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SUSTAINABILITY, RESOURCE SUBSTITUTION IN ENERGY INPUTS AND LEARNING

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Abstract: We assess the impact of the existence of a costly energy substitute (like wind, solar) for a non-renewable resource (like oil, coal) on the sustainability of consumption. The prospects for sustainability depend crucially on the costs of this substitute. If one can reduce the costs of the resource substitute via learning-by-using then we find that still this does not guarantee sustainability. Also, the poorer a country the less it will take the learning-by-using effect into account and the more likely it will be unsustainable.

Key Words : Renewable resource, non-renewable resource, substitution, sustainability, learning-by-using.

Classification JEL: Q21, Q32, Q42, Q56.

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1 Introduction

Two main events, for many developed countries, once again address the important choices between several energy sources and energy dependencies. Firstly, at the end of 1997, 160 nations reached an agreement in Kyoto, Japan, to limit their production of carbon dioxide and other greenhouse gases. Secondly, the recent rise in oil prices. The energy independence argument was, at the beginning, one of the two reasons explaining the growth of nuclear energy production in the USA or France during the sixties and the growth of Ethanol production in Brazil through the "Pro-alcohol" plan in the seventies. The other argument was the relatively low price of those alternative energy sources. For example, in February 2008, the price of Ethanol for Brazilian consumers was 0.769US $ per liter against 1.443 US $ for petrol. In Brazil, Ethanol became the main substitute for petrol (85% of cars are flex-fuel in Brazil with the obligation to include 22% of Ethanol per liter of petrol), mainly due to the low costs of producing Ethanol. Similarly, the nuclear industry has gradually become the main source of electricity production in France where the nuclear industry currently covers 86% of French production of electricity and 20% of the total final energy consumption (Stenzel *et al.* (2003)).

If we accept Ethanol and nuclear as renewable resources (in the sense of non-depletable energy which also includes hydro power, wind energy, solar energy, biomass and geothermal energy), they are the two, consistent and real, examples of substitution between non-renewable fossil fuels and renewable energy resources. As said André and Cerda (2006), the main reason is certainly that in some situations it is easy, from a technological perspective, to replace a non-renewable resource with a renewable one. Indeed, it is relatively easy to substitute nuclear electricity or biofuels for coal and oil in electricity production or transportation (IEA (2007a) for electricity production and Harrington and McConnell, (2003) for transportation). In other cases, the replacement is currently too costly and difficult to perform (see for example Darmstadter (2001)). This is mainly due to the currently higher costs of alternative energy sources like solar, hydro or biomass (IEA (2007b)).

In fact, most renewable energy, with the exception of large hydro-electric power, nuclear and ethanol, represents a range of technologies still in their infancy. Due to R&D inputs, learning-by-using and wider commercial application, the capital costs of renewable energy are expected to fall substantially, making production from renewable energy sources increasingly competitive. If we look at our two examples, it is clear that substitution between renewable and non-renewable resources requires time, money and a voluntary policy (Steenblik, R. (2003), Oosterhuis, F. (2001)). The Brazilian government, since 1975, introduced the "Pro-alcohol" plan which implies substantial subsidies on production and for subvention. This, among others, includes the obligation to include 22% of Ethanol per liter of petrol. As a result we can now see that the productivity of Ethanol in Brazil is 30% higher compared to other countries (Chade (2006), Guimaraes (2007)). Research undertaken in the United States provides a useful indicator of the respective levels of total subsidy support for nuclear power and wind power at similar stages of technological development. Goldberg (2000) estimates that the nuclear industry in the USA received about 30 times more support per kWh output than wind power in the first 15

years of the industry's development(EEA (2004)). The comparative advantage of nuclear power in France stems from early investments with a monopoly structure in production .

These policies, through their impact on the relative price of non-renewable and renewable resources, lead to a much larger supply of the otherwise too costly renewable substitute. Setting aside possible effects of these policies on pollution and climate change, then one of the main reasons for this kind of subsidy policies is the potential learning-by-using effect. It is clear that a product, when in its infancy of development, can improve fastest through a sort of improvement process which one might call trial and error. It is however not clear whether these subsidy policies necessarily provide a socially valuable investment at these early stages. This question can however be addressed by looking at the conditions under which a rational policy maker would start to invest in learning-by-using.

The contribution of this article is thus twofold. Firstly, we assess how and when a costly energy substitute should be substituted for a non-renewable input and under what conditions this is likely to lead to a sustainable consumption path. Secondly, we address the impact of learning-by-using on the costs of the energy substitutes and how this is likely to augment our previous results. With this we also intend to add to the understanding of potential incentives to induce learning-by-using. We base our analysis on the now seminal contribution by Dasgupta and Heal (1974) but allow for a costly alternative energy production which is a perfect substitute for the non-renewable resource.

Our work here is similar to Tahvonen and Salo (2001) but differs in several crucial aspects. We firstly assume that the extraction of non-renewable resources is costless, and we secondly assume constant per unit costs of the energy substitute. Though we lose some generality we gain the advantage of explicit solutions for some stages of the optimal path. We also believe to obtain some results which seem slightly more realistic. For example, due to their assumptions they find that both types of resources should be used simultaneously. What we, in reality, however observe is that renewable resources like solar and wind energy are only used since they are so strongly subsidized. Indeed, without that level of subsidies, we would not see any solar or wind energy in today's energy production (see e.g. US Department of Energy, (2000)).

In addition, many empirical articles suggest that the use of non-renewable resources has recently been increasing and is on a rising trend (e.g. Berk and Roberts, (1996)). Our model predicts that it would be optimal, under certain conditions, to increase the extraction of the non-renewable resource shortly before one switches to the energy substitute. This result mainly hinges on the wealth of the nation in question and its valuation of the future.

The article is organized as follows. In section 2 we introduce the model with an exogenous cost of the substitute and obtain conditions for the resource use, the transition between resources and the sustainability of consumption. In section 3 we introduce endogenous learning-by-using and compare the results to the exogenous cost case. Section 4 concludes.

2 The Model

Our modeling approach here closely follows the basic idea of Dasgupta and Heal (1979) with several significant changes. We follow Dasgupta and Heal (1979) by assuming the existence of a representative agent who maximizes his stream of utility subject to a capital accumulation constraint, where income comes from a non-renewable energy input and capital. Our extension is that the agent can also make use of a costly energy substitute, which is a perfect substitute for non-renewable resources in energy production.[1] Our agent therefore solves the subsequent optimal control problem.

$$\max_{\{C(t),R(t),M(t)\}} W = \int_0^\infty e^{-\rho t} u(C(t))dt \tag{1}$$

subject to

$$\begin{aligned} \dot{K}(t) &= F(K(t), R(t) + M(t)) - C(t) - \gamma M(t), & (2)\\ \dot{S}(t) &= -R(t), & (3)\\ & C(t), K(t), S(t), R(t), M(t) \geq 0, \\ & K(0), S(0) \text{ given.} \end{aligned}$$

Here, consumption at time t is represented by $C(t)$; capital by $K(t)$; the flow of non-renewable resources by $R(t)$ and the stock thereof by $S(t)$; the costly substitute by $M(t)$ with a price γ; $u(\cdot)$ is the utility function; $\rho > 0$ the discount rate and $F(K, R+M)$ is the production function for capital accumulation. [2] We impose the following assumptions.

Assumption 1: The utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is at least twice continuously differentiable and has the standard properties of $u'(C) > 0$, $u''(C) < 0$ $\forall C$. We assume $u'(0) = +\infty$.

Assumption 2: The production function $F : \mathbb{R}^2_+ \rightarrow \mathbb{R}_+$ is concave in both arguments with $F_K \geq 0$ and $F_R \geq 0$, and verifies $F(0, R + M) = F(K, 0) = 0$.

An *admissible path* is defined as a trajectory $\{C(t), K(t), R(t), M(t), S(t)\}_{0 \geq t \geq \infty}$ which meets the constraints (2) and (3) with the states $K(t)$and $S(t)$ being piecewise continuous and the controls $C(t)$, $R(t)$, $M(t)$ piecewise continuous. A path $\{C(t)^*, K(t)^*, R(t)^*, M(t)^*, S(t)^*, t^*\}$ is an *optimal path* if it is admissible and $\forall\{C(t), K(t), R(t), M(t), S(t), t\}$ admissible paths we have $\int_0^\infty e^{-\rho t} u(C(t)^*)dt \geq \int_0^\infty e^{-\rho t} u(C(t))dt$. The optimization problem can be rewritten in Lagrangian form as follows:

$$\mathcal{L}(t) = \mathcal{H}(t) + \omega_R(t)R(t) + \omega_M(t)M(t),$$

where $\mathcal{H}(t)$ is the constant value Hamiltonian and given by

$$\mathcal{H}(t) = u(C(t)) + q(t)\big(F(K(t), R(t) + M(t)) - \gamma M(t) - C(t)\big) - \lambda(t)R(t),$$

[1] The costs of this substitute are assumed constant in this section but are endogenized in the section on technical change.

[2] We use $dx/dt \equiv \dot{x}$ and we denote the partial derivative of a function $G(x, y)$ with respect to x by G_x.

where $q(t)$ is the shadow value of capital and $\lambda(t)$ the one of the non-renewable resource. The first order conditions give us

$$\begin{aligned} u'(C(t)) &= q(t), && (4)\\ q(t)F'_R - \lambda(t) + \omega_R(t) &= 0, && (5)\\ q(t)(F'_R - \gamma) + \omega_M(t) &= 0, && (6)\\ -q(t)F'_K &= \dot{q}(t) - \rho q, && (7)\\ \rho\lambda(t) &= \dot{\lambda}(t). && (8) \end{aligned}$$

The complementarity slackness conditions are given by

$$\begin{aligned} \omega_R(t)R(t) &= 0, \quad R(t) \geq 0, \quad \omega_R(t) \geq 0,\\ \omega_M(t)M(t) &= 0, \quad M(t) \geq 0, \quad \omega_M(t) \geq 0. \end{aligned}$$

Finally, the transversality conditions read

$$\begin{aligned} \lim_{t\to\infty} q(t)K(t)e^{-\rho t} &= 0,\\ \lim_{t\to\infty} \lambda(t)S(t)e^{-\rho t} &= 0, \end{aligned}$$

and, for now, we assume that the utility integral is bounded for any optimal path, such that $W = \int_0^\infty u(C^*(t))e^{-\rho t}dt < \infty$. Furthermore, both the utility function and the set of constraints are concave in states and controls, wherefore the Mangasarian sufficiency conditions are fullfilled. We now analyze the basic properties of this model before we take a look at the conditions for dynamics and sustainability.

Lemma 1 *Given the policy maker's problem* (1)*, it is never optimal to utilize non-renewable resources and a costly substitute at the same time.*

Assume that $R > 0$ and $M > 0$. This implies $qF_R = \lambda$ and $F_R = \gamma$, which implies $q\gamma = \lambda$. This condition can only be satisfied if $F_K = 0$, thus if $K \to \infty$, which implies a contradiction.■

Even allowing for depreciation will only result in a degenerate set of time points under which both energy inputs will be used simultaneously. This is a direct result of the assumption of perfect substitutability in energy inputs. This Lemma states that given the structure of the control problem (1), both energy inputs should never be used simultaneously. The reason why we currently see different types of energy inputs being used simultaneously should then either be due to vintage technologies, such that it is not feasible to immediately switch from one energy input to another on an aggregate scale; or because there exists an underlying mechanism which changes the incentive for using non-renewable and a more costly renewable energy input simultaneously. In section 3 we propose technical change via learning-by-using as one of possible mechanism which may lead to the simultaneous use of both energy inputs.

Lemma 2 *Given the policy maker's problem* (1) *the non-renewable resource will be used until it becomes efficient to use the costly substitute, or never at all.*

We take the case of $R > 0$ and $M = 0$. This implies $qF_R = \lambda$, $q(F_R - \gamma) = -\omega_M$, and therefore $F_R < \gamma$ and we obtain a Hotelling rule, $\hat{F}_R = F_K$. Since this implies that $\hat{F}_R > 0$, there exists a T s.th. $\lim_{t\to T} F_R = \gamma$, which implies that $M > 0$. The next case is $R = 0$ and $M > 0$. Again, from the Kuhn-Tucker condition we calculate that $qF_R + \omega_R = \lambda$ and $F_R = \gamma$, which implies $q\gamma < \lambda$. Since $\hat{\lambda} > \hat{q}$, we know $\exists T \geq 0$ s.th. $q\gamma < \lambda$ and therefore $R = 0$, $M > 0$, $\forall t$.■

This result, in essence, boils down to one of comparing relative efficiencies. Do we still have sufficiently many non-renewable resources in order to meet the energy requirements of the current production process? Does the marginal increase in production from the renewable substitute already cover its costs? For example, Canada obtains currently over 55% of its electricity from renewable energy inputs, whereas the US obtains the same amount instead from non-renewable energy (especially coal). With similar economic and tax structures these differences can thus be explained by price differences in the energy production. The country which virtually exclusively relies on renewable resources in its energy generation is Brazil with approximately 90% of its energy generation coming from renewable energy resources. This is clearly due to the fact that Brazil firstly has an abundance of renewable resources and secondly that they come much more cheaply than non-renewable inputs. Thus, given that we currently still live in a world where non-renewable resources are abundant enough to keep production sufficiently high, it is no wonder that so few countries choose to use the more costly renewable energy sources. Therefore, today, the two main factors which lead to the choice of non-renewable resources over renewable ones are simply their relative abundance and their generally lower costs.

Lemma 3 *The policy maker will choose $R(0)$ such that when he switches energy inputs at time $t = T$, we have $S(T) = 0$.*

Proof by contradiction. Assume $\int_0^T S(t)dt < S(0)$. Since $R(t) = 0\ \forall t \geq T$, then $S(T) > 0$. But by the transversality condition $\lim_{t\to\infty} \lambda S(t)e^{-\rho t} = 0$, and since $\lambda(t) = \lambda(0)e^{\rho t}$ where $\lambda(0) > 0$, this would imply $\lim_{t\to\infty} \lambda(t)S(t) > 0$. Therefore we have a contradiction which implies $S(T) = 0$. ■

This result is useful since it shows that the (costless) non-renewable resources should ideally be fully depleted before one moves on to a costly substitute. Several modifications ought to augment this basic result. Firstly, if the extraction of non-renewable resources becomes increasingly costly the further the resource gets depleted. For example, it becomes more and more costly to mine the lower layers of coal or to withdraw oil from remote areas. Secondly, one could generally assume that the non-renewable resource is polluting and therefore imposes another externality on the agent. In these (non-exclusive) cases our results would be augmented.

2.1 Dynamics

We are essentially dealing with a problem that can be separated into two stages. The first stage with $R > 0$ has been well studied (e.g. Dasgupta and Heal, 1974), the link between the first and second stage has been studied in the previous section and the second stage will be studied here. We know that $R > 0$, $M = 0$ before T and $R = 0$, $M > 0$ after T. Since we know that $F_R(K, M) = \gamma$ after some time T, we use the implicit function theorem to write $M = h(K)$, where $h'(K) > 0$ if $F_{RR} < 0$ and $F_{KR} > 0$. This allows us to reduce the system to one consisting of two variables only, namely $C(t)$ and $K(t)$.

$$\dot{C} = -\frac{u'(C)}{u''(C)}\Big[F_K(K, h(K)) - \rho\Big], \tag{9}$$

$$\dot{K} = F(K, h(K)) - C - \gamma h(K). \tag{10}$$

In order to compare to the literature along the lines of Dasgupta and Heal (1979), we shall from now on focus on the case of a constant-elasticity of substitution production function.

We are particularly interested in assessing when a costly resource substitute will be used and whether a costly resource substitute allows for sustainable consumption. Since this question is most interesting if one assumes the realistic case of energy as an essential input in production, we shall assume this from now on. We answered the first question in the previous section and now deal with the second one. Since we know that $\dot{C} = -\frac{u'(C)}{u''(C)}\Big[F_K(K, h(K)) - \rho\Big]$, we know that $\dot{C} > 0$ if $F_K > \rho$. From our previous analysis we know that the costly substitute will be used if $F_R = \gamma$. We now wish to understand what kind of production technology allows for both conditions to hold simultaneously. We shall here take the case of the constant elasticity of substitution (CES) production function, as introduced by Arrow, Chenery, Minhas and Solow (1961), with constant returns to scale. The CES production function is given by $F(K, R + M) = A[\alpha K^\theta + (1 - \alpha)(R + M)^\theta]^{\frac{1}{\theta}}$. From $F_R = \gamma$ we can then solve for M as a function of K, which gives

$$M = \left[\frac{\alpha}{[\frac{\gamma}{(1-\alpha)A}]^{\frac{\theta}{1-\theta}} - (1 - \alpha)}\right]^{\frac{1}{\theta}} K \equiv \psi^{\frac{1}{\theta}} K.$$

Important here is the linearity between M and K. As we can easily calculate, an interior solution requires $\gamma < (1 - \alpha)^{\frac{1}{\theta}} A$, where we remind that we assume complementarity between capital and energy input (s.th. $\theta < 0$). This condition is more likely to be satisfied the lower the cost of the energy substitute; the larger the share of capital in production; the better the substitution between capital and energy; and the higher the exogenously given level of technology. Substituting the optimal relationship between M and K into equation (9) and (10) we obtain

$$\dot{C}(t) = -\frac{u'(C(t))}{u''(C(t))}\Big[\alpha A[\alpha + (1 - \alpha)\psi]^{\frac{1-\theta}{\theta}} - \rho\Big] \equiv -\frac{u'(C(t))}{u''(C(t))}\big[\Phi - \rho\big], \tag{11}$$

$$\dot{K}(t) = \big(A[\alpha + (1 - \alpha)\psi]^{\frac{1}{\theta}} - \gamma\psi^{\frac{1}{\theta}}\big)K(t) - C(t) \equiv \Psi K(t) - C(t). \tag{12}$$

With some manipulations it can be shown that $\Psi > 0$ iff $\gamma < (1-\alpha)^{\frac{1}{\theta}} A$. Thus, an interior solution for M also implies a positive Ψ. The next observation is that an interior steady state in consumption will only exist in a deteriorate set of parameter configurations, given by $\alpha A[\alpha + (1-\alpha)\psi]^{\frac{1-\theta}{\theta}} = \rho$. We will neglect this possibility in the further analysis. We now define $-\frac{cu''(c)}{u'(c)} \equiv 1/\sigma$, the elasticity of intertemporal substitution, which allows to retrieve an explicit solution for $C(t)$ and $K(t)$ in terms of exogenous parameters. We find that $\forall t \geq T$, the dynamics are characterized by

$$C(t) = C(T)e^{\sigma\left(\Phi-\rho\right)(t-T)}, \tag{13}$$

$$K(t) = \Sigma e^{\Psi(t-T)} - \frac{C(T)e^{\sigma(\Phi-\rho)(t-T)}}{\sigma\left(\Phi-\rho\right)-\Psi}. \tag{14}$$

T is determined by the point in time when $\lim_{t\to T} F_R = \gamma$. For γ sufficiently big we could have $T = \infty$, whereas for γ sufficiently small, we would have $T = 0$. However, we are more interested in whether consumption can be positive or non-decreasing over the whole time horizon. A necessary condition for non-decreasing consumption is $\Phi > \rho$. Comparative static exercises show that Φ increases with increases in A and α but decreases with a lower γ. Thus, if capital is more important for production or the larger the (currently) exogenously given level of technology, the more likely will consumption be non-decreasing. On the other hand, a more expensive costly substitute makes it more likely to have an unsustainable consumption path. We also obtain the standard result that a stronger preference towards today increases the likelihood of unsustainable consumption.

In order to study the full system we now need to find Σ. We can calculate its value with the aid of the transversality condition for capital. The transversality condition $\lim_{t\to\infty} q(t)K(t)e^{-\rho t} = 0$ can be re-written using the results from the first-order condition as well as equations (13) and (14) for $t \geq T$ as

$$\lim_{t\to\infty}\left\{\frac{C_T}{\Psi-\sigma(\Phi-\rho)}e^{(\sigma-1)(\Phi-\rho)(t-T)} + \Sigma e^{(\Psi-(\Phi-\rho))(t-T)}\right\}e^{-\rho(t-T)} = 0. \tag{15}$$

For the empirically relevant case of $\sigma < 1$ we obtain several crucial conditions from the transversality condition.

Firstly, one may assume that $\Phi > \rho$ which implies positive consumption growth. Then $\Sigma = 0$ is a sufficient condition for the TVC to hold. This implies $K(t) = -\frac{C(T)e^{\sigma(\Phi-\rho)(t-T)}}{\sigma\left(\Phi-\rho\right)-\Psi}$. Given $K(T) = K_T > 0$, then $C(T) = (\Psi - \sigma(\Phi-\rho))K_T$ and we require $\Psi > \sigma(\Phi-\rho)$ for an interior solution. In that case, capital will accumulate according to $K(t) = K_T e^{\sigma(\Phi-\rho)(t-T)}$ and consumption will be a constant fraction $(\Psi - \sigma\left(\Phi-\rho\right))^{-1}$ of capital. In this case we obtain endogenous growth where the energy input is sufficiently cheap such that enough of it can be produced in order to overcome the ever increasing energy demands from the increasing production.

Secondly, assume $\Phi < \rho$ which implies a continuous decline in consumption over time. Then $\Sigma = 0$ is again a sufficient condition for the TVC to be satisfied since

$\sigma(\Phi - \rho) - \Psi < 0$. We again obtain that capital will accumulate according to $K(t) = K_T e^{\sigma(\Phi-\rho)(t-T)}$, but this time capital will decline with consumption over time. The renewable energy substitute is too expensive to keep capital non-declining.

2.2 Further questions

We have not been able to answer a few crucial questions analytically since it is not possible to obtain the explicit solution of the first stage of the model when $R > 0$. Several questions are therefore still outstanding. Firstly, what determines the switching time? Secondly, how does the non-renewable resource get depleted?

The switching time

The analytical results show that $F_R = \gamma$ is the switching condition for the non-renewable resource. It is thus evident that the size of γ is crucial for the switching time. However, so are the initial values of the stocks $K(0)$ and $S(0)$. Indeed, the agent is required to build up a sufficient amount of capital in order to satisfy the switching condition. We therefore simulate the model (we provide a description of the simulation in the Appendix) in order to understand how both γ and the initial conditions for the state variables affect the switching time. In terms of the stocks, it suffices to look at only one of them and we choose $K(0)$ for convenience. Figure 1 shows how the switching time is affected by changes in γ and $K(0)$.

> Figure 1 about here <

What we observe is that for a large enough initial value of capital and a low enough cost of the substitute, the stock of the non-renewable resource will be used up completely in the first period. Furthermore, for a given value of γ, a smaller initial value of the capital stock implies that the switch is postponed further. For a given initial value of capital, the switching time is a monotonic, convex function of γ. Also, the smaller is the initial value of capital the more convex the relationship between γ and the switching time.

In terms of policy implications this suggests that, ceteris paribus, poorer countries should switch to a costly energy substitute later than richer ones. This is a pure level effect and does not change the subsequent growth rate of consumption or capital.

The depletion of the non-renewable resource

In the standard Dasgupta-Heal (1974)-Solow (1974)-Stiglitz (1974) model, if the resource inputs are complements, then the non-renewable resource is monotonically depleted and its extraction tends to zero over time. However, in this model it is possible that the non-renewable resource is depleted in a non-monotonic fashion. We provide an analytical condition in the subsequent proposition and then show a numerical example.

Proposition 1 *The non-renewable resource may be depleted non-monotonically if* $\exists t'$ *such that for some* $t' < T$, *sign*$(C_{t'}/Y_{t'} - 1 + \upsilon) \neq$ *sign*$(C_t/Y_t - 1 + \upsilon)$.

The proof of this uses the first phase of the control problem. We already showed that Hotelling's rule holds for $t < T$, such that $\hat{F}_R = F_K$. We can rewrite this as

$$\dot{R} = \frac{1}{F_{RR}}\big[F_R F_K - F_{RK}\dot{K}\big].$$

By the assumption of constant returns to scale in the production function we know that the elasticity of substitution in a CES function is given by $\upsilon \equiv \frac{F_R F_K}{F_{RK} F}$, with $\theta \equiv \frac{\upsilon - 1}{\upsilon}$. From the capital accumulation we know that $\dot{K} = Y - C$, where $Y = F(K, R)$. Thus, $\dot{R} > 0$ if $F_R F_K < F_{RK}\dot{K}$, which is equivalent to $C/Y < 1 - \upsilon$. Thus, if there exists an interval of t' in the phase where $R > 0$, ie. for $t < T$, then the sign of $\dot{R}$ changes only if there is a change in the sign of $C/Y - 1 + \upsilon$, where $\text{sign}(\dot{R}) = -\text{sign}(C_t/Y_t - 1 + \upsilon)$. ■

This result comes about since the agent firstly discounts the future and therefore uses an initially large but declining amount of the non-renewable resource, and then secondly wishes to build up a sufficient capital stock in order to satisfy the switching condition.

> Figure 2 about here <

In Figure 2 we show how the non-monotonicity of the non-renewable resource depletion depends on the costs of the energy substitute. For given initial conditions $K(0)$ and $S(0)$ we notice that the higher is cost of the substitute, the lower will be the non-renewable resource extraction. Furthermore, the non-monotonicity result does not prevail for all values of γ. The more costly the substitute the more time is needed to build up capital and therefore the longer will the non-renewable resource be in use.

2.3 When is consumption sustainable?

An important lesson from this model is the potential non-sustainability of consumption despite the existence of an energy substitute. It is clear now that this result crucially hinges on the costs of the energy substitute. Our benchmark calibration in Figures 3 and 4 shows a potential variety of consumption and capital paths for an interior solution of the energy substitute for different levels of γ.

> Figures 3 and 4 about here <

As can easily be seen, different levels of γ can lead from endogenous growth to endogenous decline. We can easily obtain that the effect of γ on the growth rate of capital and consumption is negative since $d\Phi/d\gamma < 0$. Thus, a higher γ implies a lower growth rate of capital and consumption. One can also calculate that $\dot{C}(t) < 0$ if

$$\gamma > (1-\alpha)A\left[\frac{1-\alpha(\frac{\rho}{\alpha A})^{\frac{\theta}{\theta-1}}}{1-\alpha}\right]^{\frac{\theta-1}{\theta}} \equiv \bar{\gamma}, \tag{16}$$

and $\dot{C}(t) \geq 0$ otherwise. Comparative statics with respect to $\bar{\gamma}$ show that $d\bar{\gamma}/d\rho < 0$, $d\bar{\gamma}/dA > 0$, $d\bar{\gamma}/d\alpha < 0$. If we furthermore use standard parameter configurations of

$\rho = 0.03$, $\alpha = 0.3$, $A > 1$, then in the neighborhood of these values we have that $d\bar{\gamma}/d\theta > 0$.

Firstly, the higher ρ the lower will be threshold $\bar{\gamma}$, implying that less care about the future requires an even smaller cost of the energy substitute for sustainable consumption. The parameter ρ can, of course, capture any reason for discounting the future, from uncertainty over cultural reasons to personal characteristics of agents. The intrinsic relationship revealed here between discounting the future and the costs of the energy substitute suggests that countries with a low life-expectancy are rather likely to be unsustainable (in comparison to countries with a high life-expectancy) despite the existence of an energy substitute.

Secondly, for a higher total factor productivity we find that consumption may still be sustainable with larger costs of the energy substitute. This for example suggests that countries like the USA or UK, France and Germany, those countries that have a comparatively high total factor productivity, are able to substitute costly renewable energy inputs at lower costs and should therefore substitute relatively earlier than most developing countries.

Thirdly, the higher the distribution parameter of capital the lower must be the cost of the energy substitute in order to obtain a sustainable consumption path. Thus, countries which are heavily relying on capital in their production function (those countries where the agricultural sector plays a minor role) will be more likely to see a declining growth path.

Finally, the easier it is to substitute energy and capital the lower may be the costs of the energy substitute for long-term growth. Indeed, what we get from this condition is that the less importance energy has for production the more likely will consumption be non-declining.

3 Technical Change

From the previous analysis we derive several important results for the sustainability of consumption and for the shift between non-renewable and renewable energy inputs. The crucial parameter that we investigated was the cost of the renewable energy substitute, γ. We took γ as constant and given. However, we also know that the costs of the energy substitutes like wind or solar energy have significantly decreased over the past years. This is mostly attributed to learning-by-using or learning-by-doing (Arrow (1962), Bramoullé and Olson (2005)). This means that the more one uses of the energy substitute, the more efficient one becomes in producing and using it and therefore more experience with this substitute implies a lower cost (see for example Van der Zwaan *et al.* (2002), Gerlagh and Van der Zwaan (2003)). We therefore now address the question of whether endogenous learning-by-using can revert some of the previous results. We are especially interested in two questions. Firstly, we wish to understand whether there exist circumstances under which both the non-renewable resource and the costly substitute are used simultaneously. Our previously analysis has shown that it is, generally, not optimal to use both at the same

time.[3] Our intuition is that we should use the renewable resource directly if we believe that learning-by-using will reduce the costs of the substitute, but this may depend on how learning translates into cost reductions. Secondly, we want to know whether learning-by-using may revert the previous result of unsustainable consumption for sufficiently high costs of the energy substitute.

3.1 Learning-by-using

The best way to describe learning-by-using is via the so called 'learning curves', which explain the price of the energy substitute as a function of the cumulative capacity used (McDonald and Schrattenholzer (2001), Rubin *et al.* (2004)).

In the following paragraphs we now extend the previous model to endogenous learning-by-using. We assume that $\gamma'(B(t)) < 0$, $\gamma''(B(t)) \geq 0$ and $\dot{B}(t) = M(t)$. $B(t)$ therefore represents cumulative capacity use of $M(t)$. The Langrangian, omitting time subscripts for convenience, of this problem then becomes

$$\mathcal{L} = u(c)e^{-\rho t} + q\big(F(K, R+M) - c - \gamma(B)M\big) - \lambda R + \phi M + \omega_R R + \omega_M M. \quad (17)$$

The new term is ϕM which represents the value of M through its use in B. The first-order conditions lead to

$$\begin{aligned} u'(C)e^{-\rho t} &= q, & (18)\\ \dot{\lambda} &= 0, & (19)\\ \dot{q} &= -qF_K, & (20)\\ qF_R &= \lambda - \omega_R, & (21)\\ \dot{\phi} &= q\gamma'(B)M, & (22)\\ \omega_M &= q(\gamma - F_R) - \phi. & (23) \end{aligned}$$

In the next paragraphs we collect a number of results which one can obtain from this setup.

Result 1 *In the case of learning-by-using with a costly energy substitute the energy substitute will, in general, be used earlier than if costs are exogenously given.*

From the Kuhn-Tucker conditions we know that $(q(\gamma(B) - F_R) - \phi)M = 0$. We clearly start to use the costly substitute much earlier than in the case without learning-by-using, namely already when $F_R < \gamma(B)$, or $q(\gamma(B) - F_R) = \phi$. In addition, we know that $M = 0$ if $q(\gamma(B) - F_R) > \phi$. It seems reasonable to assume $\gamma'(B) > -\infty$. The shadow value of B is given by $\phi(t) = -\int_t^\infty q\gamma'(B)Mdt > 0$. Therefore, the more efficient is the cumulative capacity use of M in reducing the costs of M through learning-by-using the

[3] If the energy inputs are not perfect substitutes then in general both may be used simultaneously. We do not investigate this possibility here.

larger will be the shadow value of B. A larger shadow value of B implies that the difference between F_R and $\gamma(B)$ will be greater at the time of the transition to the renewable energy substitute. Conclusively, those countries that have good learning conditions should use the energy substitute much earlier than those that have worse learning conditions.

Result 2 *Countries which are very poor are less likely to consider the impact of learning.*

Assume $R > 0$ and $M = 0$. Then $q(\gamma - F_R) > \phi$, which implies $\gamma > F_R$. In that case we obtain the usual Hotelling rule, where $\hat{F}_R = F_K > 0$. This implies there exists a point in time after which $q(\gamma - F_R) = \phi$. This condition can be rewritten to

$$F_R = \gamma + \frac{\int_t^\infty q(s)\gamma'(B(s))M(s)ds}{u'(c)e^{-\rho t}}.$$

If the level of consumption is very low, i.e. close to subsistence level, then the representative agent will ignore the second term and continue to extract the non-renewable resource until $F_R \approx \gamma$. The further consumption is above subsistence level, the more important will be the second term, and the earlier will the agent start to divert capital to the energy substitute M. This gives some suggestive policy implications. If one were to continue this line of thought, then richer countries should provide stronger incentives for using the energy substitute than poor countries. In a decentralized version of this model one would therefore expect to see higher incentives for the use of the costly substitute through e.g. tax and subsidy efforts. In effect, this is a behavior that we are already able to observe nowadays. It is mostly the rich countries that subsidize renewable energy inputs and thereby help to drive their costs down.

Result 3 *The price of the energy substitute will decline over time.*

Assume $R = 0$ and $M > 0$. Then we obtain a new Hotelling rule given by

$$\hat{F}_R = F_K(1 - \gamma/F_R) < 0.$$

Therefore, if the energy substitute gets used, then the marginal product of the energy substitute will decrease over time.

Result 4 *A balanced growth path may exist with growth characteristics qualitatively equivalent to those of the exogenous cost model.*

The dynamic system of this is then given by

$$\dot{C} = -\frac{u'(C)}{u''(C)}(F_K - \rho), \tag{24}$$

$$\dot{M} = -\frac{1}{F_{RR}}\left[F_{RK}\dot{K} + F_K(\gamma - F_R)\right], \tag{25}$$

$$\dot{K} = F(K, M) - C - \gamma(B)M, \tag{26}$$

$$\dot{B} = M. \tag{27}$$

We calculate the balanced growth path (BGP) for an interior solution of $M(t)$ for the CES case. On the BGP we assume that $\gamma'(B) = 0$ and therefore constant at $\gamma_{\min}$. We can then derive the following system that describes the possible BGP, where we define $x = C/K$ and $y = M/K$.

$$\hat{C} = \sigma\Big(A\big(\alpha + (1-\alpha)y^{\theta}\big)^{\frac{1-\theta}{\theta}} - \rho\Big), \tag{28}$$

$$\hat{M} = \frac{\alpha\gamma_{\min}y^{1-\theta}}{(1-\alpha)(1-\theta)} + \frac{\gamma_{\min}\theta y - \theta A\big(\alpha + (1-\alpha)y^{\theta}\big)^{\frac{1}{\theta}}}{1-\theta} - x, \tag{29}$$

$$\hat{K} = A\big(\alpha + (1-\alpha)y^{\theta}\big)^{\frac{1}{\theta}} - x - \gamma_{\min}y, \tag{30}$$

$$\tag{31}$$

On the BGP all variables must grow at the same rate, such that $g \equiv \hat{C} = \hat{M} = \hat{K}$. We then find that, on the BGP the new variables x and y are given by

$$\begin{aligned} x = {} & \sigma\Bigg(\rho - A\alpha^{\theta}\Bigg[\frac{\big(\frac{\gamma}{A(1-\alpha)}\big)^{\frac{\theta}{1-\theta}}}{\big(\frac{\gamma}{A(1-\alpha)}\big)^{\frac{\theta}{1-\theta}} - (1-\alpha)}\Bigg]^{\frac{1-\theta}{\theta}}\Bigg) + A\alpha^{\theta}\Bigg[\frac{\big(\frac{\gamma}{A(1-\alpha)}\big)^{\frac{\theta}{1-\theta}}}{\big(\frac{\gamma}{A(1-\alpha)}\big)^{\frac{\theta}{1-\theta}} - (1-\alpha)}\Bigg]^{\frac{1}{\theta}} \\ & -\gamma\alpha^{\frac{1}{\theta}}\Bigg[\Bigg(\frac{\gamma}{A(1-\alpha)}\Bigg)^{\frac{\theta}{1-\theta}} - (1-\alpha)\Bigg]^{-\frac{1}{\theta}}, \end{aligned} \tag{32}$$

$$y = \alpha^{1/\theta}\Bigg[\Bigg(\frac{\gamma}{(1-\alpha)A}\Bigg)^{\frac{\theta}{1-\theta}} - (1-\alpha)\Bigg]^{-1/\theta}. \tag{33}$$

Therefore, on the BGP, all variables grow at the rate

$$g = \sigma\Bigg(A\alpha^{\theta}\Bigg[\frac{\big(\frac{\gamma}{A(1-\alpha)}\big)^{\frac{\theta}{1-\theta}}}{\big(\frac{\gamma}{A(1-\alpha)}\big)^{\frac{\theta}{1-\theta}} - (1-\alpha)}\Bigg]^{\frac{1-\theta}{\theta}} - \rho\Bigg). \tag{34}$$

Comparative static exercises show that $dg/d\gamma < 0$, suggesting that a larger cost of the energy substitute lowers the growth rate on the BGP as one would expect. Indeed, one can calculate that the level of γ which leads to a negative growth rate in consumption is exactly the same as that obtained in equation (16). Thus, the previous long-run results carry through to the endogenous learning-by-using case.

However, we are still interested in whether consumption is monotonic or not during transition, that means whether the existence of an energy substitute with learning-by-using implies sustainability. For this we shall simulate the transition period under several scenarios that only differ in their respective impacts of the learning-by-using on the cost of the energy substitute.

Result 5 *A costly energy substitute does not guarantee a monotonic consumption profile despite the possibility of learning-by-using.*

This is shown in Figures 5 to 10, with different values of x reflecting different strengths of learning (the lower is x the weaker is the learning). Figure 10 clearly shows that learning-by-using may help in obtaining sustainability in comparison to an exogenously given level of γ (case $x = 0$). However, we also see that consumption can be non-monotonic. Since we start to use the energy substitute now earlier, this implies that we reduce the capital stock since the substitute is too expensive. However, with a higher capacity use the agent learns and therefore the costs of the energy substitute diminish, which eventually leads to a situation where the costs may be low enough to imply endogenous growth.

$>$ Figures 5 to 10 about here $<$

4 Conclusion

In this article we investigate the impact of a costly energy substitute on the sustainability of consumption and on the transition between non-renewable and renewable resources. We can summarize our results as follows.

Due to the cost structures it is never optimal to use both non-renewable resources and the costly substitute at the same time. Indeed, we show that the non-renewable resource will be used until becomes efficient to shift to the costly substitute, or never at all. We also derive that if the non-renewable resource gets used then it will be fully depleted at the time of the switch. The time of the switch is analyzed numerically and we find that the switching time increases convexly with the level of the cost of the substitute. We also find that initially poorer countries should shift towards the costly substitute later than richer ones. Finally, the non-renewable resource may be depleted non-monotonically (u-shaped extraction) depending on whether the savings ratio exceeds the elasticity of substitution or not.

We then investigate learning-by-using. In that case the energy substitute will, in general, be used earlier than if costs are exogenously given. This opens up a great deal of possible policy implications. For example, firms which are producing in a competitive structure may not be able to invest in a technology which is too costly now and therefore when the non-renewable resources are depleted, the substitutes may be too expensive. This point therefore gives support to the current subsidy policies that we see everywhere. Helping firms to bring energy substitutes on the market via subsidies implies that when the non-renewable resources will be finally used up, the substitutes may be cheap enough to guarantee long-run growth.

We notice that the costs of the energy substitute is crucial for the sustainability of consumption and derive explicit conditions under which optimal consumption may be non-declining. We then investigate the role of learning-by-using in the energy substitute for the sustainability of consumption and the transition between the resources. Our result is that learning-by-using may help sustainability but it does not guarantee it. Though it is more likely that learning-by-using may lead to endogenous growth due to its effect on the cost of the energy substitute, during the transition period consumption may be changing in a non-monotonic fashion.

Another relevant result is that countries, which are very poor, are less likely to consider the impact of learning. This suggests that current wealth plays a significant role when it comes to evaluating future options. Richer countries whose inhabitants do not live on the brink of starvation will be more inclined to rely on the non-renewable resource and when it is finally depleted they might be faced with a cost of the energy substitute which is too high for a sustainable path of consumption.

5 Appendix

We use the following configurations for the simulations. Simulations are done over a horizon of 250 time periods. Utility is of the constant relative risk aversion type with $\sigma = 0.5$. Production is a CES function with a distribution parameter of $\alpha = 0.33$, total factor productivity of $A = 1$. In the baseline cases γ is kept constant and varied between 1.3 and 1.8. When γ is endogenous we choose the functional form $(\gamma_{\min}+1/(1+xB(t)))$. We assume $gmin = 0.4$ and vary x from 0 to 0.0014. Initial conditions are $S(0) = 200$ and $K(0) = 0.2$, the discount rate is chosen at $\rho = 0.03$ and the elasticity of substitution in the production function is $\theta = -1$.

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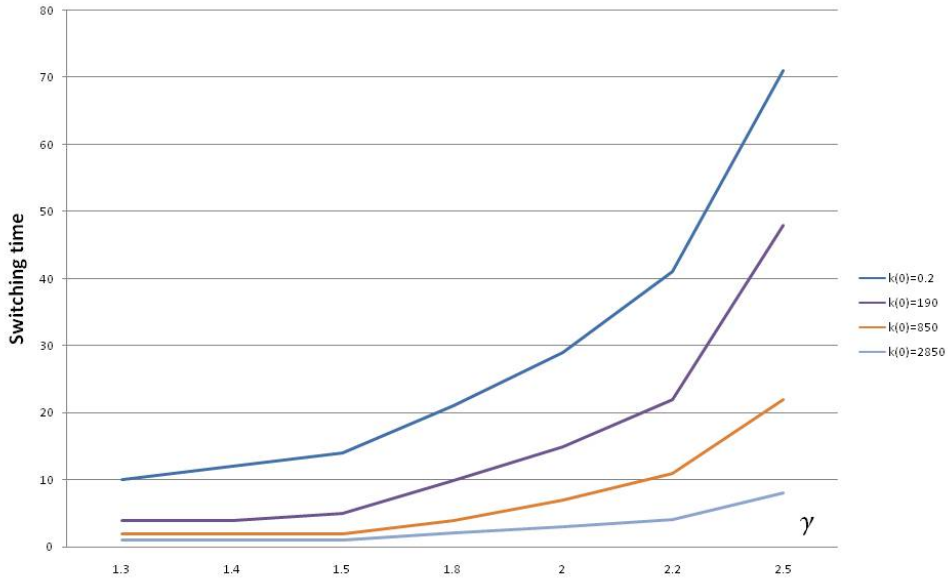


Figure 1: The switching time as a function of γ and $K(0)$

6 Figures

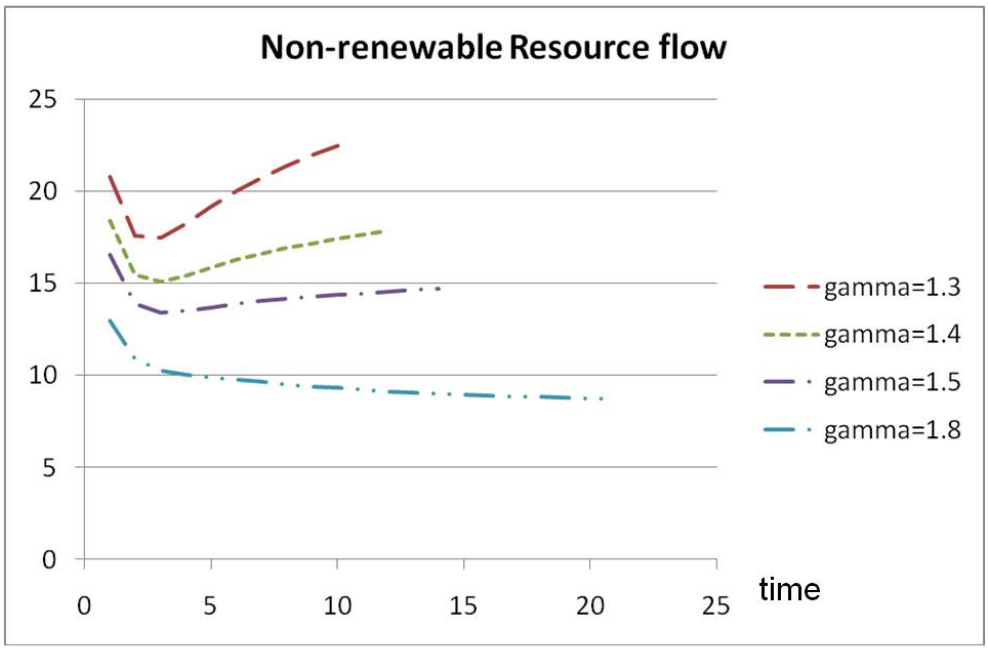


Figure 2: Non-renewable resource flow

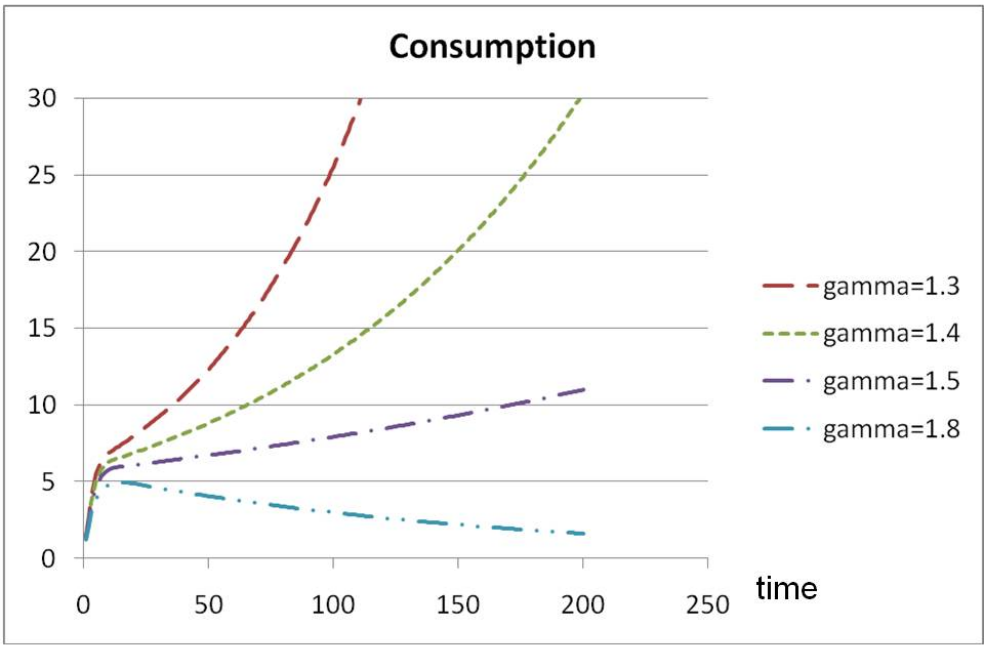


Figure 3: Evolution of consumption

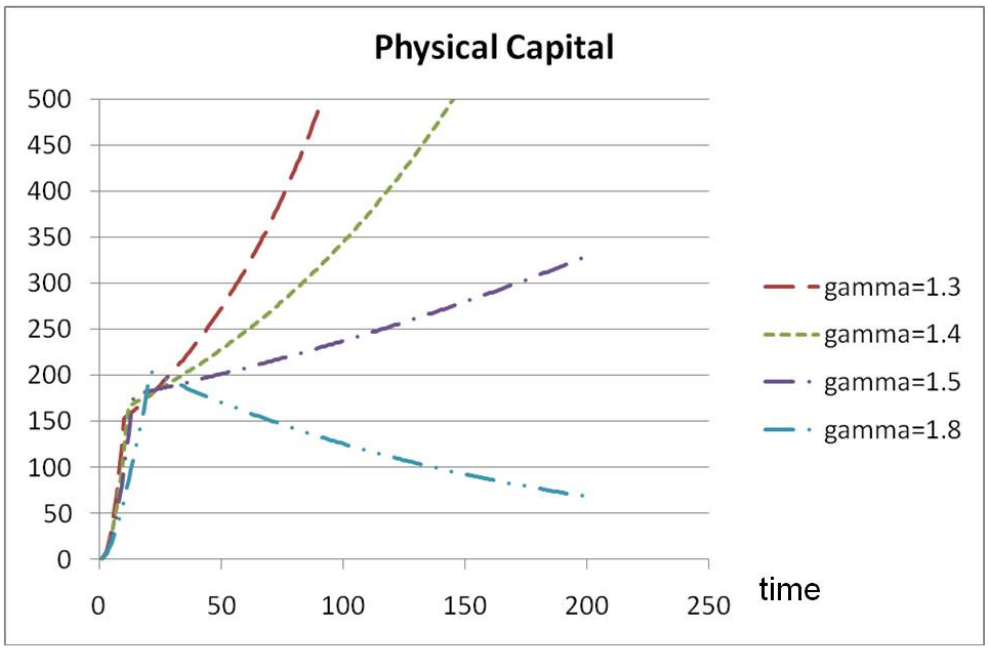


Figure 4: Evolution of the capital stock

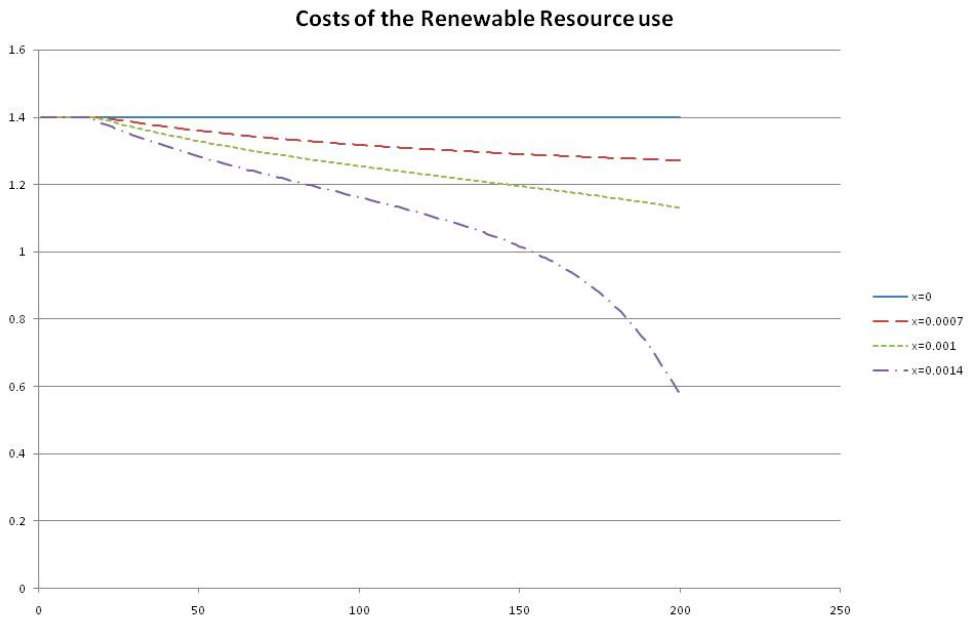


Figure 5: Costs of the energy substitute under learning-by-using

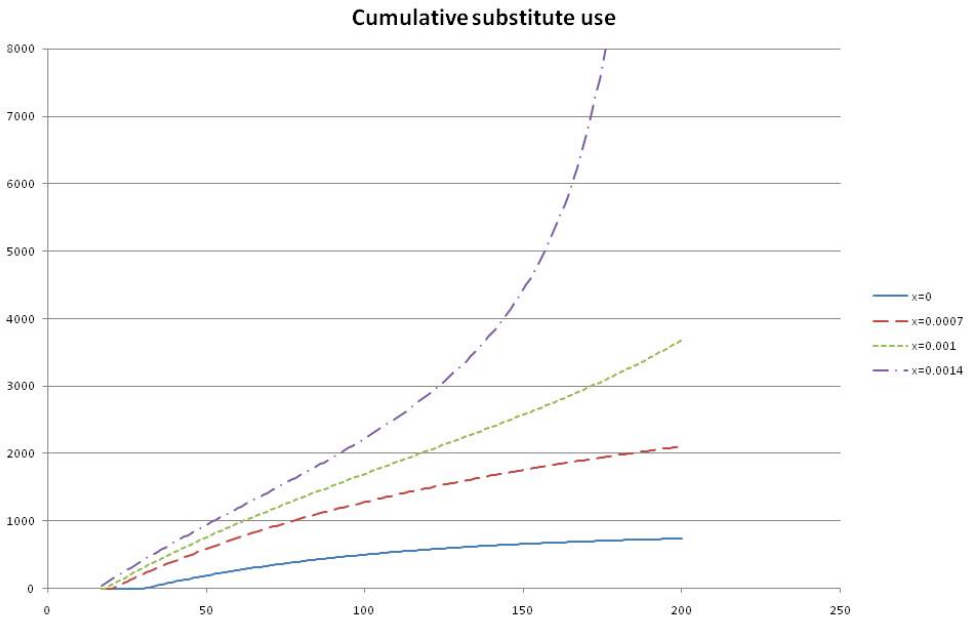


Figure 6: Cumulative substitute use under learning-by-using

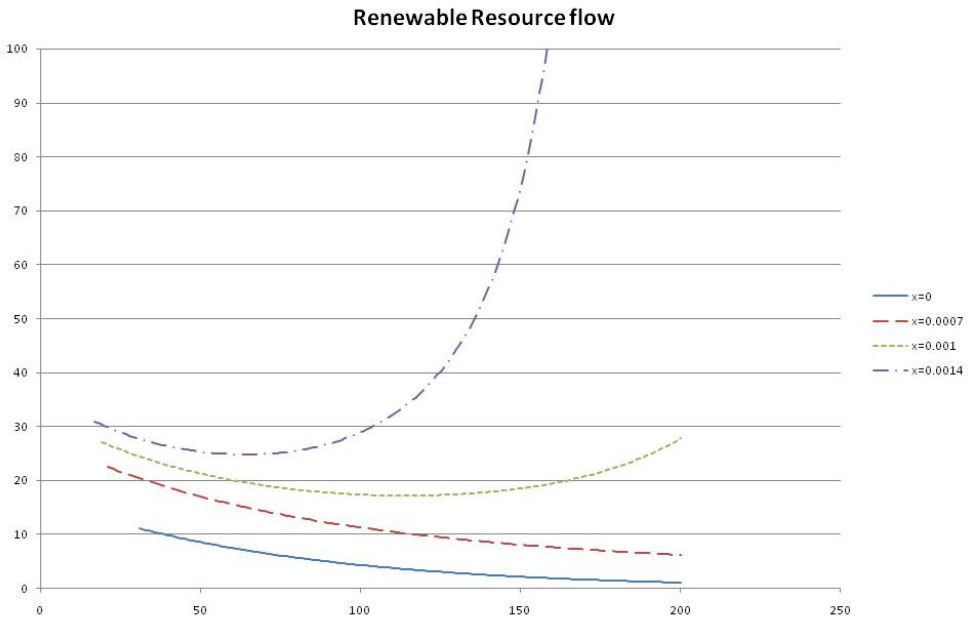


Figure 7: Renewable resource under learning-by-using

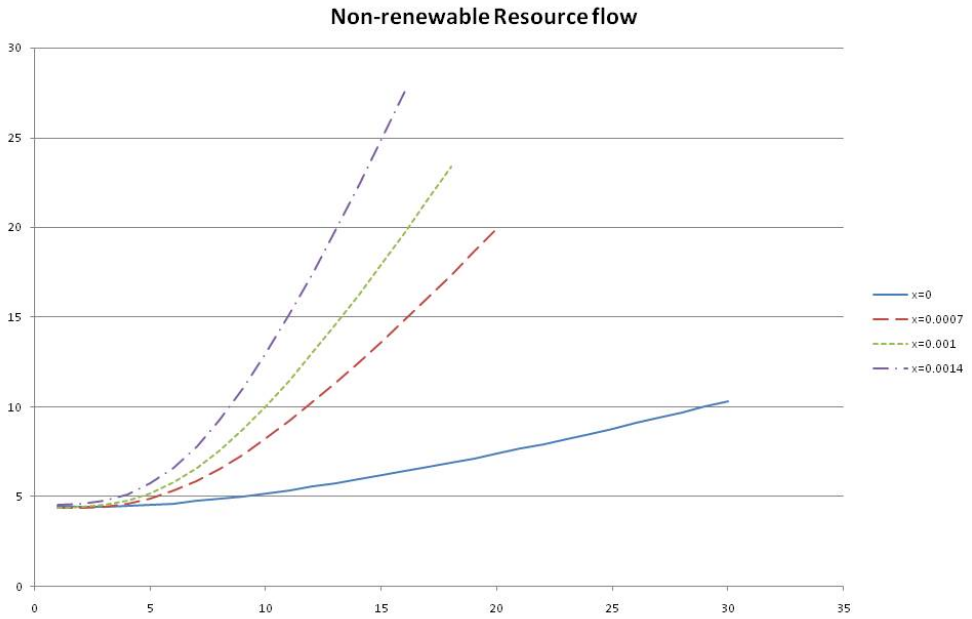


Figure 8: Non-renewable resource under learning-by-using

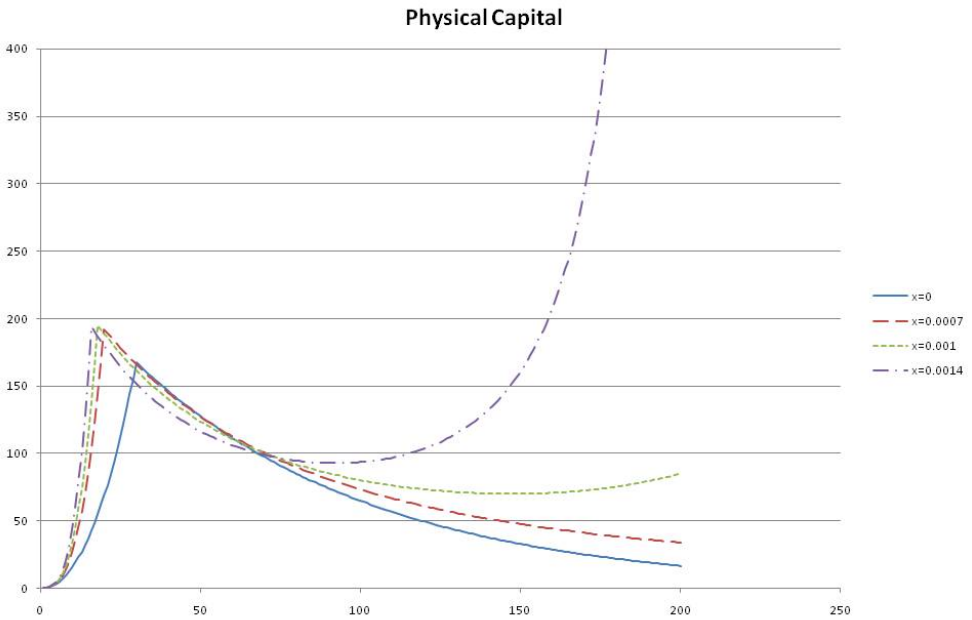


Figure 9: Capital stock under learning-by-using

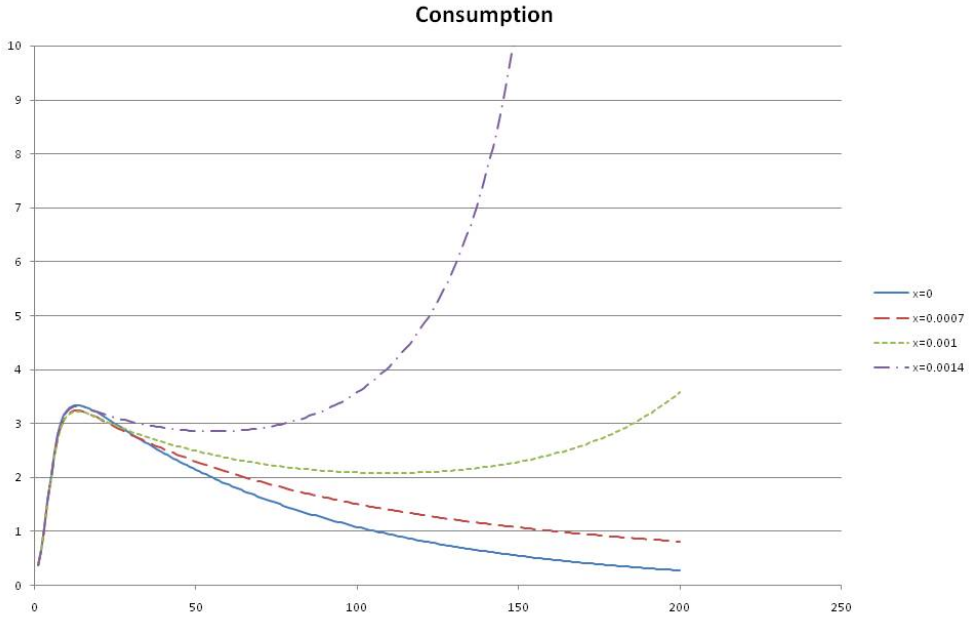


Figure 10: Consumption under learning-by-using