Transmission gaps and Fano resonances in an acoustic waveguide: analytical model

El Houssaine El Boudouti, Tarik Mrabti, Housni Al-Wahsh, Bahram Djafari-Rouhani, Abdellatif Akjouj, L. Dobrzynski

Transmission gaps and Fano resonances in an acoustic waveguide: analytical model

E H El Boudouti[1], T Mrabti[1], H Al-Wahsh[2], B Djafari-Rouhani[3], A Akjouj[3,4] and L Dobrzynski[3]

[1] Laboratoire de Dynamique et d'optique des matériaux, Département de Physique, Faculté des Sciences, Université Mohamed I, Oujda, Morocco
[2] Faculty of Engineering, Benha University, 11241 Cairo, Egypt
[3] Institut d'Electronique, de Microélectronique et de Nanotechnologie (IEMN), UMR CNRS 8520, UFR de Physique, Université des Sciences et Technologies de Lille, 59655 Villeneuve d'Ascq Cédex, France

E-mail: Abdellatif.Akjouj@univ-lille1.fr

Abstract
A simple acoustic device consisting of two dangling side resonators grafted at two sites on a slender tube is designed possibly to obtain transmission stop bands (where the propagation of longitudinal acoustic waves is forbidden). In contrast to all known systems of this kind, a spectral transmission gap of nonzero width occurs here even with this simple structure. This is obtained by combining appropriately the zeros of transmission of the side resonators. Sharp resonant states inside the gaps can be achieved without introducing any defects in the structure. This results from an internal resonance of the structure when such a resonance is situated in the vicinity of a zero of transmission or placed between two zeros of transmission, the so-called Fano resonances. A general analytical expression for the transmission coefficient is given for various systems of this kind within the framework of the Green's function method. The amplitude and the phase of the transmission are discussed as a function of frequency and it is shown that the width of the stop bands is very sensitive to the number of side resonators. These results should have important consequences for the suppression of low-frequency noise and for designing filters.

1. Introduction

Low dimensional structures which have a regular distribution of scattering centers have been seen to possess a distinct and interesting array of acoustical properties, perhaps most strikingly frequency band gaps within which acoustic waves cannot propagate through the structure—a so-called phononic band gap. In recent years, such phononic crystals have been the subject of intense theoretical [1] and experimental [2] investigations. Two-and three-dimensional composite systems constituted by periodic inclusions of a given material in a host matrix can exhibit an absolute acoustic band gap where the propagation of sound waves and ultrasonic vibrations is inhibited in any direction of space [3]. These acoustic band gap materials can have many practical applications such as elastic/acoustic filters [4]. Studies of lower dimensional systems such as 1D periodic layered media [5, 6] and periodic waveguide systems with different geometries [7–13] are conducted as analogs of 2D and 3D systems and for applications in their own right. These structures are attractive since their production is more feasible at any wavelength scale and they require only simple analytical and numerical calculations.

In a previous publication [9], some of the authors reported that the acoustic transmission spectrum of 1D comb structures exhibits large gaps. These structures, called *star wave guides*, are composed of N' dangling side branches (DSBs) periodically grafted at each of the N equidistant sites on slender tubes. The gaps originate from the periodicity of the system determined by the distance between two neighboring sites and from the eigenfrequencies of the DSB, which play the role of resonators. These theoretical results are confirmed by experiments using an impulse response technique in the

[4] Author to whom any correspondence should be addressed.

interval from 650 to 1100 Hz [12]. Unlike other 1D (e.g. Bragg lattices), 2D or 3D phononic crystals, in which the contrast between the constituents is a critical parameter for the stop band existence, this star waveguide exhibits relatively large forbidden bands even if the backbone and the resonators are made of the same material [9].

On the other hand, we have proposed [13] an asymmetric serial loop structure made of asymmetric slender tube loops pasted together with slender tubes of finite length; the loops play the role of resonators. Such a structure exhibits new features, in comparison with the star waveguide [9]. In the case of symmetric loops, the system becomes equivalent to a diameter modulated slender tube [8]. Recently, we have also investigated theoretically the propagation and localization of acoustic waves in quasi-periodic [14] slender tubes and these results have been confirmed experimentally [15]. In addition, we have shown the existence of transmission zeros in the case of asymmetric loop structure [14]. These transmission zeros may lead to a phase drop of π and therefore negative phase time [14]. A recent experiment by Robertson *et al* [16] on one asymmetric loop filter made of two different tubes has shown clearly the existence of such negative phase time and therefore negative group velocity.

In the last few years, the low-frequency band gap (called the locally resonant band gap) of a phononic crystal with small dimension has attracted much attention [17, 18]. 3D and 2D systems consisting of rubber-coated lead spheres and cylinders embedded in epoxy have shown the existence of such band gaps and their interest for blocking low-frequency sound [19, 20]. In addition to these gaps, great interest has been paid to the so-called Fano resonances that may be introduced in such gaps. Some analytical models have been proposed to explain the origin and the behavior of such resonances [18–20]. These resonances were first theoretically described by Fano [21] when he studied the inelastic auto-ionizing resonances in atoms. The asymmetry (Fano profile) was explained as a result of the interference between the discrete resonance and the smooth continuum background in which the former is embedded. The symmetric and asymmetric Fano line shapes have been extensively reported in the electronic transport in mesoscopic systems [22–24]. Mainly, the subject of these studies was to use the so-called Aharonov–Bohm interferometric systems to show the conditions for the existence and the collapse of Fano resonances as functions of the applied current voltage and magnetic flux. These studies are also related to the investigation of the electronic states of quantum dots [22, 23] as well as to the understanding [25, 26] of the transmission phase jumps by π between two adjacent resonances in relation to the experiments of Yacoby *et al* [27]. The analogy between scattering properties of electrons and phonons suggests that this type of feature can also appear in other vibrational systems [28].

The motivation behind the work presented in this paper is to introduce a design of a simple acoustic filter consisting of two slender side tubes, which play the role of resonators, grafted at two sites on an infinite slender tube (see figure 1). We show analytically and numerically that this simple structure can exhibit transmission gaps (their widths depend on the number of dangling resonators) and Fano-like resonances. In particular, we show that the transmission amplitude through such a system can be written following the Fano-like shape around these resonances. In addition, we give an explicit expression of the Fano parameter [21] as well as the position and the width of the Fano resonances [21] as a function of the geometrical parameters of the system. It is worthwhile to notice that such stop bands and resonances could be observed experimentally by using simple slender tubes as in the recent experiments by Robertson *et al* [12, 16].

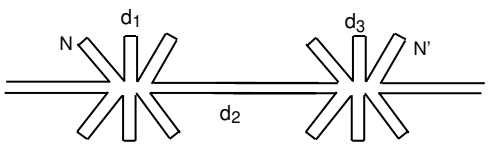

Figure 1. Schematic illustration of the one-dimensional slender tube waveguide of length d_2 with dangling resonators on both sides. The whole structure is inserted between two semi-infinite tubes. The lengths of the dangling resonators are d_1 and d_3 and their numbers are N and N', respectively (here $N = N' = 6$). The boundary conditions at the end of the resonators are rigid.

This paper is organized as follows. In section 2, we give a brief review of the theoretical model used in this work as well as the analytical results of the structure depicted above. These results are necessary for an analytical understanding of the new phenomenon obtained for the structure proposed in this work. Section 3 is devoted to the transmission gaps and Fano resonances. The conclusions and some implications for future experiments are presented in section 4.

2. Method of theoretical and numerical calculation

2.1. Interface response theory of continuous media

Our theoretical analysis is performed with the help of the interface response theory of continuous media, which allows calculation of the Green's function of any composite material. In what follows, we present the basic concept and the fundamental equations of this theory [35]. Let us consider any composite material contained in its space of definition D and formed out of N different homogeneous pieces located in their domains D_i. Each piece is bounded by an interface M_i, adjacent in general to j $(1 \leqslant j \leqslant J)$ other pieces through subinterface domains M_{ij}. The ensemble of all these interface spaces M_i will be called the interface space M of the composite material. The elements of the Green's function $g(DD)$ of any composite material can be obtained from [35]

$$g(DD) = G(DD) - G(DM)G^{-1}(MM)G(MD) + G(DM)G^{-1}(MM)g(MM)G^{-1}(MM)G(MD), \quad (1)$$

where $G(DD)$ is the reference Green's function formed out of truncated pieces in D_i of the bulk Green's functions of the infinite continuous media and $g(MM)$ the interface element of the Green's function of the composite system. The knowledge of the inverse of $g(MM)$ is sufficient to calculate the interface states of a composite system through the relation [35]

$$\det[g^{-1}(MM)] = 0. \quad (2)$$

Moreover, if $U(D)$ represents an eigenvector of the reference system, equation (1) enables the calculation of the eigenvectors $u(D)$ of the composite material and

$$u(D) = U(D) - U(M)G^{-1}(MM)G(MD) + U(M)G^{-1}(MM)g(MM)G^{-1}(MM)G(MD). \quad (3)$$

In equation (3), $U(D)$, $U(M)$, and $u(D)$ are row vectors. Equation (3) provides a description of all the waves reflected and transmitted by the interfaces, as well as the reflection and transmission coefficients of the composite system. In this case, $U(D)$ is a bulk wave launched in one homogeneous piece of the composite material [17].

2.2. Inverse surface Green's functions of the elementary constituents

For the sake of simplicity, we embark on the simpler one-dimensional system made of nonviscous fluid tubes. The calculations at hand simplify because the transverse speed of sound v_t is zero in nonviscous fluids. Nevertheless, the ordinary wave equation is inapplicable to inhomogeneous system. The correct wave equation, namely the equation of motion in the framework of the acoustic approximation, is

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla[(\rho/a)v^2 \nabla \cdot (a\mathbf{u})] \quad (4)$$

where $\rho(\mathbf{r})$ is the mass density, $a(\mathbf{r})$ the cross section and $v(\mathbf{r})$ the longitudinal speed of sound. In the case of acoustic approximation, $\nabla \times (\rho \mathbf{u}) = 0$. Therefore, it is possible to define a scalar potential $\Phi(\mathbf{r}, t)$ such that $\rho \mathbf{u} = \nabla \Phi$. Then equation (4) may be cast in the form of a scalar equation

$$\left[\frac{\rho}{a} v^2\right]^{-1} \frac{\partial^2 \Phi}{\partial t^2} = \nabla \cdot [(\rho/a)^{-1} \nabla \Phi]. \quad (5)$$

This is the starting point to be followed by the interface response theory [29].

In the following we consider a system made of homogeneous isotropic slender tubes i full of the same fluid of mass density ρ and velocity of sound v_0. Each tube is characterized by its impedance $Z_i = \rho v_0 / a_i$, where a_i is the cross section.

In the case of an infinite homogeneous one-dimensional slender tube along the x-axis, equation (5) becomes

$$\left(\frac{\rho}{a}\right)^{-1} \left[\frac{\partial^2 \Phi}{\partial x^2} - \alpha^2\right] \Phi(x) = 0, \quad (6)$$

where $\alpha^2 = -\frac{\omega^2}{v_0^2}$ and ω is the angular frequency of the wave. Then, the corresponding Green's function is defined by [29]

$$\left(\frac{\rho}{a}\right)^{-1} \left[\frac{\partial^2 \Phi}{\partial x^2} - \alpha^2\right] G(x, x') = \delta(x - x'), \quad (7)$$

whose solution is given by

$$G_i(x, x') = \frac{-\mathrm{j}}{2\omega} Z_i \mathrm{e}^{-\alpha |x - x'|} \quad (8)$$

where $\alpha = -\mathrm{j}k$, $k = \omega / v_0$ and $\mathrm{j} = \sqrt{-1}$.

Before addressing the problem of the simple structure presented in this work (see figure 1), it is helpful to know the surface elements of its elementary constituents, namely, the Green function of a finite slender tube of length d_i, $i = 1, 2, 3$, and of a semi-infinite tube. The finite slender tube of length d_2 is bounded by two free surfaces located at $x = 0$ and $x = d_2$. These surface elements can be written in the form of a (2×2) matrix $g_2(MM)$, within the interface space $M = \{0, +d_2\}$. The inverse of this matrix takes the following form [29]:

$$g_2^{-1}(MM) = \begin{pmatrix} -\frac{\omega C_2}{Z_2 S_2} & \frac{\omega}{Z_2 S_2} \\ \frac{\omega}{Z_2 S_2} & -\frac{\omega C_2}{Z_2 S_2} \end{pmatrix} \quad (9)$$

$C_2 = \cos(kd_2)$, $S_2 = \sin(kd_2)$. The inverse of the surface Green's functions of the dangling resonators grafted at the sites $\{0\}$ and $\{d_2\}$ with rigid boundary conditions at their ends is given by $g_1^{-1}(0, 0) = -N\omega C_1 / Z_1 S_1$ and $g_3^{-1}(d_2, d_2) = -N'\omega C_3 / Z_3 S_3$, where $C_i = \cos(kd_i)$, $S_i = \sin(kd_i)$, $i = 1, 3$. N and N' are the number of side branches on both sides of the finite slender tube of length d_2. The inverse of the surface Green's functions of the two semi-infinite tubes surrounding the whole structure is given by $g_s^{-1}(0, 0) = g_s^{-1}(d_2, d_2) = j\omega / Z_s$, where $Z_s = \rho v_0 / a_s$. In what follows, we suppose that all the tubes have the same cross section (i.e. $a_1 = a_2 = a_3 = a_s = a$), or equivalently the same impedances (i.e. $Z_1 = Z_2 = Z_3 = Z_s = Z = \rho v_0 / a$). We report on results of calculated transmission coefficients and phase or phase time as a function of frequency. Using the Green's function method [29], the expression giving the inverse of the Green's function of the whole system given in the inset of figure 2(c) can be obtained from a linear superposition of the above inverse Green's functions of the constituent, namely

$$g^{-1}(MM) = \frac{-\omega}{Z} \begin{pmatrix} \frac{C_2}{S_2} + \frac{NC_1}{S_1} - \mathrm{j} & -\frac{1}{S_2} \\ -\frac{1}{S_2} & \frac{C_2}{S_2} + \frac{N'C_3}{S_3} - \mathrm{j} \end{pmatrix}. \quad (10)$$

2.3. Transmission coefficient

Let us consider an incident wave $U(x) = \mathrm{e}^{-\mathrm{j}\alpha x}$ launched in the left semi-infinite waveguide (figure 1). With the help of equations (3), (8) and (10), one easily finds the transmission wave in the right semi-infinite waveguide (figure 1), namely [28, 29] $t = (2\mathrm{j}\omega / Z) g(0, d_2)$, or equivalently

$$t = \frac{2 S_1 S_3}{\chi_1 - \mathrm{j}\chi_2} \quad (11)$$

where

$$\chi_1 = S_2(N S_3 C_1 + N' S_1 C_3) + 2 C_2 S_1 S_3 \quad (12)$$

and

$$\chi_2 = -N C_1 (S_3 C_2 + N' S_2 C_3) - N' C_2 C_3 S_1 + 2 S_1 S_2 S_3. \quad (13)$$

From the expression of t (equation (4)), one can deduce the transmission coefficient

$$T = \frac{4 S_1^2 S_3^2}{\chi_1^2 + \chi_2^2} \quad (14)$$

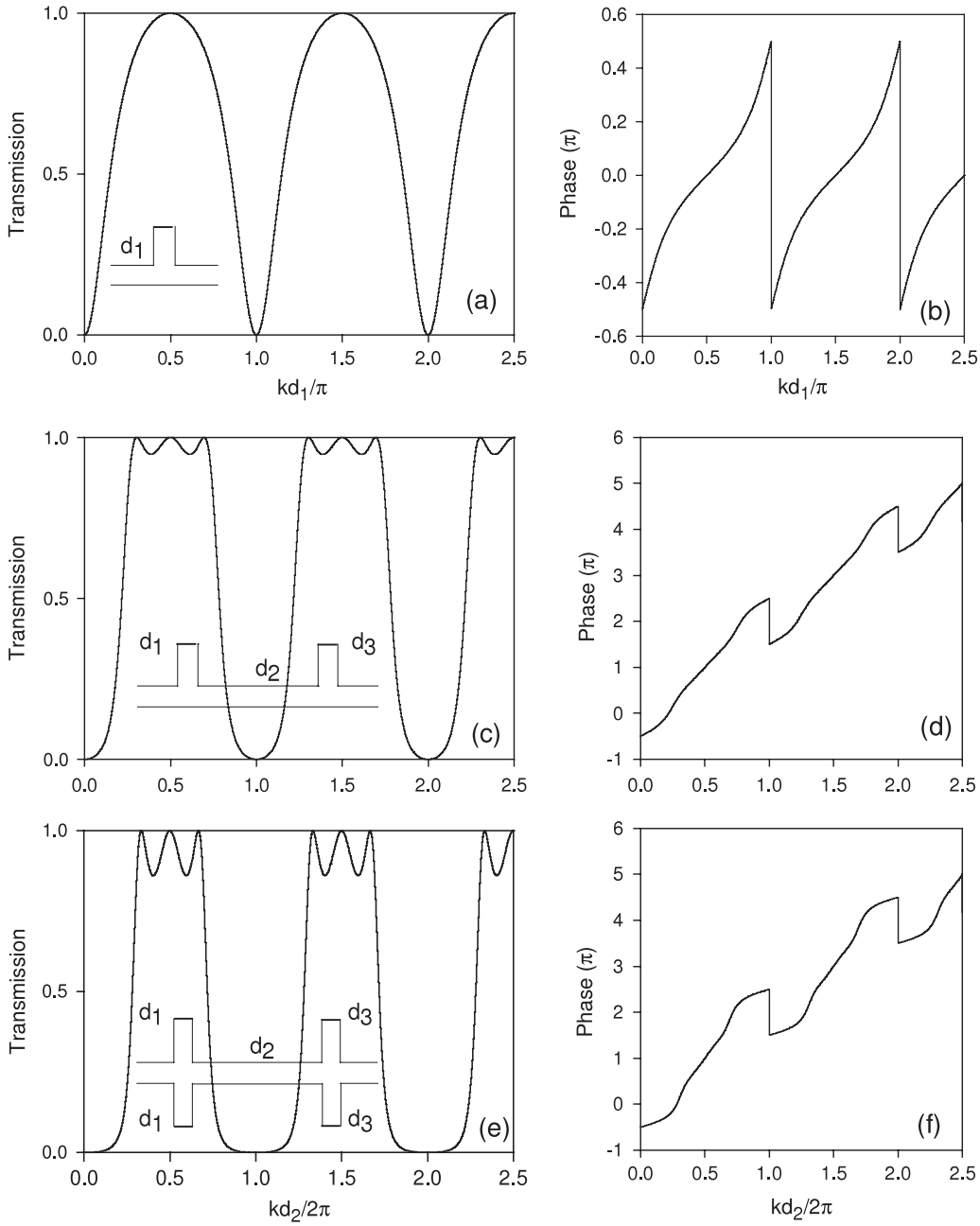

Figure 2. Left panel. (a) Transmission coefficient versus the reduced wavevector kd_1/π for a structure (depicted in the inset) consisting of an infinite line with one grafted segment of length d_1. The one-dimensional tubes constituting the infinite line and the finite segment are assumed to be of the same cross section. The boundary conditions at the end of the stubs are rigid. (c) Transmission coefficient versus the reduced wavevector $kd_2/2\pi$ for the structure depicted in the inset with $d_2 = 2d_3 = 2d_1$. (e) The same as in (c) but for the structure depicted in the inset. Right panel. (b), (d), (f) The same as in the left panel but for the variation of the phase.

as well as the phase

$$\varphi = \arctan(\chi_2/\chi_1) + \pi\ \Theta[S_1 S_3] \tag{15}$$

where Θ means the Heaviside function. From equations (4) and (7) one can notice that the transmission zeros are induced by the side branches (i.e. $S_1 = 0$ or $S_3 = 0$). When the expression $S_1 S_3$ changes sign at some frequencies denoted by ω_n, then the phase (equation (8)) exhibits a jump of π.

Another interesting quantity is the first derivative of φ with respect to the frequency, which is related to the delay time taken by the phonons to traverse the structure. This quantity, called phase time, is defined by [30, 31]

$$\tau_\varphi = \frac{\mathrm{d}\varphi}{\mathrm{d}\omega} \tag{16}$$

and can be written as

$$\tau_\varphi = \frac{\mathrm{d}}{\mathrm{d}\omega}\arctan(\chi_2/\chi_1) + \pi \sum_n \mathrm{sgn}\left[\frac{\mathrm{d}}{\mathrm{d}\omega}(S_1 S_3)_{\omega=\omega_n}\right]\delta(\omega - \omega_n) \tag{17}$$

where sgn means the sign function. Furthermore, the density of states (DOS) of the present composite system from which we have subtracted the DOS of the semi-infinite tubes is given by [31]

$$\Delta n(\omega) = \frac{1}{\pi}\frac{\mathrm{d}}{\mathrm{d}\omega}\arctan(\chi_2/\chi_1). \tag{18}$$

Because of the second term in the right-hand side of equation (10), one can deduce that $\tau_\varphi \neq \pi\Delta n(\omega)$, as τ_φ (equation (10)) may exhibit δ functions at the transmission zeros that do not exist in the variation of the DOS (equation (11)). However, if the system does not exhibit transmission zeros, then $\Theta[S_1 S_3] = 0$ and $\tau_\varphi = \pi\Delta n(\omega)$. Let us mention that phase time and negative δ functions associated with the transmission zeros in asymmetric loop structures made of two different coaxial cables and slender tubes have been observed, respectively, in recent experiments by El Boudouti *et al* [32] and Robertson *et al* [16].

It should be pointed out again that the validity of our results is subject to the requirement $\sqrt{a_i} \ll d_i, \lambda$, i.e. the cross section of the slender tubes being negligible compared to their length and to the propagation wavelength λ. The assumption of monomode propagation is then satisfied. It is worthwhile to notice that this model has been applied successfully to reproduce experimental measurements on band gaps, defect modes and group velocities in structured acoustic waveguides [7, 8, 12, 16].

3. Transmission gaps and Fano resonances

Before addressing the problem of the whole structure described above (figure 1), let us first recall briefly the results of a particular case necessary for the understanding of wave propagation in the structures shown in figure 1; namely, if $d_2 = 0, N = 1$ and $N' = 0$, we obtain the transmission function of a simple structure consisting of one resonator grafted on an infinite guide (see the inset of figure 2(a)): $t = S_1/(S_1 + \mathrm{j}C_1/2)$. This expression enables us to deduce the transmission coefficient $T = |t|^2 = 4S_1^2/(4S_1^2 + C_1^2)$ and the phase $\varphi = \pi\ \Theta(S_1) - \arctan(C_1/2S_1)$.

We can see that the transmission coefficient is equal to zero when $kd_1 = l'\pi$, where l' is a positive integer. The variation of T versus the reduced wavevector kd_1 (or equivalently the reduced frequency $\omega d_1/v_0$) is reported in figure 2(a). T is equal to zero for kd_1 a multiple of π and reaches its maximum value of unity for kd_1 an odd multiple of $\pi/2$. The frequencies of the transmission zeros given by f_g such as $f_\mathrm{g} = l'v_0/2d_1$ correspond to the eigenmodes of the grafted finite branch. This grafted branch behaves as a resonator and this simple composite system filters out the modes f_g. This phenomenon is related to the resonances associated with the finite additional path offered to the acoustic wave propagation. The variation of the phase versus kd_1 (figure 2(b)) shows an abrupt change of π at the transmission zeros and therefore the corresponding phase time is different from the DOS as mentioned above.

For the structure shown in the inset of figure 2(c), equation (4) clearly shows that the transmission zeros are due only to the dangling resonators (i.e. when $S_1 = 0$ or $S_3 = 0$). Figure 2(c) gives the transmission coefficient in the presence of two identical dangling resonators (i.e. $N = N' = 1$ and $d_1 = d_3 = 0.5d_2$). One can notice that the transmission coefficient presents well defined dips induced by the grafted branches. This dip transforms into a large transmission gap when the number of branches increases, as illustrated in figure 2(e) for $N = N' = 2$. It is worthwhile to mention that, because of the existence of two resonators in figure 2(c), one can expect two phase drops of π (i.e. 2π) at the transmission zeros given by $S_1 = S_3 = 0$ (i.e. $kd_2/2\pi = 1, 2, \ldots$). However, one can see in figure 2(d) that the phase presents only a phase drop of π. This is due to the existence of a resonant state with zero width at these values of $kd_2/2\pi$, which induces a phase jump of $+\pi$; these resonances collapse when $d_1 = d_3$ is taken exactly equal to $0.5d_2$. These resonances are called ghost Fano resonances in [33, 34]. To enlarge these resonances, we have to take d_1 and d_3 slightly different from $0.5d_2$. Indeed, at $kd_2/2\pi = 1, 2, \ldots$ the expression of the transmission function (equation (4)) becomes

$$t = \frac{2S_1 S_3}{2S_1 S_3 + \mathrm{j}\sin[k(d_1 + d_3)]}. \tag{19}$$

So, if $k(d_1 + d_3) = m\pi$ but $kd_1 \neq m_1\pi$ and $kd_2 \neq m_2\pi$ (m, m_1 and m_2 are integers), one obtains a resonance that reaches unity (i.e. $t = 1$). An example corresponding to this situation is given in figure 3(a), where $d_1 = 0.46d_2$ and $d_3 = 0.54d_2$ (with $d_1 + d_3 = d_2$). One can notice that the resonance at $kd_2/2\pi = 1$ is squeezed between two zeros (indicated by solid circles on the abscissa of figure 3(a)) induced by the dangling resonators, as also illustrated in the curve of the variation of the phase (figure 3(b)). The width of this resonance increases as d_1 and d_3 deviate from $0.5d_2$ (see below). In the particular case where $kd_1 = m_1\pi$ and $kd_2 = m_2\pi$, the numerator and denominator of t (equation (12)) vanishes altogether. In this case, the resonance as well as the two zeros induced by the resonators fall at the same position, so the resonance collapses, the transmission coefficient vanishes and the phase drops by π as shown in figure 2(d).

The resonance in figure 3(a) shows the same characteristics as a Fano resonance but with two zeros of transmission around the resonance instead of one as is usually the case [21, 22]. Indeed, one can obtain an approximate analytical expression for the transmission function (equation (4)) in the vicinity of the resonance. A Taylor expansion around $kd_2 = 2\pi$ (i.e. $kd_2 = 2\pi + \varepsilon$ with $\varepsilon/2\pi \ll 1$) enables us to obtain

$$t = \frac{\zeta\ \zeta'}{2\varepsilon^2 + \zeta\zeta' + \mathrm{j}\,\varepsilon(4 - \zeta\zeta'/2)} \tag{20}$$

where $\zeta = 2\Delta + \varepsilon(1 + \Delta/\pi)$, $\zeta' = -2\Delta + \varepsilon(1 - \Delta/\pi)$ and Δ is the detuning of d_1 and d_3 from $0.5d_2$ (i.e. $\Delta = 2\pi(0.5 - d_1/d_2) = 2\pi(-0.5 + d_3/d_2)$).

From equation (12), one can show that the transmission coefficient T can be written (following the Fano line shape [21]) in the form

$$T = A\,\frac{(\varepsilon + q_1\Gamma)^2(\varepsilon - q_2\Gamma)^2}{\varepsilon^2 + \Gamma^2}, \tag{21}$$

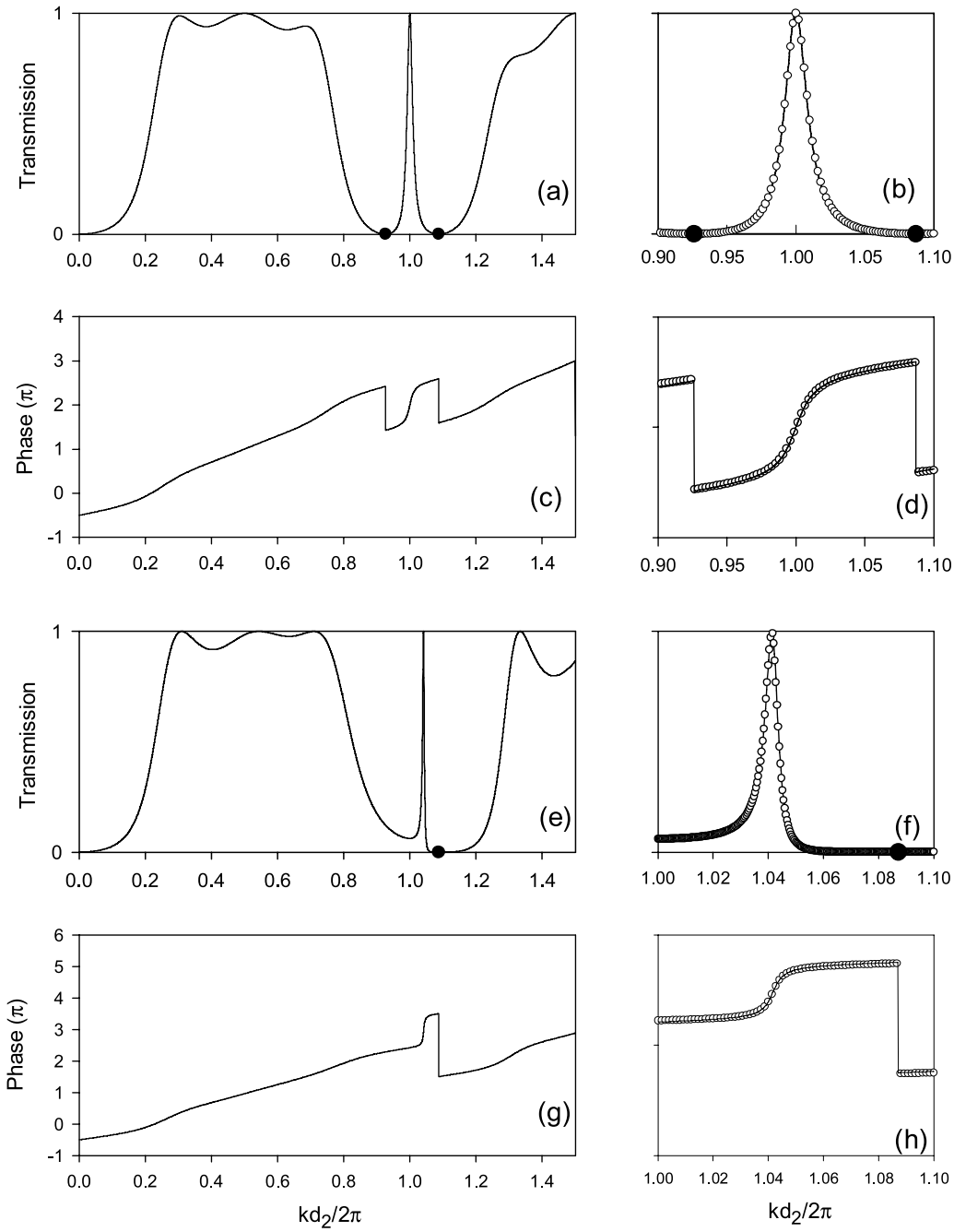

Figure 3. (a) The same as in figure 2(a), but the lengths of the resonators are taken such that $d_1 = 0.46d_2$, $d_3 = 0.54d_2$ and $N = N' = 1$. Solid circles on the abscissa indicate the positions of the transmission zeros induced by the dangling resonators on both sides of the resonance. (c) The same as in (a) but for the variation of the phase. (b), (d) The approximate results (open circles) around the resonance. (e) The same as in (a) but the resonators are taken to be of identical lengths $d_1 = d_3 = 0.46d_2$. (g) The same as in (c) but for the variation of the phase. (f), (h) The approximate results (open circles) around the resonance.

where $A = (1 - \Delta^2/\pi^2)^2/(4 + 2\Delta^2)^2$. $\Gamma = 2\Delta^2/(2 + \Delta^2)$ characterizes the width of the resonance falling at $\varepsilon = 0$. $q_1 = (2 + \Delta^2)/\Delta(1 + \Delta/\pi)$ and $q_2 = (2 + \Delta^2)/\Delta(1 - \Delta/\pi)$ are the coupling parameters; they give qualitatively the interference between the bound states and the propagating continuum states [21–23]. One can notice that when increasing Δ, Γ increases and q decreases. The results of the approximate expression (equation (14)) are shown in figure 3(b) by open circles. These results are in accordance with the exact ones (solid lines) and clearly show that the resonance is of Fano type with $q_1 \simeq 7.6$, $q_2 \simeq 8.9$ and width $2\Gamma \simeq 0.12$. The commonly studied Fano resonances are asymmetric because of the presence of only one transmission zero near the resonance (see below). In addition, in the electronic counterpart studies, a perturbation is often introduced to the system in order to create the resonance state [21–24]. However, the above calculation shows that, without introducing any perturbation in the structure, one can find a well defined symmetric

Fano resonance with a width 2Γ and coupling parameters q_1 and q_2 that can be adjusted by tailoring the lengths of the resonators (i.e. Δ). Equation (13) enables us also to deduce an approximate expression for the phase as

$$\varphi = \pi\Theta(\zeta) + \pi\Theta(\zeta') - \arctan[\varepsilon(4 - \zeta\zeta'/2)/(2\varepsilon^2 + \zeta\zeta')]. \tag{22}$$

This function is plotted by open circles in figure 3(d) and clearly shows two abrupt phase changes of π at $\zeta = 0$ and $\zeta' = 0$ (i.e. $\varepsilon_1 = -q_1\Gamma$ and $\varepsilon_2 = q_2\Gamma$) in accordance with the exact results (solid line).

One can also create an asymmetric Fano resonance by adjusting the transmission zeros on only one side of the resonance; this can be obtained by considering a structure where the resonators are supposed to be identical with lengths slightly different from $0.5d_2$. This is shown in figure 3(e) for $d_1 = d_3 = 0.46d_2$ and $N = N' = 1$. Indeed, an analytical Taylor expansion around $kd_2 = 2\pi$ enables us to write the transmission function (equation (14)) as

$$t = \frac{2\xi^2}{(\xi + \mathrm{j})(\varepsilon + 2\xi - \mathrm{j}\varepsilon\xi)} \tag{23}$$

where $\xi = \Delta + \varepsilon(1 + \Delta/\pi)/2$ and Δ is the detuning of the lengths of the two resonators from $0.5d_2$ (i.e., $\Delta = 2\pi(d_1/d_2 - 0.5)$).

From the expression of t (equation (16)), one can deduce the following Fano line shape transmission coefficient:

$$\begin{aligned} T &= B\frac{1}{1+\xi^2}\frac{(\varepsilon - \varepsilon_\mathrm{R} + q\Gamma)^4}{(\varepsilon - \varepsilon_\mathrm{R})^2 + \Gamma^2} \\ &\simeq B\frac{(\varepsilon - \varepsilon_\mathrm{R} + q\Gamma)^4}{(\varepsilon - \varepsilon_\mathrm{R})^2 + \Gamma^2} \end{aligned} \tag{24}$$

where $B = (1 + \Delta/\pi)^4/16(1 + \Delta/2\pi)^2$.

$\Gamma = \Delta^2(1 - \Delta/2\pi)/4(1 + \Delta/2\pi)^2$ and $\varepsilon_\mathrm{R} = -\Delta/(1 + \Delta/2\pi)$ characterize the width and the shift of the resonance respectively, whereas $q = 4(1 + \Delta/2\pi)/\Delta(1 + \Delta/\pi)(1 - \Delta/2\pi)$ is the Fano parameter. One can notice that the resonance shifts slightly from $kd_2 = 2\pi$ and its width is small as compared to the preceding case; this is in accordance with the numerical results of figures 3(a) and (e). Also q increases when Δ decreases and tends to infinity when Δ vanishes. In this case the resonance falls at $\varepsilon_\mathrm{R} = 0$ and its width 2Γ reduces to zero (figure 2(c)), as expected. The results of the approximate expression (equation (17)) are sketched (open circles) in figure 3(f) for $\Delta = 2\pi(d_1/d_2 - 0.5) = -0.08\pi$ (i.e. $d_1/d_2 = 0.46$). These results are in accordance with the exact ones (solid lines) and clearly show that the resonance is of Fano type with $q \simeq 16$ and width $2\Gamma \simeq 0.035$.

Concerning the evolution of the phase of the phonons in this structure, one can notice from equation (4) that the numerator of the transmission function t vanishes when $S_1 = S_3 = 0$ (or equivalently $\xi = 0$ in the approximate result (equation (16)) at $kd_2/2\pi = d_2/2d_1 = 1.086$ indicated by a filled circle on the abscissa axis of figure 3(f). The transmission zeros induced by the two identical resonators fall at the same frequency; therefore, the phase (figures 3(g) and (h)) shows a phase drop of 2π at these frequencies. Indeed, as the phase is defined modulo 2π, the 2π phase change cannot be observed if the absorption is neglected in the system. The absorption can be considered by adding for example a small imaginary part ε to the frequency ω with $\varepsilon \approx 0$, then the abrupt phase drop can be observed. The phase drops give rise to negative delta peaks in the transmission phase time as illustrated in figures 4(c) and (g) as well as in the corresponding approximative results shown by open circles in figures 4(d) and (h). A recent similar work has been performed by Rotter and Sadreev [35] on two single quantum dots coupled by a wire of finite length and neglecting the absorption. They found that at the transmission zeros the phases jump by mostly (but not always) π. However, our results show that the phase may jump by even 2π if the absorption is taken into account.

The small absorption in the tubes can also considerably affect the amplitude of the Fano resonances, which becomes less than unity as shown in figures 4(a), (b), (e) and (f). It is worthwhile to notice that recent experiments by Robertson *et al* [16] on an asymmetric loop made of slender tubes have shown the existence of negative phase times and therefore negative group velocities around the transmission zeros in analogy with our experiments on photonic crystals made of coaxial cables [32]. These results show, in accordance with section 2, that the phase time is different from the density of states by the occurrence of additional delta peaks [14].

The Green's function approach enables one also to deduce the local density of states (LDOS). The details of these calculations are given in [28]. The LDOS reflects the behavior of the square modulus of the pressure field inside the structure. An analysis of the LDOS as a function of the space position (figure 5) clearly shows that the Fano resonances in figures 3(a) and (e) are confined inside the slender tube and the resonators (see figures 5(a) and (b) respectively). In particular, the pressure is maximum in the middle of the finite tubes and vanishes at their extremities. Therefore, these resonances could be classified as local resonances.

As mentioned above, through our analysis we use the one-dimensional model for the acoustic waveguide network where the cross section of the slender tubes is negligible as compared to their length and to the propagation wavelength λ. However, if the cross section of the slender tubes is of the same order of magnitude as compared to their length and to the propagation wavelength λ, then a two-dimensional model is necessary [10, 36, 37]. Indeed, it was shown recently [36, 37] that the transmission zeros induced by the stubs in two-dimensional phononic crystals strongly depend on the number of stubs grafted at each site as well as on the distance between the stubs. Similar results to those presented in this work have been observed in quantum waveguide nanostructures [38]. In particular, when the cross sections of the wires are of nonzero width, the multimode effect and the matching of transverse modes have been considered [39]. In addition to the quantization of the conductance which is due to the lateral confinement, It has been shown that the existence of crosses and bends in the structure may also result in bound states associated with the cross sections of these contact points [40–43]. One can expect the same phenomena in acoustic waveguides due to the analogy between electronic and acoustic excitations in waveguides. Therefore, in such

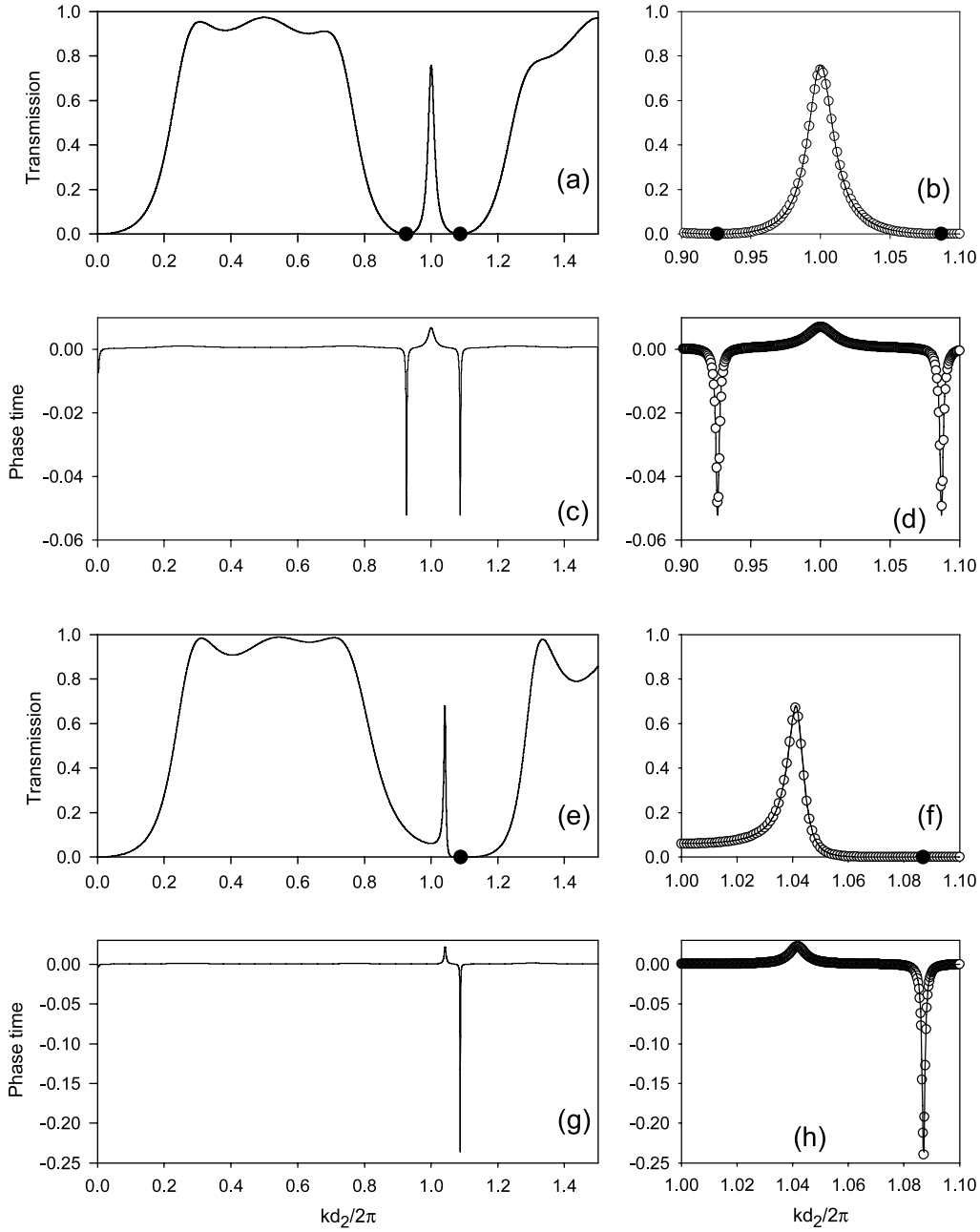

Figure 4. (a), (b), (e), (f) The same as in figures 3(a), (b), (e), (f) but here the absorption is taken into account by adding a small imaginary part to the frequency. The solid circles indicate the positions of the transmission zeros. (c), (d), (g), (h) The same as in figures 3(c), (d), (g), (h) but for the phase time.

geometries, the position and the width of the transmission gaps as well as the position and intensity of the Fano resonances in the acoustic counterpart systems could be affected.

4. Summary and conclusion

In summary, we have clearly demonstrated that a simple geometry of a slender tube with dangling side resonators on both sides can pave the way to the obtention of gaps in the sound propagation. The existence of the stop bands in the spectrum is attributed to the zero of transmission associated with the dangling resonators. The width of the transmission gaps depends on the number of the side resonators grafted on both sides of the slender tube. Besides the transmission gaps, we have shown the existence of asymmetric and symmetric Fano resonances that may lie near the vicinity of a transmission zero or be squeezed between two transmission zeros. These resonances are obtained by tailoring the lengths of the different branches constituting the structure. A study of the phase of the transmission function enables us to deduce several properties of the wave propagation through such structures as the phase times and therefore the density of states. For the system studied here, the phase time calculation is different from the density of states. The resonant modes give rise to well defined peaks in the phase time.

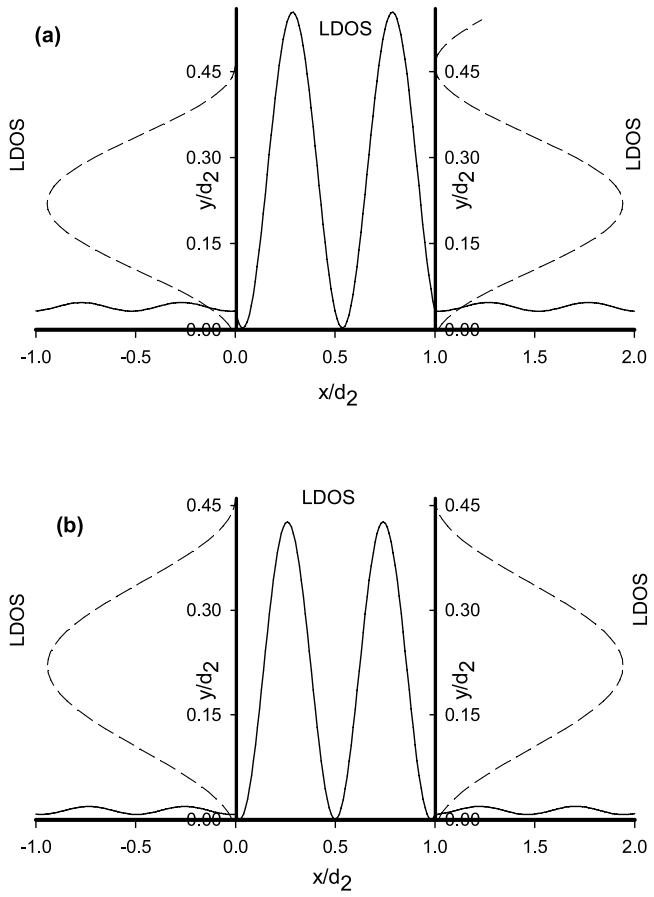

Figure 5. The local density of states (LDOS) (in arbitrary units) as a function of the space positions x/d_2 and y/d_2 along the horizontal waveguide (full curve) and vertical waveguide (dashed curves) respectively. (a) The LDOS corresponds to the Fano resonance in figure 3(a); (b) the same as in (a) but for the resonance in figure 3(e).

The advantage of the simple acoustic waveguide model presented in this work consists in finding simple analytical expressions that enable us to discuss the existence of Fano resonances as well as the effect of the different tube lengths in tailoring these resonances without incorporating a defect in the structure as is usually the case in the electronic counterpart studies [21–24].

We believe that this paper brings a new piece of work in the field of acoustic wave transport in 1D waveguide structures and we hope that it will stimulate an experimental observation [12, 16] of the transmission gaps and Fano resonances exhibited by the simple acoustic waveguide described in this work.

Acknowledgments

E H El Boudouti and H Al-Wahsh gratefully acknowledge the hospitality of the Institut d'Electronique, de Microélectronique et de Nanotechnologie (IEMN), UMR CNRS 8520 and UFR de Physique, Université de Lille 1. This work was supported by 'le Fond Européen de Développement Régional' (FEDER), INTERREG III France–Wallonie–Flandre (PREMIO) and 'le Conseil Régional Nord-Pas de Calais'.

References

[1] Kushwaha M S, Halevi P, Dobrzynski L and Djafari-Rouhani B 1993 *Phys. Rev. Lett.* **71** 2022
Sigalas M and Economou E N 1993 *Solid State Commun.* **86** 141
Sainidou R, Stefanou N, Psarobas I E and Modinos A 2002 *Phys. Rev.* B. **66** 024303
[2] Montero de Espinosa F R, Jimenez E and Torres M 1998 *Phys. Rev. Lett.* **80** 1208
Vasseur J O, Deymier P A, Frantziskonis G, Hong G and Djafari-Rouhani B 1998 *J. Phys.: Condens. Matter* **10** 6051
[3] Sigalas M, Kushwaha M S, Economou E N, Kafesaki M, Psarobas I E and Steurer W 2005 *Z. Kristallogr.* **220** 765
[4] Narayanamurti V 1981 *Science* **213** 717
[5] Bria D and Djafari-Rouhani B 2002 *Phys. Rev.* E **66** 056609 and references therein
[6] Manzanarez-Martinez B, Sanchez-Dehesa J, Hakansson A, Cervera F and Ramos-Mendieta F 2004 *Appl. Phys. Lett.* **85** 154
[7] Bradley C E 1994 *J. Acoust. Soc. Am.* **96** 1844
Bradley C E 1994 *J. Acoust. Soc. Am.* **96** 1854
Bradley C E 1995 *J. Acoust. Soc. Am.* **188** 717
Bradley C E 1996 *J. Acoust. Soc. Am.* **100** 304
[8] Munday J N, Bennett C B and Robertson W M 2002 *J. Acoust. Soc. Am.* **112** 1353
Robertson W M, Baker C and Bennett C B 2004 *Am. J. Phys.* **72** 255
[9] Kushwaha M S, Akjouj A, Djafari-Rouhani B, Dobrzynski L and Vasseur J O 1998 *Solid State Commun.* **106** 659
[10] Wang X F, Kushwaha M S and Vasilopoulos P 2001 *Phys. Rev.* B **65** 035107
[11] Carr C and Yu R 2002 *Am. J. Phys.* **70** 1154
[12] Robertson W M, Ash J and McGaugh J M 2002 *Am. J. Phys.* **70** 689
[13] Akjouj A, Al-Wahsh H, Sylla B, Djafari-Rouhani B and Dobrzynski L 2004 *J. Phys.: Condens. Matter* **16** 37
[14] Aynaou H, El Boudouti E H, Djafari-Rouhani B, Akjouj A and Velasco V R 2005 *J. Phys.: Condens. Matter* **17** 4245
[15] Kinga P D C and Cox T J 2007 *J. Appl. Phys.* **102** 014902
[16] Robertson W M, Pappafotis J and Flannigan P 2007 *Appl. Phys. Lett.* **90** 014102
[17] Liu Z, Zhang X, Mao Y, Zhu Y Y, Yang Z, Chan C T and Sheng P 2000 *Science* **289** 1734
Yang S, Page J H, Liu Z, Cowan M L, Chan C T and Sheng P 2002 *Phys. Rev. Lett.* **88** 104301
[18] Goffaux C, Sánchez-Dehesa J, Yeyati A L, Lambin P, Khelif A, Vasseur J O and Djafari-Rouhani B 2002 *Phys. Rev. Lett.* **88** 225502
Goffaux C and Sánchez-Dehesa J 2003 *Phys. Rev.* B **67** 144301
[19] Liu Z, Chan C T and Sheng P 2005 *Phys. Rev.* B **71** 014103
[20] Kosevich Y A, Goffaux C and Sánchez-Dehesa J 2006 *Phys. Rev.* B **74** 012301
[21] Fano U 1961 *Phys. Rev.* **124** 1866
[22] Goldhaber-Gordon D, Shtrikman H, Mahalu D, Abush-Magder D, Meirav U and Kastner M A 1988 *Nature* **391** 156
Gores J, Goldhaber-Gordon D, Heemeyer S, Kastner M A, Shtrikman H, Mahalu D and Meirav U 2000 *Phys. Rev.* B **62** 2188
Ji Y, Heiblum M and Shtrikman H 2002 *Phys. Rev. Lett.* **88** 076601
[23] Kobayashi K, Aikawa H, Katsumoto S and Iye Y 2002 *Phys. Rev. Lett.* **88** 256806
Kim J, Kim J R, Lee J-O, Park J W, So H M, Kim N, Kang K, Yoo K-H and Kim J-J 2003 *Phys. Rev. Lett.* **90** 166403
Babic B and Schönenberger C 2004 *Phys. Rev.* B **70** 195408

[24] Ladron de Guevara M L, Claro F and Orellana P A 2003 *Phys. Rev.* B **67** 195335
Voo K K and Chu C S 2005 *Phys. Rev.* B **72** 165307
Hackenbroich G and Weidenmuller H A 1996 *Phys. Rev. Lett.* **76** 110
Joe Y S, Satanin A M and Klimeck G 2005 *Phys. Rev.* B **72** 115310
[25] Wu J, Gu B L, Chen H, Duan W and Kawazoe Y 1998 *Phys. Rev. Lett.* **80** 1952
Taniguchi T and Buttiker M 1999 *Phys. Rev.* B **60** 13814
Lee H W 1999 *Phys. Rev. Lett.* **82** 2358
[26] Al-Wahsh H, El Boudouti E H, Djafari-Rouhani B, Akjouj A and Dobrzynski L 2007 *Phys. Rev.* B **75** 125313
[27] Yacoby A, Heiblum M, Mahalu D and Shtrikman H 1995 *Phys. Rev. Lett.* **74** 4047
Schuster R, Buks E, Heiblum M, Mahalu D, Umansky V and Shtrikman H 1997 *Nature* **385** 417
[28] Vasseur J O, Akjouj A, Dobrzynski L, Djafari-Rouhani B and El Boudouti E H 2004 *Surf. Sci. Rep.* **54** 1
[29] Dobrzynski L 1990 *Surf. Sci. Rep.* **11** 139
[30] Büttiker M and Landauer R 1982 *Phys. Rev. Lett.* **49** 1739
Büttiker M 1983 *Phys. Rev.* B **27** 6178
Hauge E H and Stovneng J A 1989 *Rev. Mod. Phys.* **61** 917
[31] Lahlaouti M L H, Akjouj A, Djafari-Rouhani B, Dobrzynski L, Hammouchi M, El Boudouti E H, Nougaoui A and Kharbouch B 2001 *Phys. Rev.* B **63** 35312
[32] El Boudouti E H, Fettouhi N, Akjouj A, Djafari-Rouhani B, Mir A, Vasseur J, Dobrzynski L and Zammouri J 2004 *J. Appl. Phys.* **95** 1102
[33] Ladrón de Guevara M L, Claro F and Orellana Pedro A 2003 *Phys. Rev.* B **67** 195335
[34] Barkay H, Narevicius E and Moiseyev N 2003 *Phys. Rev.* B **67** 045322
[35] Rotter I and Sadreev A F 2005 *Phys. Rev.* E **71** 046204
[36] Benchabane S, Khelik A, Choujaa A, Djafari-Rouhani B and Laude V 2005 *Europhys. Lett.* **71** 570
[37] Noual A 2006 *MS Thesis* University of Lille, p 1
[38] Levy L P, Dolan G, Dunsmuir J and Bouchiat H 1990 *Phys. Rev. Lett.* **64** 2074
Deo P S and Jayannavar A M 1990 *Mod. Phys. Lett.* B **7** 2074
Deo P S and Jayannavar A M 1994 *Phys. Rev.* B **50** 11629
[39] Van Wees B J, van Houten H, Beenakker C W J, Williamson J G, Kouwenhoven L P, Van der Marel D and Foxon C T 1988 *Phys. Rev. Lett.* **60** 848
Debray P, Raichev O E, Vasilopoulos P, Rahman M, Perrin R and Mitchell W C 2000 *Phys. Rev.* B **61** 10950
[40] Berggren K F and liJi Z- 1991 *Phys. Rev.* B **43** 4760
[41] Carini J P, Londergan J T and Murdock D P 1997 *Phys. Rev.* B **55** 9852
Carini J P, Londergan J T, Murdock D P, Trinkle D and Yung C S 1997 *Phys. Rev.* B **55** 9842
[42] Avishai Y, Bessis D, Giraud B G and Mantica G 1991 *Phys. Rev.* B **44** 8028
[43] For a review, see Londergan J T, Carini J P and Murdock D P 1999 *Binding and Scattering in Two-Dimensional Systems: Application to Quantum Wires, Waveguides and Photonic Crystals* (*LNP* vol m60) (Berlin: Springer)