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Optical precursors in transparent media

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We theoretically study the linear propagation of a stepwise pulse through a dilute dispersive medium when the frequency of the optical carrier coincides with the center of a natural or electromagnetically induced transparency window of the medium (slow-light systems). We obtain *explicit analytical expressions* of the entirety of the step response and show that, for parameters representative of real experiments, Sommerfeld-Brillouin precursors, main field and second precursors (“postcursors”) can be distinctly observed, all with amplitudes comparable to that of the incident step. This behavior strongly contrasts with that of the systems generally considered up to now.

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As far back as 1914, Sommerfeld and Brillouin theoretically studied the propagation of a stepwise pulse through a linear dispersive medium [1, 2, 3]. They showed in particular [2] that the arrival of the main signal is preceded by that of two successive transients they named forerunners. The first one (now called the Sommerfeld precursor) arrives with the velocity c of light in vacuum. Its instantaneous frequency, initially higher than the frequency ω_C of the optical carrier, decreases as a function of time whereas that of the second one (the Brillouin precursor), initially lower than ω_C , evolves in the opposite direction. Sommerfeld and Brillouin considered a single-resonance Lorentz medium and made their calculation by using the newly developed saddle-point method of integration. Since their pioneering work, optical precursors have become a canonical problem in electromagnetics and optics [4, 5, 6]. The problem examined by Sommerfeld and Brillouin have been revisited numerous times by using more and more sophisticated asymptotic methods. It appears that the original results are “qualitatively accurate but quantitatively inaccurate” [7], in particular concerning the amplitude of the precursors. Other models of medium have obviously been considered and the theoretical literature on precursors is very abundant. See [8] for a recent review.

As intuitively expected, the precursors will be observed only if the rise-time of the incident step is short compared to the response time of the medium [9]. Most of the theoretical papers consider dense media with very short response time (< 1 fs) and the fulfillment of the previous condition raises serious experimental difficulties. This explains the dramatic dearth of papers reporting direct demonstrations of precursors. A first experiment was achieved in the microwave region with waveguides whose dispersion mimics that of the Lorentz medium [10]. In the optical domain, Aaviksoo *et al.* studied the propagation of single-ended exponential pulses with 0.4 ps rise-time through a GaAs crystal [11]. The optical carrier frequency ω_C was tuned close to the exciton resonance

and the precursors appeared as a spike superimposed on the main pulse (see also [12]). A discussion on the observability of optical precursors in dense media (including liquids) can be found in [13].

Much more favorable time scales are obtained by exploiting the narrowness of atomic or molecular lines in vapors or gases. The switching times of the incident field may then be very long compared to the optical period without washing out the transients. In such conditions, the slowly varying envelope approximation (SVEA) is absolutely justified [14]. The medium is fully characterized by its system function $H(\Omega)$ connecting the Fourier transforms of the envelopes of the transmitted and incident fields [15]. Ω designates the deviation of the current optical frequency ω from ω_C and the envelope of the optical step response reads

$$a(t) = \int_{\Gamma} H(\Omega) \exp(i\Omega t) d\Omega / 2i\pi\Omega \quad (1)$$

where the contour Γ is a straight line parallel to the real axis passing under the pole at $\Omega = 0$. We consider in the following systems such that $H(-\Omega) = H^*(\Omega)$. $a(t)$ is then real. In the basic case of a medium with a single Lorentzian absorption-line, Varoquaux *et al.* [16] established an exact expression of $a(t)$, which, on resonance and for large optical thickness, is reduced to

$$a(t \geq 0) = e^{-\gamma t} J_0 \left(\sqrt{2\alpha L \gamma t} \right) \quad (2)$$

where L is the medium thickness, t (as in all the following) is a *local time* (real time minus L/c), α is the resonant absorption-coefficient for the intensity ($\alpha/2$ for the amplitude) and γ is the half width at half maximum of the line. This signal, experimentally evidenced in [17], may be formally analyzed in terms of Sommerfeld and Brillouin precursors, which are temporally superimposed in dilute media [18, 19]. However we remark that these “precursors” precede nothing since the medium is then opaque for the “main field”. In order to obtain true pre-

cursors we examine in this letter the much richer case where the medium is (nearly) transparent at ω_C .

We consider first a medium with a natural transparency window between two identical absorption-lines of intensity optical thickness $\alpha L/2 \gg 1$ located at $\omega_C \pm \Delta$. Such a medium has proved to be a very efficient slow-light system [20, 21, 22, 23]. Its system function reads [24, 25]

$$H(\Omega) = \exp \left\{ -\frac{\alpha L \gamma}{4} \left[\frac{1}{\gamma + i(\Omega + \Delta)} + \frac{1}{\gamma + i(\Omega - \Delta)} \right] \right\} \quad (3)$$

A good transparency at $\Omega = 0$ is achieved if $\gamma \ll \Delta$ and $\alpha L \gamma^2 / \Delta^2 < 1$. The group delay then reads $\tau_g = \alpha L \gamma / 2 \Delta^2$ [24] and $H(0) = \exp(-\alpha L \gamma^2 / 2 \Delta^2) = \exp(-\gamma \tau_g)$. Fig.1 shows the step response obtained for parameters representative of the slow-light experiments achieved on a cesium vapor [22]. In its very far wings,

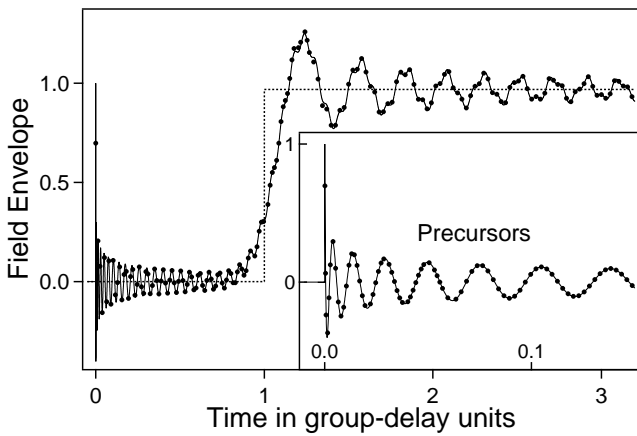


Figure 1: Comparison of the exact step-response $a(t)$ of a medium with a natural transparency window obtained by fast Fourier transform (full line) to its explicit analytic approximations (\bullet). The step of amplitude $H(0)$ retarded by τ_g is given for reference (dotted line). Inset : enlargement of the precursors. The parameters are $\Delta = 28.9 \text{ ns}^{-1}$, $\gamma = 0.0164 \text{ ns}^{-1}$ and $\alpha L = 2 \cdot 10^3$, leading to $\tau_g = 1.96 \text{ ns}$, $H(0) = \exp(-\tau_g) = 0.968$ and $b = 5.22 \text{ ns}^{-1}$.

$H(\Omega)$ equals the system function of a medium with a single line of intensity optical thickness αL and, as expected, the short time behavior of $a(t)$ is well described by Eq.(2). For $t \geq t_1 = \frac{1}{2\alpha L \gamma} = \frac{1}{4\Delta^2 \tau_g}$, $a(t)$ can be entirely calculated by the saddle point method [26, 27]. Introducing the phase function $\Psi(\Omega) = i\Omega t + \ln[H(\Omega)]$, Eq.(1) reads

$$a(t) = \int_{\Gamma} \exp[\Psi(\Omega)] d\Omega / 2i\pi\Omega \quad (4)$$

The integral is calculated by deforming Γ in a contour Γ' traveling along lines of steepest descent of the function $\Psi(\Omega)$ from the saddle points where $\Psi'(\Omega) = 0$. The contribution of a non degenerate saddle point at Ω_S to

the integral reads

$$a_S(t) = \left(i\Omega_S \sqrt{2\pi |\Psi''(\Omega_S)|} \right)^{-1} \exp[\Psi(\Omega_S) + i\theta_S] \quad (5)$$

where θ_S is the angle of the direction of steepest descent with the real axis. Note that the instantaneous frequency of $a_S(t)$, defined as $d(\text{Im}\Psi)/dt$, equals $\text{Re}(\Omega_S)$.

In the present problem, the equation $\Psi'(\Omega) = 0$ giving the saddle points can be reduced to a biquadratic equation with exact analytic solutions. The latter can be regrouped in two pairs $\Omega_n^{\pm}(t) = i\gamma \pm \Omega_n(t)$ with $n = 1, 2$ and

$$\Omega_n(t) = \Delta \sqrt{1 + \left[1 - (-1)^n \sqrt{1 + 8t/\tau_g} \right] \frac{\tau_g}{2t}} \quad (6)$$

$\Omega_1(t)$ is everywhere real and very large compared to γ , decreasing from $\Delta \sqrt{\tau_g/t}$ for $t \ll \tau_g$ to Δ for $t \rightarrow \infty$. The contribution $a_1(t) = a_1^+(t) + a_1^-(t)$ to $a(t)$ is easily derived from Eq.(5) with $\theta_{\pm}^{\pm} = \pm\pi/4$. It reads

$$a_1(t) \approx \sqrt{\frac{2}{\pi}} e^{-\gamma t} \frac{\cos \left\{ \Omega_1 t + \Delta^2 \tau_g \Omega_1 / (\Omega_1^2 - \Delta^2) - \pi/4 \right\}}{\Omega_1 \Delta \sqrt{\tau_g} \left[(\Omega_1 + \Delta)^{-3} + (\Omega_1 - \Delta)^{-3} \right]} \quad (7)$$

For $t \ll \tau_g$, $a_1(t) \approx \sqrt{\frac{2}{\pi}} e^{-\gamma t} \frac{\cos(\sqrt{2\alpha L \gamma t} - \pi/4)}{(2\alpha L \gamma t)^{1/4}}$ which is a good approximation of $e^{-\gamma t} J_0(\sqrt{2\alpha L \gamma t})$ for $t \geq t_1$. For $t < \tau_g$, Ω_2 is purely imaginary and, in a wide time-domain, $a_1(t)$ is the dominant contribution to $a(t)$. The corresponding optical field reads $E_1(t) = E_1^+(t) + E_1^-(t)$ where $E_1^{\pm}(t) = \text{Re}[a_1^{\pm}(t) \exp(i\omega t)]$ have instantaneous frequencies $\omega_1^{\pm}(t) = \omega_C \pm \Omega_1(t)$. Due to the time dependence of these frequencies, $E_1^+(t)$ and $E_1^-(t)$ may be identified respectively to the Sommerfeld precursor and to the Brillouin precursor [19]. Note that the precursors have been overlooked in [28] due to an insufficient time-resolution of the calculations. The rise of $a(t)$ around $t = \tau_g$ originates from the saddle points at Ω_2^{\pm} , which are degenerate at $t = \tau_g$. The calculation of their contribution $a_d(t)$ to $a(t)$ then requires to use an uniform asymptotic method [26]. It is convenient to determine $a_d(t)$ through the corresponding contribution $h_d(t)$ to the impulse response $h(t) = \int_{-\infty}^{+\infty} \exp[\Psi(\Omega)] d\Omega / 2\pi$. Following the procedure of [26, 27], we get $h_d(t) \approx b e^{-\gamma t} \text{Ai}[-b(t - \tau_g)]$ where $\text{Ai}(x)$ is the Airy function and $b = (\Delta^2/3\tau_g)^{1/3}$. Finally $a_d(t) = \int_{-\infty}^t h_d(x) dx$ reads

$$a_d(t) = e^{-\gamma \tau_g} \int_{-\infty}^{b(t-\tau_g)} e^{-\gamma x/b} \text{Ai}(-x) dx \approx e^{-\gamma t} \int_{-\infty}^{b(t-\tau_g)} \text{Ai}(-x) dx \quad (8)$$

the 2^{nd} form holding when $\gamma \ll b$ [28]. $a_d(t)$ attains its absolute maximum at the first zero of $\text{Ai}(-x)$, that is for

$x \approx 2.3$ or $t = t_2 = \tau_g + 2.3/b$. For $t_1 < t < t_2$, $a(t)$ is well fitted by $a_1(t) + a_d(t)$ (Fig.1). For $t > \tau_g$, Ω_2 is real and, for $t > t_2$, the frequencies Ω_2^\pm are well separated ($\Omega_2 \gg \gamma$). The contribution $a_2(t)$ of the two saddle points to $a(t)$ can then again be derived from Eq.(5) with $\theta_2^\pm = \mp\pi/4$. It reads

$$a_2(t) \approx -\sqrt{\frac{2}{\pi}} e^{-\gamma t} \frac{\cos\{\Omega_2 t + \Delta^2 \tau_g \Omega_2 / (\Omega_2^2 - \Delta^2) + \pi/4\}}{\Omega_2 \Delta \sqrt{-\tau_g} [(\Omega_2 + \Delta)^{-3} + (\Omega_2 - \Delta)^{-3}]} \quad (9)$$

The steepest descent contour Γ' passing through the four saddle points is such that $\Gamma + \Gamma'$ encircles the pole in $\Omega = 0$. For $t > t_2$, we get thus $a(t) = e^{-\gamma t} + a_1(t) + a_2(t)$ where $e^{-\gamma t} = H(0)$ is the residue of the pole. Again the agreement with the exact result is very good (Fig.1). The optical fields associated with $a_2(t)$ may be considered as second precursors but, since they arrive after the rise of the main field, we suggest to name them *postcursors*. Contrary to those of the precursors, their instantaneous frequencies $\omega_2^\pm(t) = \omega_C \pm \Omega_2(t)$ are initially close to ω_C before deviating from this frequency.

We will now examine the important case of a medium with an electromagnetically induced transparency (EIT) window [29, 30, 31]. Precursors have been indirectly demonstrated in such a medium by a method of two-photon coincidence [32] and a first theoretical study of propagation is reported in [33]. We consider the simplest Λ arrangement with a resonant control field. If the coherence relaxation rate for the forbidden transition is small enough, the medium may be transparent at ω_C and its system function reads

$$H(\Omega) = \exp\left\{\frac{-\alpha L \gamma / 2}{i\Omega + \gamma + \Omega_r^2 / 4i\Omega}\right\} \quad (10)$$

where Ω_r is the modulus of the Rabi frequency of the coupling field [24, 31]. We get then $\tau_g = 2\alpha L \gamma / \Omega_r^2$. Fig.2 shows the step responses $a(t)$ obtained for different Ω_r and for a value of αL intermediate between those of the celebrated experiments achieved on a lead vapor [29] and on an ultracold gas of atomic sodium [30]. As previously and for the same reasons, the very short term behavior of $a(t)$ (up to $t_1 = \frac{1}{2\alpha L \gamma} = \frac{1}{\Omega_r^2 \tau_g}$) is given by Eq.(2). In general, the 4th degree equation giving the saddle point frequencies has no simple solutions but the following properties are easily demonstrated. Irrespective of Ω_r , $\Omega_2^-(\tau_g) = 0$ and, for $t \rightarrow 0$, $\Omega_1^\pm(t) \rightarrow i\gamma \pm \Omega_r \sqrt{\tau_g / 4t}$ while $\Omega_2^\pm(t) \rightarrow \pm i\Omega_r / 2$. When $\Omega_r < \gamma$, $\Omega_1^+(t)$ and $\Omega_2^-(t)$ keep non degenerate and purely imaginary at every time. If on the contrary $\Omega_r > \gamma$, these two frequencies coalesce at a time $t_d > \tau_g$ in $\Omega_d = i\Omega_r \sin[\sin^{-1}(\gamma/\Omega_r)/3]$. For $\Omega_r > 4\gamma$, $\Omega_d \approx i\gamma/3$ and $t_d = \tau_g(1 + 4\gamma^2/3\Omega_r^2)$. Explicit analytical expressions of $a(t)$ can be obtained when Ω_r/γ is moderate [30] or large [29]. In the first case, $\gamma\tau_g = 2\alpha L \gamma^2 / \Omega_r^2 \gg 1$ and the precursors will have

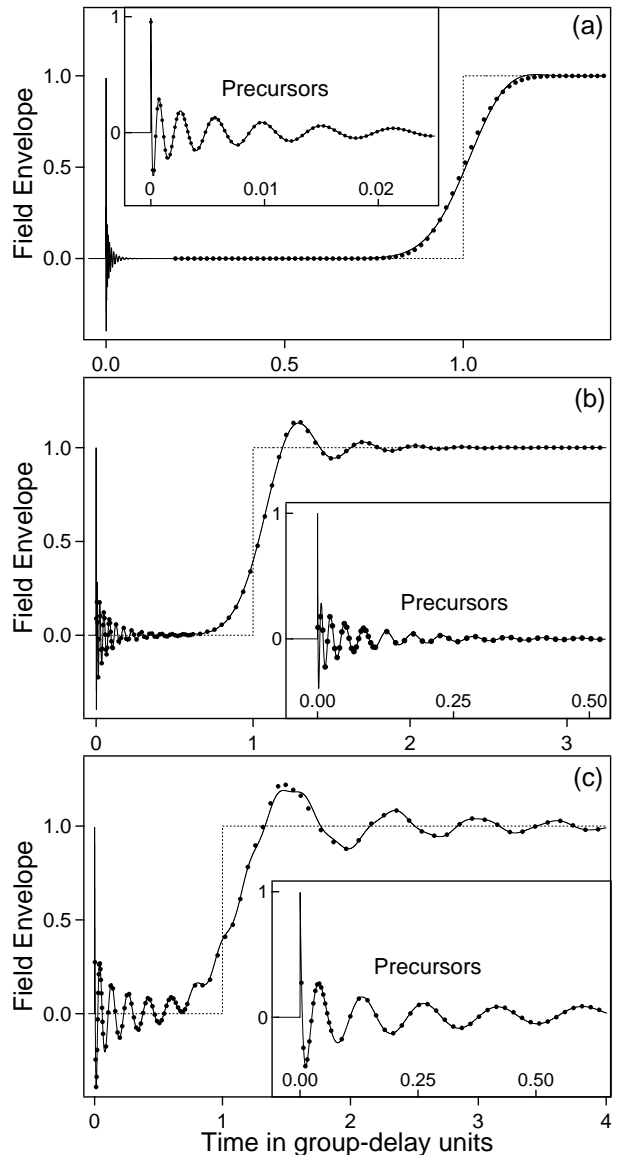


Figure 2: Same as Fig.1 for a medium with an electromagnetically induced transparency window. The parameters are $\alpha L = 600$ and $\Omega_r/\gamma =$ (a) 4.60 (b) 14.0 (c) 34.6, leading to (a) $\gamma\tau_g = 56.7$ (b) $\gamma\tau_g = 6.12$ and $b = 1.39\gamma$ (c) $\gamma\tau_g = 1.00$ and $b = 4.64\gamma$.

a short duration compared to τ_g . In this time domain $\Omega_1^\pm(t) \approx i\gamma t \pm \Omega_r(1 + 3t/2\tau_g)\sqrt{\tau_g/4t}$ and

$$a_1(t) \approx \sqrt{\frac{2}{\pi}} e^{-\gamma t} \frac{\cos[\Omega_r(1 + t/2\tau_g)\sqrt{t\tau_g} - \pi/4]}{(\Omega_r\sqrt{t\tau_g})^{1/2}} \quad (11)$$

When $\Omega_r \leq 1$ or when $\Omega_r > 1$ with $\gamma(t_d - \tau_g) \gg 1$ (Fig.2a), the only other significant contribution to $a(t)$ is $a_2(t)$ associated with the saddle point at $\Omega_2^-(t)$ which tends to 0 for $t \rightarrow \tau_g$. We circumvent the difficulty due to the coincidence of the saddle point with a pole by passing through the associated impulse response $h_2^-(t)$. It

reads $h_2^-(t) = \left(\sqrt{2\pi|\Psi''(\Omega_2^-)|}\right)^{-1} \exp[\Psi(\Omega_2^-) + i\theta_2^-]$ with $\theta_2^- = 0$, $\Psi(\Omega_2^-) \approx -[\Omega_r(t - \tau_g)/4\sqrt{\gamma\tau_g}]^2$ and $\Psi''(\Omega_2^-) \approx -8\gamma\tau_g/\Omega_r^2$. We finally get

$$a_2^-(t) = \frac{1}{2} (1 + \operatorname{erf}[\Omega_r(t - \tau_g)/4\sqrt{\gamma\tau_g}]) \quad (12)$$

where $\operatorname{erf}(x)$ is the error function. $a_2^-(t) \rightarrow 1$ when $\Omega_r(t - \tau_g) \gg 4\sqrt{\gamma\tau_g}$ and $a_1(t) + a_2^-(t)$ provides a good approximation of the exact step response at every time (Fig.2a).

When $\Omega_r \gg \gamma$ the coupling field splits the original line in a doublet of lines approximately centered at $\omega_C \pm \Omega_r/2$. If, in addition, $(\Omega_r/\gamma)^4 \gg 8\alpha L/3$, then $\gamma(t_d - \tau_g) \ll 1$ and the situation is analogous (but not identical) to that encountered with a natural transparency window. The frequencies of the saddle points approximately equal $\Omega_n^\pm \approx i\gamma_n(t) \pm \Omega_n(t)$ where $\gamma_n(t) \approx (\gamma/2) \left[1 - (-1)^n (1 + 8t/\tau_g)^{-1/2}\right]$ and where $\Omega_n(t)$ is given by Eq.(6), with $\Delta = \Omega_r/2$. The different contributions to $a(t)$ then read

$$a_1(t) \approx \sqrt{\frac{2}{|\pi\Psi''(\Omega_1^+)|}} \operatorname{Re} \left\{ \frac{1}{\Omega_1^+} e^{[\Psi(\Omega_1^+) - i\pi/4]} \right\} \quad (13)$$

$$a_2(t) \approx -\sqrt{\frac{2}{|\pi\Psi''(\Omega_2^+)|}} \operatorname{Re} \left\{ \frac{1}{\Omega_2^+} e^{[\Psi(\Omega_2^+) + i\pi/4]} \right\} \quad (14)$$

$$a_d(t) \approx \int_{-\infty}^{b(t-\tau_g)} \operatorname{Ai}(-x) \exp(-\gamma x/3b) dx \quad (15)$$

where $b = (\Omega_r^2/12\tau_g)^{1/3}$. As in the case of the natural frequency window, $a_1(t) + a_d(t)$ and $a_1(t) + a_2(t) + H(0)$ fit very well the exact step response, respectively for $t_1 < t < t_2 = \tau_g + 2.3/b$ and for $t > t_2$ (Fig.2b and Fig.2c). The main difference is that a significant damping of the precursors is now compatible with a good transparency at ω_C . For intermediate values of Ω_r it is so possible to observe both well developed precursors and postcursors without overlapping (Fig.2b). On the contrary, the tail of the precursors again partially interferes with the postcursors for very large Ω_r (Fig.2c). Note that a step response similar to that of Fig.2b has been demonstrated in a single-line medium (without EIT) when the field amplitude is large enough to saturate the absorption [34]. At short term, the linear approximation keeps valid (small pulse-area limit) and the signal given by Eq.(2) is retrieved but the subsequent evolution of the signal, despite a similar look, obviously results from nonlinear effects [35], quite different from the purely linear effects considered in the present letter.

To conclude, we have obtained explicit analytic expressions of the step response of linear media with a transparency window. Our results show that these media, contrary to those generally considered, are well adapted to

observe in a same experiment the precursors, the main field and the postcursors, all well distinguishable from each other and having comparable amplitudes. Insofar as the parameters used in the calculations are representative of real experiments, we think that our work might stimulate an experimental observation of these rich dynamics, which would, in turn, stimulate new theoretical investigations on related slow-light systems.

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