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NUMERICAL SIMULATION OF THE CONDUCTING SURFACE  
OF HIGH-VOLTAGE INSULATING SYSTEMS IN 3D

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ABSTRACT

The numerical simulation described in this paper is based upon the boundary element method. The effects of the surface conducting film on the potential distribution of insulating systems are considered. A particular interface condition is used for the resistive surface of insulators. This approach allows fast subsequent analysis for different positions and resistances of the surface layers.

1. INTRODUCTION

For the design of high-voltage insulating systems, it is very important to analyse the conducting film effects over the surface of insulators (pollution, semi-conducting glaze for anti-pollution).

The presence of the surface current throughout the insulating system distorts the capacitive potential distribution. There are many published papers presenting a numerical analysis of this effect, in which the finite difference method, the finite element method and the charge simulation method are used [1],[2].

These methods, however, suffer from some inconveniences: the charge simulation method is very sensitive to the number of charges chosen for a particular problem; the suggestion that the solution quality increases with the number of charge is not necessarily true [3]. For the finite element or finite difference method, the entire volume of the problem must be subdivided, making it difficult to treat arbitrarily thin conducting film.

Since the phenomenon can be considered an interfacial one, it is logical to use the boundary element method (BEM) for the simulation of this problem. In this paper we present a numerical method based on BEM for the simulation of conducting film effects.

2. BOUNDARY INTEGRAL EQUATION

The calculation of electric fields requires the solution of the Laplace's equation in one or several region(s)  $V$  with the boundary conditions. The equivalent formula of the Laplace's equation is:

$$C_p \Phi(P) = - \int_{\Sigma} \left( \phi \frac{\partial G}{\partial n} - \frac{1}{\epsilon_r} \psi G \right) ds \quad (1)$$

with:

$$G = \frac{1}{4\pi r} \quad \psi = \epsilon_r \frac{\partial \Phi}{\partial n}$$

as shown in the previous publications [4], where:

$\Sigma$  is the boundary of  $V$   
 $\vec{n}$  is the normal vector of  $\Sigma$   
 $r$  is the distance between the point  $P$  and the current point of integration  
 $C_p$  is the solid angle viewing from the point  $P$   
 $\phi$  is the potential  
 $\epsilon_r$  is the relative permittivity.

By choosing a set of points  $P$  on  $\Sigma$  and using the classical finite element technique, the equation (1) can be written as:

$$C_p \Phi(P) = \sum_k \sum_i \left( \phi_{ki} A_{ki} - \frac{1}{\epsilon_r} \psi_{ki} B_{ki} \right) \quad (2)$$

where:

$k$  is the subscript of the boundary element  
 $i$  is the subscript of the discrete node of an element  
 $\phi_{ki}$  is the value of  $\phi$  at the discrete node  
 $\psi_{ki}$  is the value of  $\psi$  at the discrete node

$A_{ki}$  and  $B_{ki}$  depend only on the geometry and can be calculated.

To solve (2) three types of boundary conditions have to be considered:

- \* Dirichlet:  $\phi = V$  known for a conductor
- \* Neumann:  $\psi = 0$  for a plan of symmetry
- \*  $\phi$  and  $\psi$  are both unknown for an interface

The equation (2) is established on each discrete point of the boundary with the first two types of boundary conditions.

On interfaces, equation (2) is established twice on each discrete point for the two concerned regions. So we can establish equation (2) as many as the number of unknowns for a problem with several different dielectric mediums. After resolution, one obtains the distributions of  $\phi$  and  $\psi$  on  $\Sigma$  and  $\Phi$  anywhere by reusing (2).

The field is obtained in the same manner by using the gradients of the two weighting functions ( $G, \partial G / \partial n$ ).

$\psi$  is continuous through the interface of two dielectrics, on condition that there is no conducting film on this interface, otherwise a particular interface condition should be used.

### 3. INTERFACE CONDITION FOR THE CONDUCTING SURFACE

Let  $S$  be a pillbox-shaped closed surface (figure 1) of end areas  $F$ , over the surface, whose height  $dl$  is equal to the conducting film's thickness.

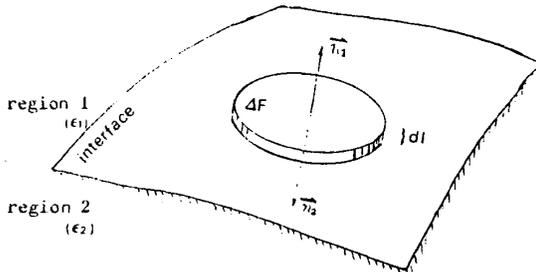


Figure 1: The pillbox constructed for deriving the boundary condition of the conducting surface

The charge conservation gives the following relation:

$$\int_S (\vec{J} + j\omega \vec{D}) \cdot d\vec{s} = 0 \quad (3)$$

where:

$\vec{J}$  is the current density  
 $\vec{D}$  is the displacement

$\frac{\omega}{2\pi}$  is the frequency

Assuming  $dl$  is negligible in comparison with the  $\Delta F$  radius and  $\Delta F$  is also very small, (3) can be written as follows:

$$\int_S \vec{J} \cdot d\vec{s} = j\omega \epsilon_0 (\Psi_1 + \Psi_2) \Delta F \quad (4)$$

where  $\Psi_1$  and  $\Psi_2$  are the values of  $\Psi$  on  $\Sigma$ , but defined in different media.

Let's  $\vec{s} = dl \cdot \vec{J}$  be the surface current density, we have:

$$\lim_{\substack{\Delta F \rightarrow 0 \\ dl \rightarrow 0}} \frac{\int_S \vec{J} \cdot d\vec{s}}{\Delta F \cdot dl} = \nabla \cdot \vec{s} = j\omega \epsilon_0 (\Psi_1 + \Psi_2) \quad (5)$$

Using the Ohm's law, one obtains the boundary condition for conducting surface:

$$\Psi_1 = -\Psi_2 + j \frac{\sigma}{\omega \epsilon_0} \nabla \cdot \vec{E}_s \quad (6)$$

where:

$\vec{E}_s$  is the tangential field on the interface  
 $\sigma$  is the surface conductivity.

As we have defined the interpolation expression of  $\Psi$  on the boundary,  $\vec{E}$  can be calculated.

### 4. CALCULATION OF $\nabla \cdot \vec{E}_s$

On the boundary element (3D), one has:

$$\Phi = \sum_i P_i(u, v) \Phi_i \quad (7)$$

where  $P_i$  is the interpolating function;  $u$  and  $v$  are the local curvilinear coordinates.

By using differential operators in 2d curvilinear space, one obtains:

$$\nabla \cdot \vec{E}_s = -\frac{1}{\sqrt{g}} \left[ \frac{\partial}{\partial u} (\sqrt{g} a_u) + \frac{\partial}{\partial v} (\sqrt{g} a_v) \right] \quad (8)$$

$$\begin{aligned} \text{with: } a_u &= g^{uv} \frac{\partial \Phi}{\partial u} + g^{vv} \frac{\partial \Phi}{\partial v} \\ a_v &= g^{vv} \frac{\partial \Phi}{\partial v} + g^{uv} \frac{\partial \Phi}{\partial u} \end{aligned} \quad (9)$$

where:

$g$  is the determinant of the covariant metrical matrix

$g^{uv}$ ,  $g^{uv}$  and  $g^{vv}$  are the elements of the contravariant metrical matrix:

$$\vec{e}_v = \frac{\partial x}{\partial v} \vec{i} + \frac{\partial y}{\partial v} \vec{j} + \frac{\partial z}{\partial v} \vec{k} \quad \epsilon_{uu} = \vec{e}_u \cdot \vec{e}_u$$

$$\vec{e}_u = \frac{\partial x}{\partial u} \vec{i} + \frac{\partial y}{\partial u} \vec{j} + \frac{\partial z}{\partial u} \vec{k} \quad \epsilon_{vv} = \epsilon_{vu} = \vec{e}_v \cdot \vec{e}_v \quad (10)$$

$$\epsilon_{vv} = \vec{e}_v \cdot \vec{e}_v$$

$$[g^{uv}] = [g_{uv}]^{-1}$$

For the orthogonal local coordinates:

$$\begin{aligned} \nabla \cdot \vec{E}_s &= \frac{1}{g_{uu}} \frac{\partial^2 \Phi}{\partial u^2} + \frac{1}{g_{vv}} \frac{\partial^2 \Phi}{\partial v^2} + \frac{1}{2} \frac{\partial \Phi}{\partial u} \left( \frac{1}{g} \frac{\partial g_{vv}}{\partial u} - \frac{1}{g_{vv}} \frac{\partial g_{uu}}{\partial u} \right) \\ &+ \frac{1}{2} \frac{\partial \Phi}{\partial v} \left( \frac{1}{g} \frac{\partial g_{uu}}{\partial v} - \frac{1}{g_{vv}} \frac{\partial g_{vv}}{\partial v} \right) \end{aligned} \quad (11)$$

where  $g_{uu}$ ,  $g_{uv}$  and  $g_{vv}$  are the elements of the covariant metrical matrix.

For the axisymmetric case, the expression is more simple:

$$\nabla \cdot \vec{E}_s = \frac{1}{J^2} \frac{\partial^2 \Phi}{\partial U^2} + \left[ \frac{1}{J^2 R} \frac{\partial R}{\partial U} - \frac{1}{J^2} \left( \frac{\partial R}{\partial U} \frac{\partial^2 R}{\partial U^2} + \frac{\partial z}{\partial U} \frac{\partial^2 z}{\partial U^2} \right) \right] \frac{\partial \Phi}{\partial U} \quad (12)$$

where:

$$J = \sqrt{\left( \frac{\partial R}{\partial U} \right)^2 + \left( \frac{\partial z}{\partial U} \right)^2} \quad (13)$$

The relation (6) is true under the condition that the surface current is not zero. If there is no surface current, the conducting film has to be treated by an other method.

### 5. CONDUCTING FILM WITH FLOATING POTENTIAL

When the conducting film is not connected by electrodes, there is no surface current (for example, the surface pollution film is interrupted by the dry bands), in this case, the conducting film behaves itself like a good conductor with floating potential. (3) can be written as:

$$\int_S \Psi ds = 0 \quad (14)$$

where  $S$  is the two surfaces of the conducting film.

This relation allows us to establish a supplementary equation and the two surfaces of the conducting film can be treated as the boundary with the Dirichlet's boundary condition.

Numerically, we double the discrete elements of conducting part of insulator's surface (these elements constitute a closed surface), so that (14) can be imposed over these elements. Figure 2 shows an example in the axisymmetric case.

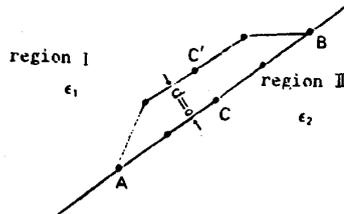


Figure 2: AB-conducting film with floating potential on which the discrete elements are doubled

The integration of equation (1) established at the point C' is over the boundary of the medium I; the integration of this equation established at C is over the boundary of the medium II. Although C and C' are geometrically the same point, we can obtain two different equations because the boundaries of the two media are not the same. The points A, B become the geometrical singular points and  $\Psi$  has not a unique value on these points.

The numerical method proposed in [4] is very convenient for treating these points.

It is obvious that this technique can be used for treating metallic films. If the metallic film is in a medium, not over the surface of insulator, we can add an artificial interface in order that the metallic film becomes a part of this interface (figure 3).

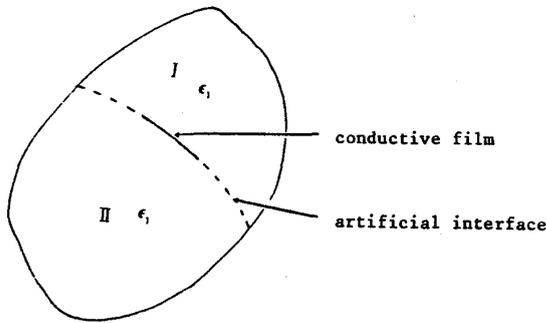


Figure 3: Artificial interface

The metallic film can also be treated as source. In such region, (1) becomes:

$$\epsilon_0 \phi(P) = - \int_{\Sigma} \left( \phi \frac{\delta G}{\delta n} - \frac{1}{\epsilon_1} \psi G \right) ds - \frac{1}{\epsilon} \int_{S_c} \tau G ds \quad (15)$$

where:

$\tau$  is the surface charge density  
 $S_c$  is the surface of the metallic film.

This treatment can reduce the number of unknowns, but complicates programming.

G. SOME EXAMPLES

The above development is now applied in PHI3D program [4] for the simulation in 3D space and in PHIAX program [5] for axisymmetric cases.

The illustrating results of some examples are given here.

Figure 4 shows the potential distribution over the surfaces of an insulator between two electrodes. Of 4 surfaces, only one is conductive. It can be seen that the presence of the surface current distorts the potential distribution:

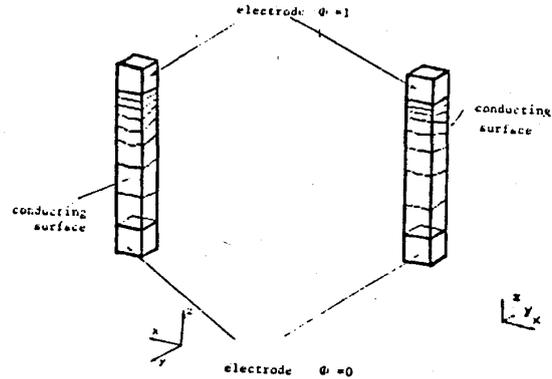


Figure 4: Potential distribution over the surfaces of an insulator

Figure 5 gives an example for an axisymmetric case. It shows the effect of the floating potential film on the potential distribution:

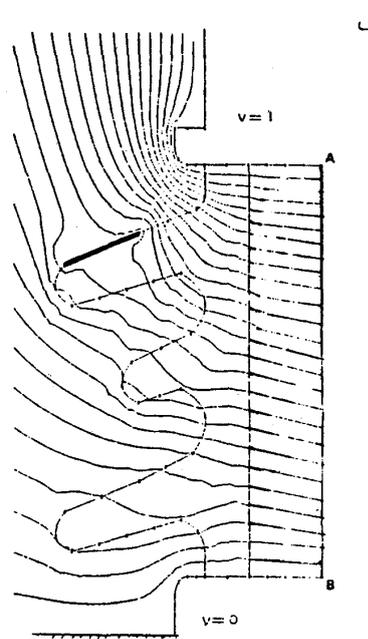


Figure 5: Potential distribution for an axisymmetric insulator where AB is the resistance surface and the dark-line segment the floating potential film

## 7. CONCLUSION

Conducting film effects are very easy to be implemented with the boundary integral equation method using a particular boundary condition.

This approach allows fast subsequent analysis for different positions and resistances of the conducting films. In addition, the influence of the metallic foils with floating potential can be easily simulated.

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