[11,2]-2) >= f[11,10] (22.567818>21.882506)

Case of a cut of value 10, with maxu= 6 and maxv= 4 There are 21 dependent pairs incident at u1 or v1 (15 at u1 and 6 at v1) For u greedy partition is (6, 2, 1, 1); for v greedy partition is (4, 2, 1, 1, 1, 1); 22 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 0+f[11,6]+f[11,4]+(2*22+2)(f[11,2]-2) >= f[11,10] (22.827022>21.882506)

Case of a cut of value 10, with maxu= 7 and maxv= 2 There are 22 dependent pairs incident at u1 or v1 (21 at u1 and 1 at v1) For u greedy partition is (7, 3); for v greedy partition is (2, 2, 2, 1, 1, 1, 1); 18 independent and 5 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 1+f[11,7]+f[11,2]+(2*18+5)(f[11,2]-2) >= f[11,10] (22.434927>21.882506)

Case of a cut of value 10, with maxu= 7 and maxv= 3 There are 24 dependent pairs incident at u1 or v1 (21 at u1 and 3 at v1) For u greedy partition is (7, 2, 1); for v greedy partition is (3, 2, 1, 1, 1, 1, 1); 19 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 0+f[11,7]+f[11,3]+(2*19+2)(f[11,2]-2) >= f[11,10] (22.641539>21.882506)

Case of a cut of value 10, with maxu= 8 and maxv= 2 There are 29 dependent pairs incident at u1 or v1 (28 at u1 and 1 at v1) For u greedy partition is (8, 2); for v greedy partition is (2, 2, 1, 1, 1, 1, 1, 1); 14 independent and 2 dependent pairs (excluding pairs incident at u1 or v1) Degree bound 11: 0+f[11,8]+f[11,2]+(2*14+2)(f[11,2]-2) >= f[11,10] (22.346265>21.882506)