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***Abstract*— Model refinements of magnetic circuits are performed via a subproblem finite element method based on a perturbation technique. An approximate problem considering ideal flux tubes and simplified air-gap models is first solved. It gives the sources for a finite element perturbation problem considering the actual air gaps and flux tubes geometries with the exterior regions. The procedure simplifies both meshing and solving processes, and allows to quantify the gain given by each model refinement.**

I. Introduction

The perturbation of finite element (FE) solutions provides clear advantages in repetitive analyses [1] and helps improving the solution accuracy [2]. It allows to benefit from previous computations instead of starting a new complete FE solution for any variation of geometrical or physical data. It also allows different problem-adapted meshes and computational efficiency due to the reduced size of each subproblem.

A perturbation FE method is herein developed for refining the magnetic flux distribution in magnetic circuits starting from simplified FE models. These are based on both ideal flux tubes [3] and thin-shell air-gap models [4]. The developments are performed for the magnetic vector potential FE magnetostatic formulation, paying special attention to the proper discretization of the source constraints involved in the perturbation subproblems. The method is validated on a test problem.

II. Approximate and Perturbation Problems

A canonical magnetostatic problem p is defined in a domain Ω_p. Some related quantities are the magnetic field $\boldsymbol{h}_p$, the magnetic flux density $\boldsymbol{b}_p$ and the magnetic permeability μ_p. The objective is solving a sequence of such subproblems, each one defining a perturbation of others, and the superposition of which giving the solution $\boldsymbol{u} = \Sigma_p \boldsymbol{u}_p$ of a complete problem (with possible inter-problem iterations for nonlinear analyses). Each problem is defined in its own domain and mesh, which decreases the problem complexity and allows distinct mesh refinements.

Both volume and surface sources can be involved in each subproblem [2]. On the one hand, a change of the permeability μ in a volume region, e.g. from μ_1 to μ_2, is defined via a volume source $\boldsymbol{h}_{s,2} = (\mu_2^{-1} - \mu_1^{-1})\, \boldsymbol{b}_1$ in the $\boldsymbol{h}$-$\boldsymbol{b}$ material relation $\boldsymbol{h}_2 = \mu_2^{-1}\, \boldsymbol{b}_2 + \boldsymbol{h}_{s,2}$. On the other hand, a change of boundary or interface conditions is defined via surface sources fixing the possible trace discontinuities of $\boldsymbol{h}_2$ and $\boldsymbol{b}_2$.

In an approximate problem $p = 1$, the magnetic flux is forced to flow only in a subregion Ω_1 with perfect flux wall, i.e. a set of flux tubes of the whole domain Ω. Moreover, the possible air gaps in the tubes are approximated by surface (thin shell) FEs [4]. The perturbation problem $p = 2$ considers then the actual extension of the air gaps with volume FEs, and also that the flux walls become permeable. This allows leakage flux in $\Omega \backslash \Omega_1$ and leads to a change of the flux distribution in Ω_1. A solution refinement is thus obtained.

All the constraints involved in the subproblems have to be carefully defined in the resulting FE formulations, respecting their inherent strong and weak natures. They will be detailed, justified and validated in the extended paper. As a significant result, an efficient and accurate computation of local fields and global quantities (e.g., flux, magnetomotive force, reluctance) can be performed.

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III. Application Example

A magnetic circuit with an air gap is considered as a test problem (Fig. 1). An approximate solution is first calculated in an idealized flux tube comprising the thin-shell air gap (Fig. 1, *left*), with a fixed magnetomotive force as excitation and a coarse mesh of the tube. This solution then serves as a source for a perturbation problem defined in the vicinity of the actual volume air gap (Fig. 1, *middle*) with a locally refined mesh, giving the proper correction. The importance of this correction versus the gap thickness is shown in Fig. 2. The corrected solution has been checked to be in perfect accordance with the one-step complete FE solution. A significant gain will be shown to be given for both meshing and solving processes of complex geometries. The method naturally allows additional refinements towards eddy current or 3-D effects. It allows to quantify the gain given by each model refinement to justify its usefulness.

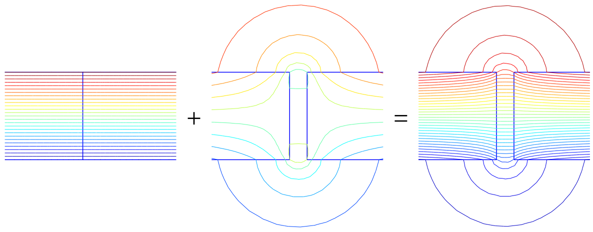

Fig. 1. Flux tube portion with air gap: approximate solution (field lines) in an idealized tube and surface air gap (*left*), perturbation solution with a volume air gap and leakage flux (*middle*), complete (corrected) solution (*right*).

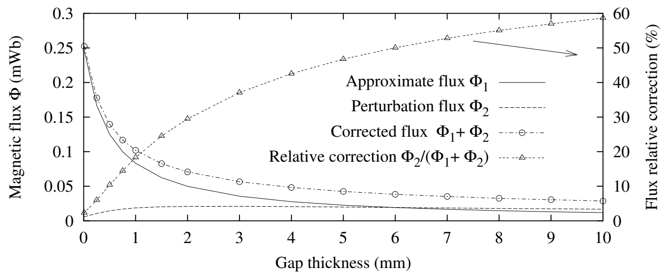

Fig. 2. Importance of the perturbation flux versus the air-gap thickness.

References

[1] Z. Badics *et al.*, “An effective 3-D finite element scheme for computing electromagnetic field distorsions due to defects in eddy-current nondestructive evaluation,” *IEEE Trans. Magn.*, Vol. 33, No. 2, pp. 1012-1020, 1997.

[2] P. Dular, R. V. Sabariego, J. Gyselinck, L. Krähenbühl, “Sub-domain finite element method for efficiently considering strong skin and proximity effects,” *COMPEL*, Vol. 26, No. 4, pp. 974-985, 2007.

[3] P. Dular, J. Gyselinck, T. Henneron, F. Piriou, “Dual finite element formulations for lumped reluctances coupling,” *IEEE Trans. Magn.*, Vol. 41, No. 5, pp. 1396-1399, 2005.

[4] C. Guerin, G. Tanneau, G. Meunier, X. Brunotte, J.B. Albertini, “Three dimensional magnetostatic finite elements for gaps and iron shells using magnetic scalar potentials,” *IEEE Trans. Magn.*, Vol. 30, No. 5, pp. 2885-2888, 1994.

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