



**HAL**  
open science

## Numerical simulation of powder-snow avalanche interaction with an obstacle

Denys Dutykh, Céline Acary-Robert, Didier Bresch

► **To cite this version:**

Denys Dutykh, Céline Acary-Robert, Didier Bresch. Numerical simulation of powder-snow avalanche interaction with an obstacle. 2009. hal-00354000v2

**HAL Id: hal-00354000**

**<https://hal.science/hal-00354000v2>**

Preprint submitted on 4 Feb 2009 (v2), last revised 9 Nov 2010 (v4)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Numerical simulation of powder-snow avalanche interaction with an obstacle

Denys Dutykh <sup>a,\*</sup>

<sup>a</sup>*LAMA, UMR 5127 CNRS, Université de Savoie, 73376 Le Bourget-du-Lac  
Cedex, France*

Céline Acary-Robert <sup>b</sup>

<sup>b</sup>*LAMA, UMR 5127 CNRS, Université de Savoie, 73376 Le Bourget-du-Lac  
Cedex, France*

Didier Bresch <sup>c</sup>

<sup>c</sup>*LAMA, UMR 5127 CNRS, Université de Savoie, 73376 Le Bourget-du-Lac Cedex  
France*

---

## Abstract

In this paper we present direct numerical simulations of a sliding avalanche in aerosol regime. The second scope of this study is to get more insight into the interaction process between an avalanche and a rigid obstacle. An incompressible model of two miscible fluids can be successfully employed in this type of problems. We allow for mass diffusion between two phases according to the Fick's law. It is shown that the present model is consistent in the sense of kinetic energy. Some connections with Brenner-Navier-Stokes and Kazhikhov-Smagulov systems are revealed. The governing equations are discretized with a contemporary fully implicit finite volume scheme. The solver is able to deal with arbitrary density ratios. Encouraging numerical results are presented. Impact pressure profiles, avalanche front position and velocity field are extracted from numerical simulations and discussed. The influence of the bottom boundary condition onto propagation and impact processes is discussed. Finally we give some ideas of how this methodology can be used for practical engineering problems.

*Key words:* snow avalanche, gravity-driven flow, miscible fluids, two-phase flow, finite volumes, numerical simulation

---

---

\* Corresponding author.

*Email addresses:* [Denys.Dutykh@univ-savoie.fr](mailto:Denys.Dutykh@univ-savoie.fr) (Denys Dutykh),



Figure 1. Two illustrations of powder-snow avalanche flows.

## 1 Introduction

By definition, snow avalanches are abrupt and rapid gravity-driven flows of snow down a mountainside, often mixed with air, water and sometimes debris (see Figure 1). Avalanches are physical phenomena of great interest, mainly because they represent a big risk for those who live or visit areas where this natural disaster can occur. During last several decades, the risk has increased due to important recreational and construction activities in high altitude areas. We remember recent events at Val d’Isère in 1970 and in Northern Alps in 1999 [Anc01].

The avalanches arise from an instability in a pile of granular material like sand or snow. The destabilization phase of an avalanche life is still a challenging problem. There are many factors which influence the release process. One can recall snowpack structure, liquid content, shape and curvature of starting zone and many others [Anc01]. In this study we focus especially on sliding and stopping phases.

The serious research work on this natural phenomenon was preceded by the creation of scientific nivology at the end of the XIXth century. Among the pioneers we can mention Johann Coaz (swiss engineer) [Coa81] and Paul Mougin (french forest engineer, author of the first avalanche model using an analogy with a sliding block) [Mou22, Mou31]. The work of P. Maugin was ignored until the 1950s when A. Voellmy developed a similar model [Voe55] which is still used by engineers nowadays.

---

Celine.Acary-Robert@univ-savoie.fr (Céline Acary-Robert),  
Didier.Bresch@univ-savoie.fr (Didier Bresch).

We would like to point out here important contributions of the Soviet school by S.S. Grigorian, M.E. Eglit, A.G. Kulikovskiy, Y.L. Yakimov and many others [GEY67, KE73, BE73, BKK<sup>+</sup>75, KS77b, Egl83, Egl91, BL98, Egl98, BEN02]. They were at the origin of all modern avalanche models used nowadays in the engineering practice and, sometimes, in scientific research. Their works were mainly devoted to the derivation and comprehension of mathematical models while occidental scientists essentially looked for numerical solutions.

Conventionally we can divide all avalanches in two idealized types of motion: flowing and powder-snow avalanches. A flowing avalanche is characterized by a high-density core ranging from 100 to 500  $kg/m^3$  and consists of various types of snow: pasty, granular, slush, etc. The flow depth is typically about a few meters which is much smaller than the horizontal extent. This argument is often used to justify numerous depth-integrated models of the Savage-Hutter type<sup>1</sup> [SH89, SH91]. These avalanches can cause extensive damage because of the important snow masses involved in the flow in spite of their low speed.

On the other hand, powder-snow avalanches are large-scale turbidity currents descending slopes at high velocities [RH04]. They seriously differ from flowing avalanches. These clouds can reach 100 m in height and very high front velocities of the order of 100 m/s. They grow continuously and the average density is fairly low (from 4 to 25  $kg/m^3$ ). These spectacular avalanches (see Figure 1) occur only under certain conditions (after abundant fresh snowfalls, cold, dry and weakly cohesive snow on strong slopes) and they produce a devastating pressure wave which breaks the trees, buildings, tears off the roofs, etc. During the propagation stage, they are able to cross the valleys and even to climb up on the opposite slope. Hence, measurements by intrusive probes are almost impossible. Avalanches in aerosol are not very frequent events in Alps but in the same time we cannot say they are very seldom. In the technical literature there is an opinion that an avalanche in aerosol is less destructive than a flowing one since the transported mass is much smaller. Nevertheless, recent events of the winter 1999 revealed the important destructive potential of the powder-snow avalanches (see Figure 2).

Recently several systematic measurements campaigns *in situ* were conducted in Norway, Switzerland and Japan [MS84, NTK90, NMKI93, NSKL95, DGBA00]. Researchers shed some light on the internal structure of big avalanches. More precisely, they show that there exists a dense part of the avalanche which remains permanently in contact with the bed. This dense core is covered by the aerosol suspension of snow particles in the air. From these results it follows that mentioned above two types of avalanches may coexist in nature and proposed above classification is rather conventional. Perhaps, future studies will

---

<sup>1</sup> In hydrodynamics and hydraulics this type of modeling is also known as shallow water or Saint-Venant equations [dSV71].



Figure 2. Protecting wall in armed concrete at Tacconnaz (Haute-Savoie, France) destroyed by the powder-snow avalanche of the 11 February 1999. The height is 7 m and the thickness is 1.5 m (Photo by C. Ancey).

perform a coupling between the dense core and powder-snow envelope.

Let us review various existing approaches to the mathematical modeling of snow avalanches. Generally, we have two big classes of mathematical models: probabilistic and deterministic. In the present article we deal with a deterministic model and we refer to the works of K. Lied and D.M. McClung [LB80, ML87, McC00, McC01] for more information on statistical approaches to avalanche modeling. Deterministic models can be further divided into continuous and discrete ones depending whether the material under consideration can be approximated as a continuum medium or not. For the review and some recent results on dense granular flows we refer to the works of J. Rajchenbach [Raj02b, Raj02a, Raj05] and the references therein. Some promising results were obtained with discrete models based on cellular automata [DGRS<sup>+</sup>99, ADGM<sup>+</sup>00, DSI06].

The first contemporary avalanche models appeared in 1970 by soviet scientists Kulikovskiy and Sveshnikova [KS77b, BL98]. Later, their idea was exploited by Beghin [Beg79, BB83, BO91] and others [HTD77, FP90, AU99, Anc04, RH04]. We call this type of modeling 0D-models since the avalanche is assimilated to semi-elliptic cloud with variable in time volume  $V(t)$ , momentum  $(\rho U)(t)$ , etc. All quantities of interest are assimilated to the center of mass and their dynamics is governed by conservation laws expressed as Ordinary Differential Equations (ODE).

On the next complexity level we have various depth-integrated models. The governing equations are of Shallow Water (or Saint-Venant [dSV71]) type. In general, they are derived by depth averaging process or some asymptotic expansion procedure from complete set of equations. Thus, a physical 3D (or 2D) problem results in a 2D model (1D correspondingly). From computational point of view these models are very affordable even for desktop computers. On

the other hand, they provide us very approximative flow structure, especially in the vertical direction.

In France, G. Brugnot and R. Pochat were among the pioneers [BP81] while in Soviet Union this direction was explored in the beginning of 1970 by N.S. Bakhvalov et M.E. Eglit [BE73, BKK<sup>+</sup>75]. Currently, each country concerned with the avalanche hazard, has its own code based on this type of equations. Probably the most representative model is that developed by S. Savage and K. Hutter [SH89]. Nowadays this set of equations is generally referred as the Savage-Hutter model. We refer to [Hop83, Hut96] as general good reviews of existing theoretical models and laboratory experiments.

This approach was further developed by incorporating more complex rheologies and friction laws [GWH98, Hut91, HG93, MCVB<sup>+</sup>03, HWP05, FNBB<sup>+</sup>08]. To conclude on this part of our review, we have to say that this modeling is more relevant to the flowing avalanche regime which is characterized by small ratio of the depth  $h$  to the horizontal extent  $\ell$ :

$$\frac{h}{\ell} \ll 1.$$

We would like to mention impressive density currents simulations in Boussinesq regime by T. Özgökmen, P.F. Fischer and their collaborators [OFDI04, OFDI06, BDF<sup>+</sup>07, OIF<sup>+</sup>07]. They solved 2D and 3D variable density Navier-Stokes equations by a Spectral Element Method (SEM). Even if their results applied to physical oceanography, there is an analogy with the subject of our concern.

Finally, we come to the so-called *two-fluid* (or *two-phase*) models. In this paradigm both phases are resolved and, a priori, no assumption is made on the shallowness of the flow under consideration. Another advantage consists in fact that efforts exerted by the ambient air on the sliding mass are naturally taken into account. From computational point of view, these models are the most expensive [NG98, ESH04, Eti04, EHS05]. In the same time, they offer quite complete information on the flow structure.

As it follows from the title, in this study we are mainly concerned with powder-snow avalanches. We would like to underline that our modeling paradigm allows for taking into account of the dense core. The density is completely determined from the snow volume fraction distribution. This parameter can be used to introduce a stratification in the initial condition, for example. Otherwise, if we propagate our avalanche sufficiently long time, it will happen automatically due to mixing processes in the flow.

We retained a simple incompressible two-fluid model which is described in detail in Section 2. Two phases are allowed to interpenetrate, forming a mixing

zone in the vicinity of the interface. We make no Boussinesq-type hypothesis [Anc04] about the density ratio. Moreover, our solver is robust and is able to deal with high density ratios (we tested up to 3000). In the nature, the flow under consideration is highly turbulent but at the present stage we do not incorporate any turbulence modeling beyond resolved scales.

Good understanding of these natural phenomena can improve the risk assessment of such natural hazards. Numerical simulations of avalanches provide useful information on the dynamics of these flows. On the other hand, large scale experiments are not feasible<sup>2</sup>. Field measurements during the event are very hazardous and the events are rare. Laboratory experiments on powder-snow avalanches are essentially limited to Boussinesq clouds<sup>3</sup> [Kel95, NBNBH02, Pri03, NB03, PNBNF04] while it is not the case in the nature. When the Reynolds number is sufficiently high (inertial regime), two scaling parameters to be respected are the density ratio  $\frac{\Delta\rho}{\rho^+} = \frac{\rho^+ - \rho^-}{\rho^+}$  and the densimetric Froude number  $Fr := \frac{U}{\frac{\Delta\rho}{\rho^+} \sqrt{gH}}$ . In these formulas  $\rho^\pm$  are densities of the heavy and light fluids respectively,  $U$  is the characteristic flow velocity and  $H$  is the length scale. Obviously,  $g$  denotes the acceleration due to gravity. Recall that in the Boussinesq regime, only the Froude number has to be respected. We quote [PNBNF04] reporting on this important issue:

... Satisfying the Froude number and density ratio similarities in the laboratory means that a very high velocity is necessary, which calls for a very large channel. It is not possible to satisfy the density ratio similarity, because dimensionless number differs by several orders of magnitude between the processes that unfold in nature and those reproduced in the laboratory...

Thus, we do not really know how these results apply to real-world events. Recently, some progress has been made to remedy this situation [TM08]. Anyhow, computer experiments will become the main tool in studying this phenomenon. Experiments *in silico* should be complementary to those *in situ* or in laboratory. Direct Numerical Simulations (DNS) provide complete information about all flow quantities of interest such as the local density, the velocity field variations, the dynamic pressure and the energy. Recall that this information is not easily accessible by means of measurements.

The present paper is organized as follows. In Section 2 we present the govern-

---

<sup>2</sup> However, we would like to notice that there are two experimental sites: one in Switzerland (Sion Valley) and another one in France (Col d'Ornon). Unfortunately, field measurements provide very limited information nowadays [DGA01].

<sup>3</sup> The Boussinesq regime corresponds to the situation when  $\frac{\rho^+ - \rho^-}{\rho^+} \ll 1$  where  $\rho^\pm$  are densities of the heavy and light fluids respectively. This asymptotics allows to introduce the so-called Boussinesq approximation.

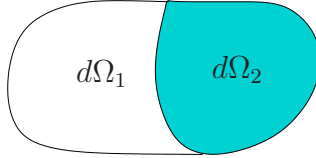


Figure 3. An elementary fluid volume  $d\Omega$  occupied by two phases.

ing equations and some constitutive relations. Special attention is paid to the kinetic energy balance properties. Some connections with Brenner-Navier-Stokes equations [Bre05b] and Kazhikhov-Smagulov systems [BES02] are also discussed. The next Section 3 contains a brief description of the numerical methods and numerous computation results are presented. Finally, this paper is ended by outlining main conclusions and a few perspectives for future studies (see Section 4).

## 2 Mathematical model

In the present study we assume that an avalanche is a two-fluid flow formed by air and snow particles in suspension. The whole system moves under the force of gravity. For simplicity we assume that the mixture is a Newtonian fluid. The last assumption is not so restrictive as it can appear. The flow under consideration is such that the Reynolds number is very high ( $\text{Re} \sim 10^9$ ). Therefore, the transient behaviour is essentially governed by the convective terms and not by the fluid rheology. On the contrary, the rheology is very important in the flowing regime.

In two-fluid flows it is natural to operate with the so-called volume fractions [Ish75, TK96, TKP99, GKC01]. consider an elementary fluid volume  $d\Omega$  surrounding an interior point  $P \in d\Omega$ . Let us assume that the first fluid occupies volume  $d\Omega_1 \subseteq d\Omega$  and the second the volume  $d\Omega_2 \subseteq d\Omega$  (see Figure 3) such that

$$|d\Omega| \equiv |d\Omega_1| + |d\Omega_2|. \quad (1)$$

The volume fraction of the fluid  $i = 1, 2$  in the point  $P$  is defined as

$$\phi_i(P) := \lim_{\substack{|d\Omega| \rightarrow 0 \\ P \in d\Omega}} \frac{|d\Omega_i|}{|d\Omega|}.$$

From relation (1) it is evident that  $\phi_1(P) + \phi_2(P) \equiv 1$ , for any point  $P$  in the fluid domain. Henceforth, it is sufficient to retain only the heavy fluid volume fraction  $\phi_1$ , for example, which will be denoted by  $\phi$ , for the sake of simplicity.

The volume fraction evolution is governed by two simultaneous processes: advection and diffusion. Very often, the diffusion is neglected. However, we cannot disregard this effect in the present study because of important mixing



processes in the aerosol flow regime. Thus, we model the fluid mixing by the Fick's law [Fic55a, Fic55b]. Two phases are constrained to have the same velocity<sup>4</sup>  $\vec{u}$  and the mixture is assumed to be incompressible. Taking into account all the hypotheses made above, we come to the following system of the governing equations:

$$\nabla \cdot \vec{u} = 0, \quad (2)$$

$$\frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi = \nabla \cdot (\nu_f \nabla \phi), \quad (3)$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} - \nu_f \nabla \log \rho \cdot \nabla \vec{u} + \frac{\nabla p}{\rho} = \vec{g} + \frac{1}{\rho} \nabla \cdot (2\mu \mathbb{D}(\vec{u})), \quad (4)$$

where  $\vec{g}$  is the acceleration due to gravity,  $\mathbb{D}(\vec{u}) = \frac{1}{2}(\nabla \vec{u} + (\nabla \vec{u})^t)$ ,  $\nu_f$  controls the mixing process between two fluids and has the dimension of the kinematic viscosity. The mixture density  $\rho$  and dynamic viscosity  $\mu$  are defined as follows:

$$\rho = \phi \rho^+ + (1 - \phi) \rho^-, \quad \mu = \phi \rho^+ \nu^+ + (1 - \phi) \rho^- \nu^-, \quad (5)$$

where  $\rho^\pm$  and  $\nu^\pm$  are constant densities and kinematic viscosities of the heavy and light fluids correspondingly.

**Remark 1** *Since we model an avalanche propagation along a sloping solid boundary, we take the gravity acceleration in the following form:*

$$\vec{g} = (g \sin \theta, -g \cos \theta),$$

where  $\theta$  is the slope of the hill and usually  $g := |\vec{g}| = 9.8 \text{ m/s}^2$ .

The snow kinematic viscosity  $\nu^+$  can be parameterized as a function of temperature  $T$  and snow density  $\rho^+$  according to [DAH<sup>+</sup>07]:

$$\nu^+ = \frac{\mu_0}{\rho^+} e^{-\alpha T} e^{\beta \rho^+},$$

where  $\mu_0 = 3.6 \times 10^6 \text{ N} \cdot \text{s} \cdot \text{m}^{-2}$ ,  $\alpha = 0.08 \text{ K}^{-1}$ ,  $\beta = 0.021 \frac{\text{m}^3}{\text{kg}}$ .

Equations (2), (3) and (4) have to be completed by appropriate initial and boundary conditions to form a well-posed problem and determine the system evolution.

Recall that governing equations can be formulated in terms of various sets of variables. For example, equation (3) is formulated in terms of the volume fraction  $\phi$ . From given above definition (5), it is straightforward to obtain

---

<sup>4</sup> This assumption is not very restrictive. Two-phase single velocity models were already be successfully applied in various situations [DDG08b, DDG08c, DDG08a, Dut07].

an evolution equation for the mixture density  $\rho$ . Using the incompressibility condition (2), it can be written in the conservative form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = \nabla \cdot (\nu_f \nabla \rho). \quad (6)$$

This equation will be used below to establish some kinetic energy properties of our system (2) – (4).

We would like to underline a certain interplay between our model (2) – (4) and very recent modification to Navier-Stokes equations proposed by Howard Brenner [Bre05a, Bre05b, Bre06, FV07]. Namely, equation (6) can be rearranged in the following way:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \quad \vec{v} := \vec{u} - \nu_f \nabla \log \rho.$$

Recall that H. Brenner introduced the notion of the volume velocity  $\vec{v}$  which is related to the classical mass velocity  $\vec{v}_m$  in a very similar way:

$$\vec{v} - \vec{v}_m = K \nabla \log \rho.$$

It is obvious that two velocities coincide for incompressible and homogeneous fluids  $\rho = \text{const}$ .

The second remark concerns the “novel” term  $\nu_f \nabla \log \rho \cdot \nabla \vec{u}$  in the momentum equation (4). In fact, this term is already known and can be encountered in the so-called Kazhikhov-Smagulov type systems which were derived for incompressible multiphasic fluid models [KS77a, AKM90, BES02, BES07].

## 2.1 Energy consistency

Consider a fluid domain  $\Omega$ . The kinetic energy  $K$  is classically defined as

$$K = \frac{1}{2} \int_{\Omega} \rho |\vec{u}|^2 d\Omega. \quad (7)$$

If we multiply equation (4) by  $\rho \vec{u}$ , equation (6) by  $\frac{|\vec{u}|^2}{2}$  and subtract them, after some simple analytical computations we get the following evolution equation for the kinetic energy:

$$\frac{d}{dt} \frac{1}{2} \int_{\Omega} \rho |\vec{u}|^2 d\Omega = \int_{\Omega} \rho \vec{g} \cdot \vec{u} d\Omega - \int_{\Omega} 2\mu |\mathbb{D}(\vec{u})|^2 d\Omega, \quad (8)$$

where  $|\mathbb{D}(\vec{u})|^2 := \mathbb{D}(\vec{u}) : \mathbb{D}(\vec{u})$ . Each term in equation (8) has a precise physical interpretation. The first term on the right-hand side is a consequence of the

kinetic energy production by the gravity force. The second term is responsible of the energy dissipation by viscous forces.

In several studies [JR93, ESH04] a slightly different model was used. Namely, the term  $\nu_f \nabla \log \rho \cdot \nabla \vec{u}$  was missing in the momentum balance equation (4). Exactly the same incomplete model was implemented in `OpenFOAM` solver `twoLiquidMixingFoam`. In this case, the energy balance equation takes the following form:

$$\frac{d}{dt} \frac{1}{2} \int_{\Omega} \rho |\vec{u}|^2 d\Omega = \int_{\Omega} \frac{|\vec{u}|^2}{2} \nabla \cdot (\nu_f \nabla \rho) d\Omega + \int_{\Omega} \rho \vec{g} \cdot \vec{u} d\Omega - \int_{\Omega} 2\mu |\mathbb{D}(\vec{u})|^2 d\Omega.$$

Since, the first term on the right-hand side is not positive-defined, the notion of kinetic energy is not well defined for the model used before.

## 2.2 Dimensional analysis

In this section we perform a dimensional analysis of the governing equations (2) – (4) in order to reveal important scaling parameters. Henceforth, starred variables denote dimensional quantities throughout this section.

The initial avalanche height  $h_0$ , the heavy fluid density  $\rho^+$  and the kinematic viscosity  $\nu^+$  are chosen to be the characteristic length, density and viscosity scales correspondingly. Velocity field is adimensionalized by  $U_0 := \sqrt{gh_0}$  as it is usually done for gravity-driven flows. Finally, from characteristic length and velocity, it is straightforward to deduce the time scale.

The scaling for the independent variables is

$$\vec{x}^* = h_0 \vec{x}, \quad t^* = \frac{h_0}{\sqrt{gh_0}} t,$$

and dimensionless dependent variables  $\rho, \vec{u}, p, \mu$  are introduced in this way:

$$\rho^* = \rho^+ \rho, \quad \vec{u}^* = \sqrt{gh_0} \vec{u}, \quad \mu^* = \rho^+ \nu^+ \mu, \quad p^* = \rho^+ U_0^2 p = \rho^+ gh_0 p.$$

We decided to adimensionalize dependent variables with respect to the heavy fluid parameters  $\rho^+$  and  $\nu^+$  but this choice is only conventional.

The governing equations (2) – (4) in dimensionless form become:

$$\begin{aligned} \nabla \cdot \vec{u} &= 0, \\ \frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi &= \nabla \cdot \left( \frac{1}{\text{ReSc}} \nabla \phi \right), \end{aligned}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} - \frac{1}{\text{ReSc}} \nabla \log \rho \cdot \nabla \vec{u} + \frac{\nabla p}{\rho} = \vec{g} + \frac{1}{\rho} \nabla \cdot \left( \frac{1}{\text{Re}} 2\mu \mathbb{D}(\vec{u}) \right).$$

This procedure reveals two important scaling parameters – the Reynolds number [Rey83]  $\text{Re}$  and the Schmidt number [ID01]  $\text{Sc}$  which are traditionally defined as

$$\text{Re} := \frac{h_0 U_0}{\nu^+} = \frac{h_0 \sqrt{g h_0}}{\nu^+}, \quad \text{Sc} := \frac{\nu^+}{\nu_f}.$$

Recall that the Reynolds number  $\text{Re}$  gives a measure of the ratio of inertial forces to viscous forces. In practice this number characterizes different flow regimes, such as laminar or turbulent flow. The Schmidt number is the ratio of fluid viscosity to mass diffusivity. This parameter is introduced used to characterize fluid flows with simultaneous momentum and mass diffusion processes [YXS02, SS03]. This number was named after the German engineer Ernst Heinrich Wilhelm Schmidt (1892 – 1975).

In fact, there are two additional scaling parameters  $\delta$  and  $\lambda$  hidden in definitions of the density  $\rho$  and viscosity  $\mu$  of the mixture:

$$\rho := \frac{\rho^*}{\rho^+} = \phi + (1 - \phi)\delta, \quad \delta := \frac{\rho^-}{\rho^+},$$

$$\mu := \frac{\mu^*}{\rho^+ \nu^+} = \phi + (1 - \phi)\delta\lambda, \quad \lambda := \frac{\nu^-}{\nu^+}.$$

Actually, the densities ratio  $\delta$  can be related to the well-known Atwood number [GGL<sup>+</sup>01, LJ05]  $\text{At}$ :

$$\text{At} := \frac{\rho^+ - \rho^-}{\rho^+ + \rho^-} = \frac{1 - \delta}{1 + \delta}.$$

The powder-snow avalanche regime is characterized by very high values of Reynolds number  $\text{Re}$  and low density ratios  $\delta$  [MS93, Anc03]:

$$\text{Re} \sim 10^6, \quad 0.05 \leq \delta \leq 0.25.$$

Thus, the flow is clearly turbulent. Nevertheless, in the present study we do not consider any turbulence modeling beyond the scales resolved by the numerical method. As we already pointed out in the Introduction section, there is an important issue with the density ratio parameter  $\delta$ . In current laboratory experiments its value is about 0.8 [NB03] which is bigger than values encountered in nature. As a result, the interpretation of laboratory results is quite ambiguous.

Regarding the Schmidt number  $\text{Sc}$ , M. Clément-Rastello reports [CR01] the following values:

$$0.5 \leq \text{Sc} \leq 1.$$

There is substantially less information on snow viscosities [DAH<sup>+</sup>07] and on the snow rheology, in general.

Thus, if an avalanche is considered as a two-phase flow, the ideal laboratory experiment has to respect four scaling parameters:  $Re$ ,  $Sc$ ,  $At$  and  $\lambda$ . Obviously, it is impossible in practice as honestly stated in many experimental studies [Pri03, NB03, PNBNF04]. At this point, numerical simulation should be considered as a complementary tool to physical modeling.

### 3 Numerical methods and simulation results

In this article we perform Direct Numerical Simulations (DNS) of a snow cloud moving down a steep slope. Our solver is based on a freely available CFD toolbox `OpenFOAM`. All computations performed in this study are 3D with only one cell in  $z$  direction for the sake of efficiency. The extension to truly 3D configurations is in progress.

In order to implement model (2) – (4) we modified the standard solver `twoLiquidMixingFoam`. The principal modification concerns the momentum balance equation. Namely, we had to incorporate a novel term  $\nu_f \nabla \log \rho \cdot \nabla \vec{u}$  which ensures the energy consistency property (8). For information, we provide here a piece of the novel code written in the internal `OpenFOAM` language:

```
fvVectorMatrix UEqn
(
    fvm::ddt(rho, U)
    + fvm::div(rhoPhi, U)
    - fvm::laplacian(muf, U)
    - Dab*(fvc::grad(U()).T() & fvc::grad(rho))
    - fvc::div(muf*(mesh.Sf())&fvc::interpolate(fvc::grad(U()).T()))
);
```

Time derivatives were discretized with the classical implicit Euler scheme. An upwind second order finite volume method was employed in space. For more details on the retained discretization scheme we refer to [Jas96, Rus02, Ope07]. The choice of the finite volume method is justified by its excellent stability and local conservation properties (especially in comparison to FEM [ESH04, Eti04, EHS05]).

#### 3.1 Test-case description

The sketch of the fluid domain and the initial condition description are given on Figure 4. We choose to simulate a classical lock-exchange type flow with an obstacle placed into the computational domain. The values of all parameters

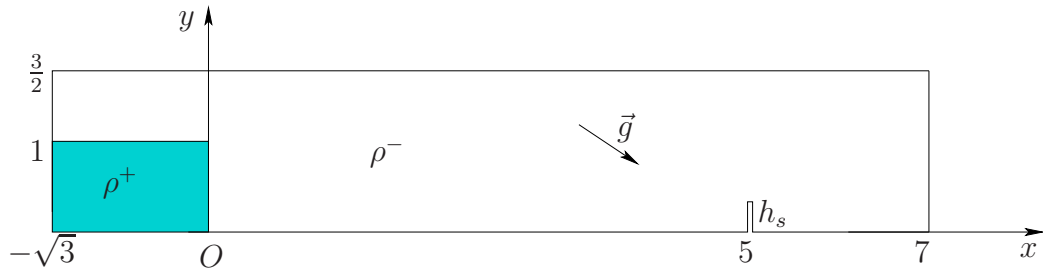


Figure 4. Sketch of the computational domain and initial condition description.

<i>parameter</i>	<i>value</i>
gravity acceleration $g$ , $m/s^2$	9.8
slope, $\theta$	$32^\circ$
Reynolds number, $Re$	$10^5$
Schmidt number, $Sc$	0.5
friction parameter <sup>5</sup> , $\alpha$	0.3
initial mass height, $h_0$	1
obstacle height, $h_s$	$0.25h_0$
obstacle thickness	$0.05h_0$

Table 1

Values of various parameters used for numerical simulations.

<i>parameter</i>	<i>heavy fluid</i>	<i>light fluid</i>
density, $\rho^\pm$ , $kg/m^3$	4	1
kinematic viscosity, $\nu^\pm$ , $m^2/s$	$10^{-5}$	$10^{-5}$

Table 2

Values of various parameters for light and heavy fluids.

are provided in Tables 1 and 2. The objective of this article is twofold. Except presenting DNS results, we also want to shed some light onto the interaction process between an avalanche and an obstacle. At the present stage we assume the obstacle to be absolutely rigid but this assumption can be relaxed in future investigations. Most of results presented here are given for the obstacle height  $h_s = 0.25h_0$ , where  $h_0$  is the initial mass height. However, we performed a few computations with bigger obstacle  $h_s = 0.6h_0$ . Interesting comparisons and discussions are presented below.

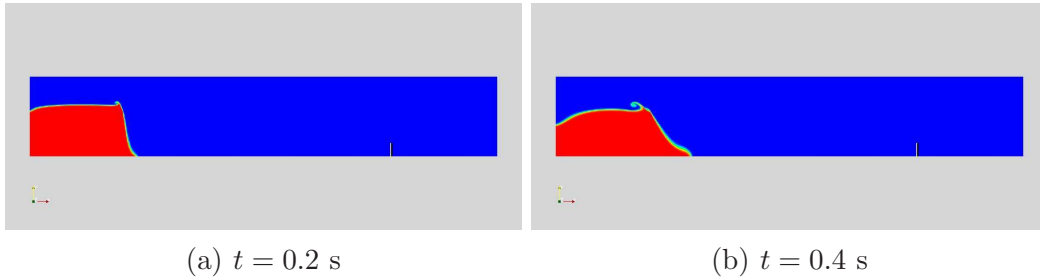


Figure 5. Sliding mass evolution just after the release. The fluid volume fraction  $\phi$  is represented. Red color corresponds to  $\phi = 1$  (snow particles) and blue to  $\phi = 0$  (ambient air).

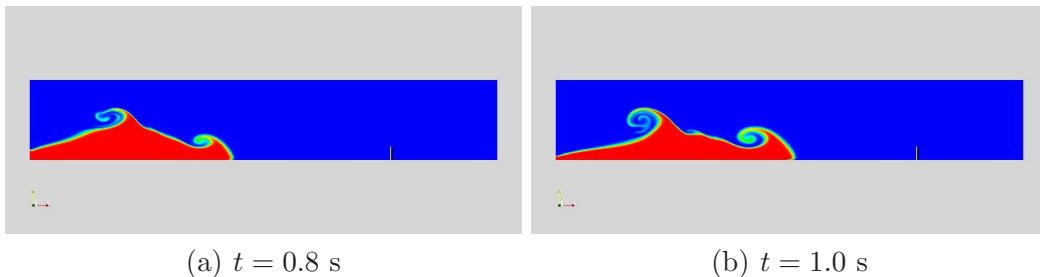


Figure 6. The avalanche formation during the sliding process.

### 3.2 Simulation results

Snapshots of the volume fraction evolution are presented on Figures 5 – 10. At the beginning, the initial rectangular mass gradually transforms into more classical elliptic form (Figures 5 – 7). Then, the sliding process continues until the interaction with the obstacle (see Figure 8). The flow deflection by is shown on Figures 9 and 10.

Our simulations clearly show that Kelvin-Helmholtz type instability [Hel68, Kel71, Cha81, DR04] develops locally during the propagation stage. For illustration, two typical roll-structures are forming on Figure 6 and they are already fully developed on Figure 7. On the other hand, the interaction process with the obstacle creates a jet directed upward. This jet has a mushroom-like shape typical for Rayleigh-Taylor instability [Ray83, Tay50, DR04]. Finally, on the last Figure 10 we see the creation of new rolling structures since the flow is deflected again by the top boundary.

Several authors pointed out an intriguing feature of the avalanche type flows [DGA01, RH04]. Namely, it was shown by radar measurements that the maximum velocity inside the avalanche exceeds the front velocity by 30% – 40%. For this purpose we visualize the velocity field magnitude during the propagation stage (see Figure 11). Qualitatively, our computations are in conformity with these experimental results. We get even more important differences between the dense core and front velocities sometimes reaching 75%.

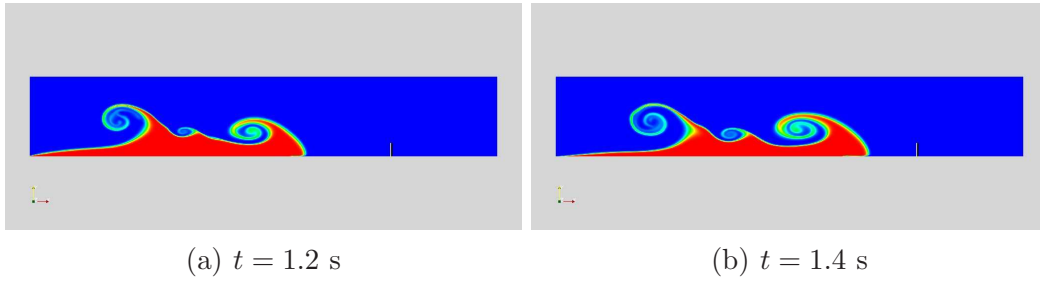


Figure 7. Fully developed gravity flow. One can already distinguish the avalanche head and the body.

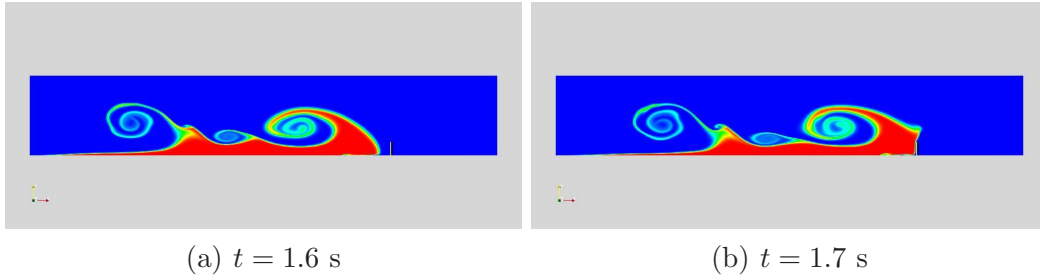


Figure 8. The avalanche just before and during the interaction with the obstacle.

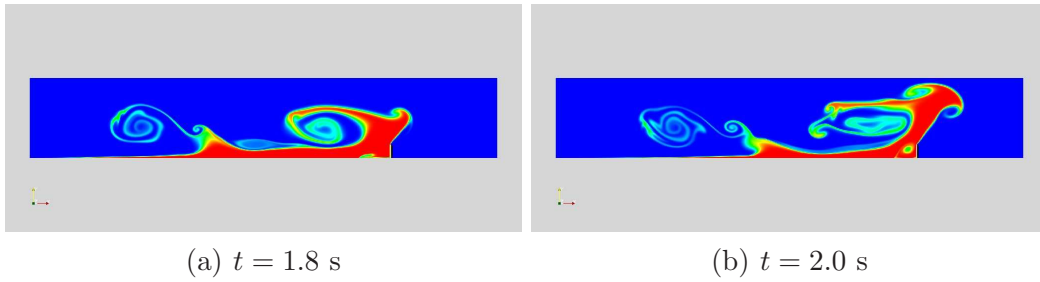


Figure 9. The flow deflection by the obstacle.

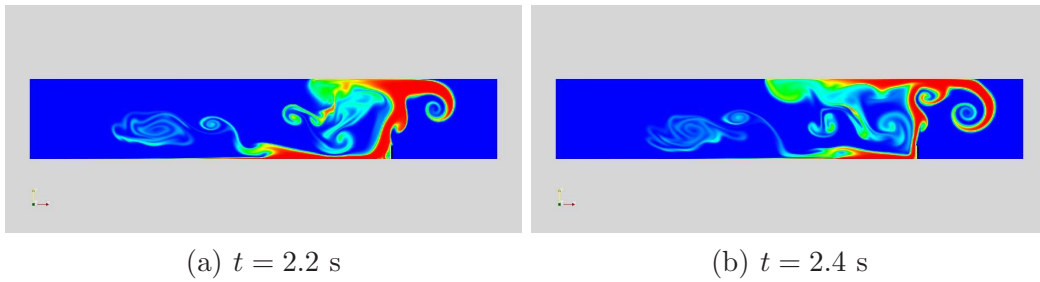


Figure 10. Interaction with the top wall.

There is another important feature of the powder-snow avalanches. In fact, the sliding mass pushes the air ahead its front. In real world events the air speed reaches 300 m/s and represents important hazard for buildings and forests even if the avalanche mass is stopped before. This destructive force cannot be computed with any single-fluid model and, thus, a two-fluid modeling is imposed. Our computations naturally demonstrate this feature since the both



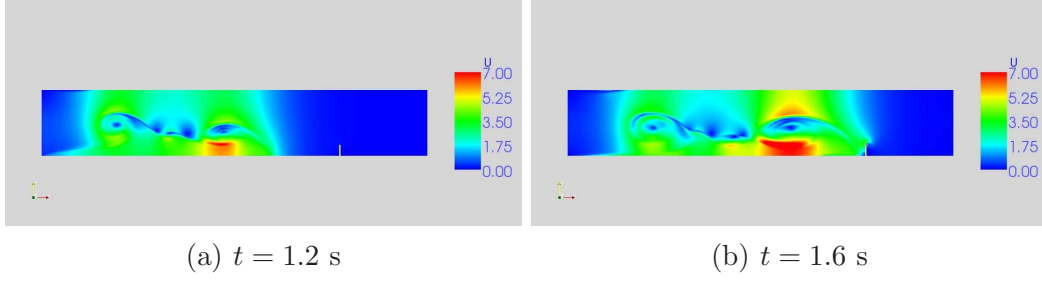


Figure 11. Velocity field magnitude. The maximum velocity inside an avalanche exceeds the speed of the front.

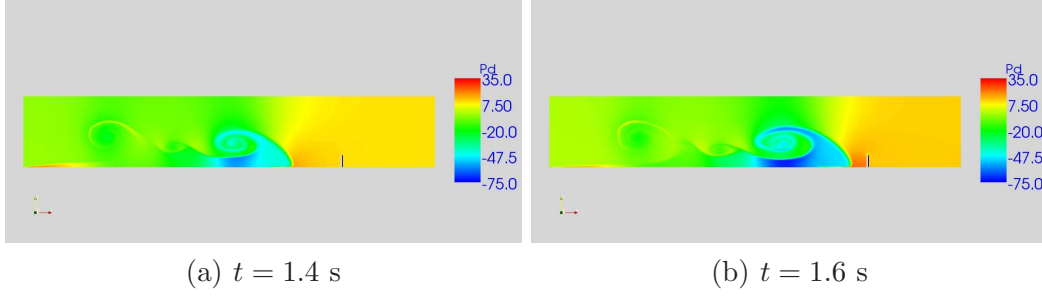


Figure 12. Dynamic pressure field during the propagation stage and just before the impact.

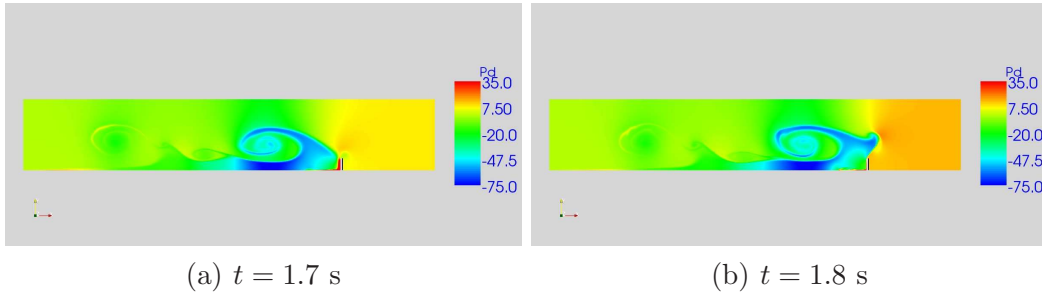


Figure 13. Dynamic pressure field during the interaction with the obstacle.

phases are resolved. An air envelope pushed by the sliding mass is illustrated on Figure 11.

The dynamic pressure field snapshots are shown on Figures 12 – 13. Recall that the dynamic pressure  $p_d$  is defined as

$$p_d := p - \rho \vec{g} \cdot \vec{r}.$$

It means, for example, that constant dynamic pressure distribution corresponds to the hydrostatic pressure field inside the fluid.

It is of big practical interest to analyze the dynamic pressures along the obstacle. Some computational results and existing engineering approaches are presented below in Section 3.3.

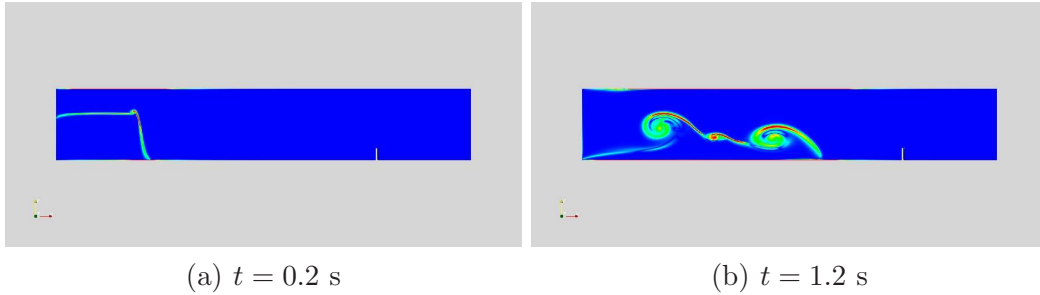


Figure 14. Vorticity field magnitude during the release and propagation stage.

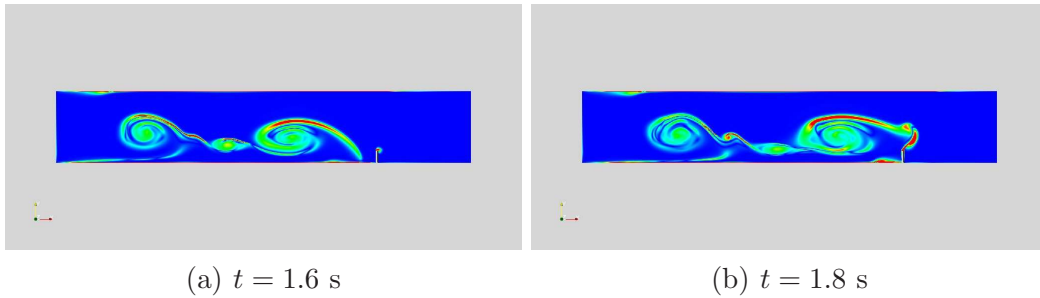


Figure 15. Vorticity field magnitude during the interaction with the obstacle.

The magnitude of the vorticity field is presented on Figures 14 – 15. From these snapshots it is clear to see that the vorticity is concentrated in regions where mixing processes take place. We would say that there is an important vorticity production across the interfaces, if the notion of an interface is appropriate in miscible fluid dynamics, of course.

During the simulation we also computed the kinetic energy according to its definition (7). We performed a little sensitivity analysis. Two different types of boundary conditions (no-slip and partial slip) and two obstacle heights were tested. As it can be seen on Figure 16, the total kinetic energy  $K$  is not very sensitive to the boundary condition type (provided that we do not have any open boundaries). The energy grows almost linearly in time during the propagation stage for all scenarios. Differences start to appear just before the interaction process with the obstacle. It is evident that bigger obstacle provides stronger attenuation of the kinetic energy. The ability to decrease the kinetic energy can be taken as a quantitative measure of a protecting structure efficiency.

**Remark 2** *The idea to use the kinetic energy loss to estimate the efficiency of a dike was already proposed by Beghin and Closet in 1990 [BC90]. However, they had very limited information on the flow structure (especially velocity and density profiles). That is why they decided to approximate this quantity by the ratio  $|U^2 - U'^2|/U^2$ . Here  $U'$  is the front velocity at certain distance below the dike and  $U$  is the front velocity of the reference avalanche measured at the same point.*

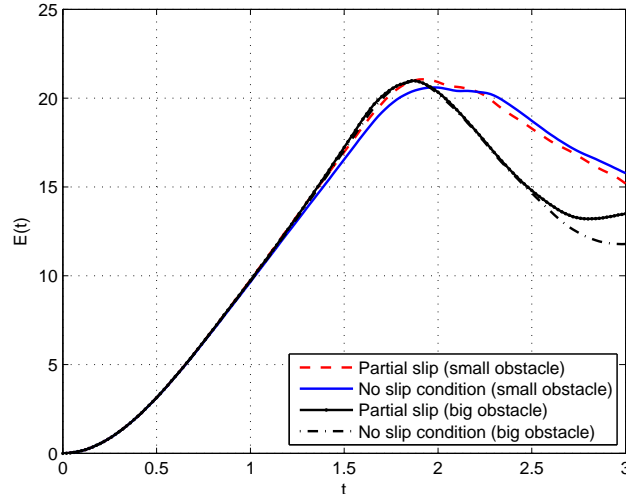


Figure 16. Kinetic energy evolution during the simulation. “Small obstacle” refers to the height  $h_s = 0.25h_0$  and “big obstacle” means  $h_s = 0.6h_0$ .

Finally, we extract from our simulations another important characterization of an avalanche flow. We speak about the front velocity  $U_f$  represented as a function of the front position  $x_f$ . It is depicted on Figure 17 for two different types of boundary conditions. This result is in accordance with earlier numerical simulations [Eti04]. However, the propagation stage is quite limited in our computations since we introduced an obstacle in the middle of the fluid domain.

### 3.3 Impact pressures

In many applications we have to estimate the loading exerted on a structure by an avalanche impact. Incidentally, the avalanche hazard level is attributed depending on the estimated impact pressure values [Lié06]. Moreover, this information is crucial for the design of buildings and other structures. In engineering practice, it is common to determine the impact pressures according to the following formula [MS93]:

$$P_d = K p_{\text{ref}} = K \bar{\rho} U_f^2 \quad (9)$$

where  $K$  is a parameter depending on the obstacle configuration,  $\bar{\rho}$  is the average avalanche density and  $U_f$  is the front velocity. For small obstacles it is advised to take  $K = 1$  and for big ones  $K = 2 \sin \alpha$ , where  $\alpha$  is the incidence angle. However, as it is pointed out in [BO91], it is difficult to estimate the maximum pressure exerted by an avalanche since we have only very limited information on the vertical structure of the flow.

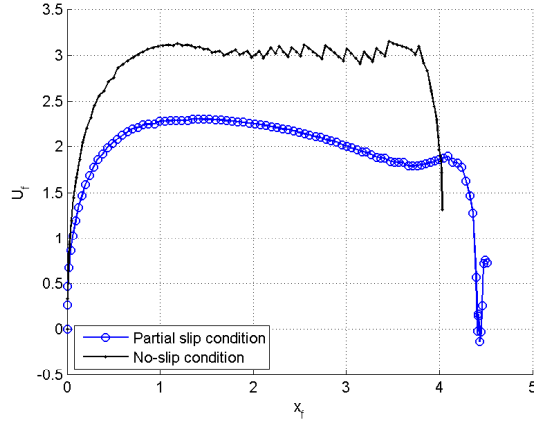


Figure 17. Avalanche front velocity as a function of the front position.

**Remark 3** *The given above formula (9) is applicable to avalanches in inertial regime. When we deal with a gravity flow regime (in Fluid Mechanics we call it the Stokes flow [HB83]), the situation is more complicated since the flow is governed by the rheology which is essentially unknown [AM04]. In this case, engineers use another expression [ABB<sup>+</sup>06]:*

$$P_d = 2\bar{\rho}g(h - z).$$

For aerosol avalanches, Beghin and Closet [BC90] proposed the following empirical law to estimate the impact pressure:

$$P_d = \frac{K}{2} K_a(z) \bar{\rho} U_f^2,$$

where  $K_a(z)$  is a dimensionless factor taking into account for the velocity variations in the upward direction. They also suggested an idealized form of

the factor  $K_a(z)$ :

$$K_a(z) = \begin{cases} 10, & z < 0.1h, \\ 19 - 90z, & 0.1h \leq z \leq 0.2h, \\ 1, & z > 0.2h, \end{cases} \quad (10)$$

where  $h$  is the impacting avalanche height. It was shown later [NB03] that this approximation underestimates the dynamic pressure in all parts of the flow.

The methodology presented in this study allows to determine the impact pressures with required accuracy. The pressure profiles at different times are presented on Figures 18 and 19. On Figure 18 we show side by side pressure profiles for the big and small obstacles. It can be seen, that the form of the vertical pressure distribution does not change drastically. It means that pre-computed pressure profiles may be scaled and reused for different obstacles within engineering accuracy [BC90, MS93].

On Figure 19 we show the comparison of impact pressures for two different boundary conditions along the bottom. Namely, we performed one run with the classical no-slip condition  $\vec{u} = \vec{0}$  and another run with the following partial slip condition:

$$\vec{u} \cdot \vec{n} = 0, \quad \left( (1 - \alpha)\vec{u} + \alpha(\mathbb{D}(\vec{u}) \cdot \vec{n}) \right) \cdot \vec{\tau} = 0, \quad (11)$$

where  $\vec{\tau}$  is the tangent vector to the boundary. We have to say that the last condition is known to be physically more relevant [Eti04].

The impact pressure is sensitive to the addition of the friction. The apparent big difference between two profiles on Figure 19 is explained by the shift in time of the impact event. However, the peak loading and the impact duration are almost the same.

Computed in the present study impact pressure profiles may be used in engineering practice. Namely, proposed earlier [BC90, Rap95] somehow idealized profiles (10) can be in principle computed for any obstacle geometry in various configurations. Once a computational tool is developed and validated, the cost of its usage is totally negligible comparing to physical experiments. It represents the major advantage of computer simulations.

## 4 Conclusions and perspectives

In the present paper we show some preliminary results on the numerical modeling of powder-snow avalanches. We compute the evolution of an avalanche

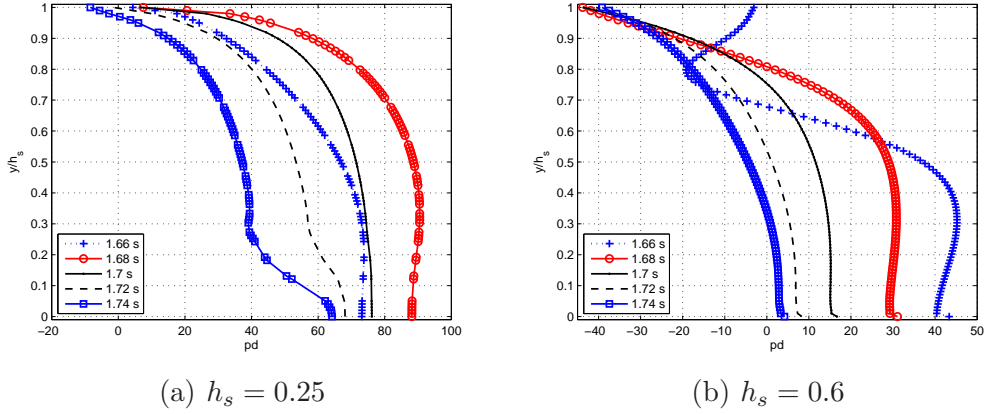


Figure 18. Dynamic pressure profiles along the obstacle at different times.

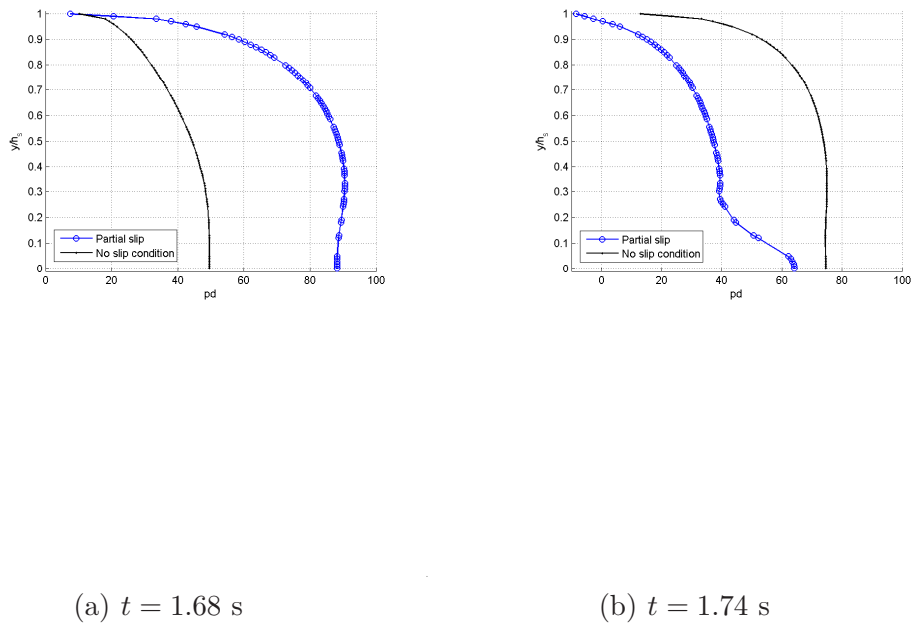


Figure 19. Comparison of impact pressures for two types of boundary conditions. Solid black line corresponds to the classical no-slip condition, while the blue line with circles refers to the partial slip condition (11).

from the beginning until hitting against the obstacle. We show impact pressure profiles (Figures 18 and 19) and kinetic energy evolution (Figure 16) for different obstacle heights. Front position and front velocities are extracted from simulations. The main goal of these computations is to prepare the tools and develop a methodology to quantify the efficiency of protecting structures.

Recall that a powder-snow avalanche front can reach the speed up to 100 m/s. Consequently, the Mach number  $Ma$  attains relatively high values:

$$Ma := \frac{u_f}{c_s} \approx 0.3,$$

where  $c_s$  is the sound speed in the air. It means that compressible effects may be important during the propagation stage. In general, impact events are followed by strong compressions. Hence, in future works we are planning to take into account the compressibility in some weak and strong senses [DDG08b, DDG08c, DDG08a, Dut07] and to perform the comparisons with the present results. Obviously, at this point a validation against experimental data is highly recommended.

Extracted impact pressures should be used to couple the present computations with a solid mechanics part [KB01]. We would like to compute deformations induced in the obstacle (later in a structure) by striking force of an avalanche. Currently we assume the obstacle to be absolutely rigid. This research axis seems to be still underexplored. However, there is already interesting experimental material on the avalanche interaction with solid obstacles [LSBH95, Pri03, PNBNF04, NB03, BR04].

The rheology of avalanches should be further investigated [AM04] and future models will take this information into account. At this stage, close collaboration between physicists and mathematicians is needed to bring the answers on challenging questions.

## Acknowledgment

The authors would like to acknowledge the University of Savoie for the PPF grant linked to the project: “Mathématiques et avalanches de neige, une rencontre possible?”. They also would like to thank Professor Carmen de JONG (head of the Institut de la Montagne - Chambéry) for interesting discussions around snow avalanches.

Finally, the authors are grateful to the developers of the freely available and open source code `OpenFOAM` which greatly facilitated their work.

## References

- [ABB<sup>+</sup>06] C. Ancey, V. Bain, E. Bardou, G. Borrel, R. Burnet, F. Jarry, O. Kolbl, and M. Meunier. *Dynamique des avalanches*. Presses polytechniques et universitaires romandes (Lausanne, Suisse), 2006. 19
- [ADGM<sup>+</sup>00] M.V. Avolio, S. Di Gregorio, F. Mantovani, A. Pasuto, R. Rongo, S. Silvano, and W. Spataro. Simulation of the 1992 Tessina landslide by a cellular automata model and future hazard scenarios.

- International Journal of Applied Earth Observation and Geoinformation*, 2:41–50, 2000. [4](#)
- [AKM90] S.N. Antonsev, A.V. Kazhikov, and V.N. Monakov. *Boundary Value Problems in Mechanics of Nonhomogeneous Fluids*. North-Holland, 1990. [9](#)
- [AM04] C. Ancey and M. Meunier. Estimating bulk rheological properties of flowing snow avalanches from field data. *J. Geophys. Res.*, 109:F01004, 2004. [19](#), [22](#)
- [Anc01] C. Ancey. *Geomorphological Fluid Mechanics*, volume 582, chapter Snow Avalanches, pages 319–338. Springer, 2001. [2](#)
- [Anc03] C. Ancey. Influence of particle entrainment from bed on the powder-snow avalanche dynamics. Technical report, Ecole Polytechnique Fédérale de Lausanne, Environmental Hydraulics Laboratory, 2003. [11](#)
- [Anc04] C. Ancey. Powder snow avalanches: Approximation as non-Boussinesq clouds with a Richardson number-dependent entrainment function. *J. Geophys. Res.*, 109:F01005, 2004. [4](#), [6](#)
- [AU99] J. Akiyama and M. Ura. Motion of 2d buoyant clouds downslope. *Journal of Hydraulic Engineering*, 125:474–480, 1999. [4](#)
- [BB83] P. Beghin and G. Brugnot. Contribution of theoretical and experimental results to powder-snow avalanche dynamics. *Cold Review Science and Technology*, 8:63–73, 1983. [4](#)
- [BC90] P. Beghin and J.-F. Closet. Effet d’une digue sur l’écoulement d’une avalanche poudreuse. Technical report, Cemagref, 1990. [17](#), [19](#), [20](#)
- [BDF<sup>+</sup>07] V.P. Boñgolan-Walsh, J. Duan, P. Fischer, T. Özgökmen, and T. Iliescu. Impact of boundary conditions on entrainment and transport in gravity currents. *Applied Mathematical Modelling*, 31:1338–1350, 2007. [5](#)
- [BE73] N.S. Bakhvalov and M.E. Eglit. Investigation of the one-dimensional motion of a snow avalanche along a flat slope. *Fluid Dynamics*, 8:683–689, 1973. [3](#), [5](#)
- [Beg79] P. Beghin. *Etude des bouffées bidimensionnelles de densité en écoulement sur pente avec application aux avalanches de neige poudreuse*. PhD thesis, Institut National Polytechnique de Grenoble, 1979. [4](#)
- [BEN02] V. Blagovechshenskiy, M. Eglit, and M. Naaim. The calibration of an avalanche mathematical model using field data. *Nat. Hazards Earth Syst. Sci.*, 2:217–220, 2002. [3](#)
- [BES02] D. Bresch, E.H. Essoufi, and M. Sy. Some new Kazhikhov-Smagulov type systems: pollutant spread and low Mach number combustion models. *C.R. Acad. Sci. Paris Ser. I*, 335:973–978, 2002. [7](#), [9](#)
- [BES07] D. Bresch, E. Essoufi, and M. Sy. Effect of density dependent viscosities on multiphasic incompressible fluid models. *J. Math.*



- Fluid Mech.*, 9:377–397, 2007. 9
- [BKK<sup>+</sup>75] N.S. Bakhvalov, A.G. Kulikovskiy, V.N. Kurkin, Y.I. Sveshnikova, and M.E. Eglit. Movement of snow avalanches. *Soviet Hydrology: Selected Papers, Rocky Mountain Station*, 4, 1975. 3, 5
- [BL98] N. Bozhinskiy and K.S. Losev. The fundamentals of avalanche science. Technical report, EISFL, Davos, 1998. 3, 4
- [BO91] P. Beghin and X. Olagne. Experimental and theoretical study of the dynamics of powder snow avalanches. *Cold Regions Science and Technology*, 19:317–326, 1991. 4, 18
- [BP81] G. Brugnot and R. Pochat. Numerical simulation study of avalanches. *Journal of Glaciology*, 27:77–88, 1981. 5
- [BR04] Ph. Berthet-Rambaud. *Structures rigides soumises aux avalanches et chutes de blocs: modélisation du comportement mécanique et caractérisation de l'interaction "phénomène-ouvrage"*. PhD thesis, Université Joseph Fourier, 2004. 22
- [Bre05a] H. Brenner. Kinematics of volume transport. *Physica A*, 349:11–59, 2005. 9
- [Bre05b] H. Brenner. Navier-Stokes revisited. *Physica A*, 349:60–132, 2005. 7, 9
- [Bre06] H. Brenner. Fluid mechanics revisited. *Physica A*, 370:190–224, 2006. 9
- [Cha81] S. Chandrasekhar. *Hydrodynamic and Hydromagnetic Stability*. Dover Publications, 1981. 14
- [Coa81] J.W. Coaz. *Die Lawinen der Schweizer Alpen*. Schmid-Franke, Bern, 1881. 2
- [CR01] M. Clément-Rastello. A study on the size of snow particles in powder-snow avalanches. *Annals of Glaciology*, 32:259–262, 2001. 11
- [DAH<sup>+</sup>07] F. Domine, M. Albert, T. Huthwelker, H.-W. Jacobi, A. A. Kokhanovsky, M. Lehning, G. Picard, and W. R. Simpson. Snow physics as relevant to snow photochemistry. *Atmos. Chem. Phys. Discuss.*, 7:5941–6036, 2007. 8, 11
- [DDG08a] F. Dias, D. Dutykh, and J.-M. Ghiadaglia. A compressible two-fluid model for the finite volume simulation of violent aerated flows. Analytical properties and numerical results. Research report, CMLA, ENS de Cachan, 2008. 8, 22
- [DDG08b] F. Dias, D. Dutykh, and J.-M. Ghiadaglia. Simulation of free surface compressible flows via a two fluid model. In *Proceedings of OMAE2008 27th International Conference on Offshore Mechanics and Arctic Engineering, June 15-20, 2008, Estoril, Portugal*, 2008. 8, 22
- [DDG08c] F. Dias, D. Dutykh, and J.-M. Ghiadaglia. A two-fluid model for violent aerated flows. *Submitted to Comput. & Fluids*, 2008. 8, 22

- [DGA01] F. Dufour, U. Gruber, and W. Ammann. Avalanches: études effectuées dans la Vallée de la Sionne en 1999. *Les Alpes*, 2:9–15, 2001. [6](#), [14](#)
- [DGBA00] F. Dufour, U. Gruber, P. Bartelt, and W.J. Ammann. Overview of the 1999 measurements at the SLF test site Vallée de la Sionne. In *Proceedings International Snow Science Workshop, Blue Sky MT, USA, October 1st-6th*, pages 527–534, 2000. [3](#)
- [DGRS<sup>+</sup>99] S. Di Gregorio, R. Rongo, C. Siciliano, M. Sorriso-Valvo, and W. Spataro. Mount Ontake landslide simulation by the cellular automata model SCIDDICA-3. *Physics and Chemistry of the Earth*, 24:97–100, 1999. [4](#)
- [DR04] P.G. Drazin and W.H. Reid. *Hydrodynamic Stability*. Cambridge University Press, 2004. [14](#)
- [DSI06] D. D’Ambrosio, W. Spataro, and G. Iovine. Parallel genetic algorithms for optimising cellular automata models of natural complex phenomena: an application to debris-flows. *Computers & Geosciences*, 32:861–875, 2006. [4](#)
- [dSV71] A.J.C. de Saint-Venant. Théorie du mouvement non-permanent des eaux, avec application aux crues des rivières et à l’introduction des marées dans leur lit. *C. R. Acad. Sc. Paris*, 73:147–154, 1871. [3](#), [4](#)
- [Dut07] D. Dutykh. *Mathematical modelling of tsunami waves*. PhD thesis, École Normale Supérieure de Cachan, 2007. [8](#), [22](#)
- [Egl83] E. M. Eglit. *Advances in the Mechanics and the Flow of Granular Materials*, chapter Some mathematical models of snow avalanches, pages 577–588. Trans Tech Publications, Clausthal-Zellerfeld, 1983. [3](#)
- [Egl91] M. Eglit. The dynamics of snow avalanches. In *Proceedings of the Steklov Institute of Mathematics*, volume 186, pages 187–193, 1991. [3](#)
- [Egl98] M. Eglit. Mathematical and physical modeling of powder-snow avalanches in Russia. *Annals of Glaciology*, 26:281–284, 1998. [3](#)
- [EHS05] J. Etienne, E.J. Hopfinger, and P. Saramito. Numerical simulations of high density ratio lock-exchange flows. *Phys. Fluids*, 17:036601, 2005. [5](#), [12](#)
- [ESH04] J. Etienne, P. Saramito, and E.J. Hopfinger. Numerical simulations of dense clouds on steep slopes: application to powder-snow avalanches. *Annals of Glaciology*, 38:379–383(5), 2004. [5](#), [10](#), [12](#)
- [Eti04] J. Etienne. *Numerical simulation of high density ratio gravity currents. Application to the avalanches*. PhD thesis, Institut National Polytechnique de Grenoble, 2004. [5](#), [12](#), [18](#), [20](#)
- [Fic55a] A. Fick. *Phil. Mag.*, 10:30, 1855. [8](#)
- [Fic55b] A. Fick. *Poggendorff’s Annel. Physik*, 94:59, 1855. [8](#)
- [FNBB<sup>+</sup>08] E.D. Fernández-Nieto, F. Bouchut, D. Bresch, M.J. Castro-Díaz, and A. Mangeney. A new Savage-Hutter type models for sub-

- marine avalanches and generated tsunamis. *J. Comput. Phys.*, 227:7720–7754, 2008. 5
- [FP90] Y. Fukushima and G. Parker. Numerical simulation of powder-snow avalanches. *Journal of Glaciology*, 36:229–237, 1990. 4
- [FV07] E. Feireisl and A. Vasseur. New perspectives in fluid dynamics: Mathematical analysis of a model proposed by Howard Brenner. *Preprint*, pages 1–31, 2007. 9
- [GEY67] S.S. Grigorian, M.E. Eglit, and Y.L. Yakimov. A new formulation and solution of the problem of a snow avalanche motion. *Trudy Vysokogornogo Geofizicheskogo Instituta*, 12:104–113, 1967. (in Russian). 3
- [GGL<sup>+</sup>01] J. Glimm, J. W. Grove, X.-L. Li, W. Oh, and D. H. Sharp. A critical analysis of Rayleigh-Taylor growth rates. *J. Comput. Phys.*, 169:652–677, 2001. 11
- [GKC01] J.-M. Ghidaglia, A. Kumbaro, and G. Le Coq. On the numerical solution to two fluid models via cell centered finite volume method. *Eur. J. Mech. B/Fluids*, 20:841–867, 2001. 7
- [GWH98] J.M.N.T. Gray, M. Wieland, and K. Hutter. Gravity-driven free surface flow of granular avalanches over complex basal topography. *Proc. R. Soc. Lond. A*, 455:1841–1874, 1998. 5
- [HB83] J. Happel and H. Brenner. *Low Reynolds number hydrodynamics*. D. Reidel Publishing Co., Hingham, MA, 1983. 19
- [Hel68] H. Helmholtz. On the discontinuous movements of fluids. *Monthly Reports of the Royal Prussian Academy of Philosophy in Berlin*, 23:215, 1868. 14
- [HG93] K. Hutter and R. Greve. Two-dimensional similarity solutions for finite-mass granular avalanches with Coulomb and viscous-type frictional resistance. *Journal of Glaciology*, 39:357–372, 1993. 5
- [Hop83] E.J. Hopfinger. Snow avalanche motion and related phenomena. *Ann. Rev. Fluid Mech.*, 15:47–76, 1983. 5
- [HTD77] E.J. Hopfinger and J.-C. Tochon-Danguy. A model study of powder-snow avalanches. *Journal of Glaciology*, 81:343–356, 1977. 4
- [Hut91] K. Hutter. Two- and three dimensional evolution of granular avalanche flow – theory and experiments revisited. *Acta Mechanica*, 1:167–181, 1991. 5
- [Hut96] K. Hutter. *Hydrology of disasters*, chapter Avalanche dynamics, pages 317–394. Kluwer Academic Publishers, 1996. 5
- [HWP05] K. Hutter, Y. Wang, and S.P. Pudasaini. The Savage-Hutter avalanche model. How far can it be pushed? *Phil. Trans. R. Soc. Lond. A*, 363:1507–1528, 2005. 5
- [ID01] F.P. Incropera and D.P. DeWitt. *Fundamentals of Heat and Mass Transfer*. Wiley; 5 edition, 2001. 11
- [Ish75] M. Ishii. *Thermo-Fluid Dynamic Theory of Two-Phase Flow*.

- Eyrolles, Paris, 1975. 7
- [Jas96] H. Jasak. *Error analysis and estimation for the finite volume method with applications to fluid flows*. PhD thesis, University of London and Imperial College, 1996. 12
- [JR93] D. D. Joseph and Y. Y. Renardy. *Fundamentals of two-fluid dynamics*. Interdisciplinary Applied Mathematics. Springer Verlag, New-York, 1993. 10
- [KB01] C.J. Keylock and M. Barbolini. Snow avalanche impact pressure - vulnerability relations for use in risk assessment. *Can. Geotech. J.*, 38(2):227–238, 2001. 22
- [KE73] A.G. Kulikovskiy and M.E. Eglit. Two-dimensional problem of the motion of a snow avalanche along a slope with smoothly changing properties. *Journal of Applied Mathematics and Mechanics*, 37:792–803, 1973. 3
- [Kel71] Lord Kelvin. Hydrokinetic solutions and observations. *Philosophical Magazine*, 42:362–377, 1871. 14
- [Kel95] S. Keller. Measurements of powder snow avalanches: Laboratory. *Surveys in Geophysics*, 16:661–670, 1995. 6
- [KS77a] A. Kazhikov and Sh. Smagulov. The correctness of boundary value problems in a diffusion problem of an homogeneous fluid. *Sov. Phys. Dokl.*, 22:249–252, 1977. 9
- [KS77b] A.G. Kulikovskiy and E.I. Sveshnikova. A model for computing a powder snow avalanche motion. *Glaciological Studies*, 31:74–80, 1977. (in Russian). 3, 4
- [LB80] K. Lied and S. Bakkehøi. Empirical calculations of snow avalanche run-out distances based on topographic parameters. *Journal of Glaciology*, 26:165–177, 1980. 4
- [Lié06] J. Liévois. *Guide méthodologique des plans de prévention des risques d’avalanches*. La Documentation Française, Paris, 2006. 18
- [LJ05] O.A. Likhachev and J.W. Jacobs. A vortex model for Richtmyer-Meshkov instability accounting for finite Atwood number. *Phys. Fluids*, 17:031704, 2005. 11
- [LSBH95] G.F. Lane-Serff, L.M. Beal, and T.D. Hadfield. Gravity current flow over obstacles. *J. Fluid Mech.*, 292:39–53, 1995. 22
- [McC00] D.M. McClung. Extreme avalanche runout in space and time. *Can. Geotech. J.*, 37:161–170, 2000. 4
- [McC01] D.M. McClung. Extreme avalanche runout: a comparison of empirical models. *Can. Geotech. J.*, 38:1254–1265, 2001. 4
- [MCVB<sup>+</sup>03] A. Mangeney-Castelnaud, J.-P. Vilotte, M.O. Bristeau, B. Perthame, F. Bouchut, C. Simeoni, and S. Yerneni. Numerical modeling of avalanches based on Saint-Venant equations using a kinetic scheme. *J. Geophys. Res.*, 108:2527, 2003. 5
- [ML87] D.M. McClung and K. Lied. Statistical and geometrical defini-

- tion of snow avalanche runout. *Cold Regions Science and Technology*, 13:107–119, 1987. [4](#)
- [Mou22] P. Mougin. *Les avalanches en Savoie IV*. Ministère de l’Agriculture, Direction Générale des Eaux et Forêts, Service des Grandes Forces Hydrauliques, Paris, 1922. [2](#)
- [Mou31] P. Mougin. *Restauration des montagnes*. Imprimerie Nationale, Paris, 1931. [2](#)
- [MS84] D. McClung and P.A. Schaerer. Determination of avalanche dynamics, friction coefficients from measured speeds. *J. Glaciology*, 20:109–120, 1984. [3](#)
- [MS93] D.M. McClung and P.A. Schaerer. *The Avalanche Handbook*. The Mountaineers, Seattle, 1993. [11](#), [18](#), [20](#)
- [NB03] F. Naaim-Bouvet. *Approche macro-structurelle des écoulements bi-phasiques turbulents de neige et de leur interaction avec des obstacles*. Habilitation à diriger les recherches, Cemagref, 2003. [6](#), [11](#), [12](#), [20](#), [22](#)
- [NBNBH02] F. Naaim-Bouvet, M. Naaim, M. Bacher, and L. Heiligenstein. Physical modelling of the interaction between powder avalanches and defence structures. *Nat. Hazards Earth Syst. Sci.*, 2:193–202, 2002. [6](#)
- [NG98] M. Naaim and I. Gurer. Two-phase numerical model of powder avalanche. Theory and application. *Natural Hazards*, 117:129–145, 1998. [5](#)
- [NMKI93] K. Nishimura, N. Maneo, K. Kawada, and K. Izumi. Structures of snow cloud in dry-snow avalanches. *Annals of Glaciology*, 18:173–178, 1993. [3](#)
- [NSKL95] K. Nishimura, F. Sandersen, K. Kristensen, and K. Lied. Measurements of powder snow avalanche – Nature. *Surveys in Geophysics*, 16:649–660, 1995. [3](#)
- [NTK90] H. Norem, K. Tronstad, and K. Kristensen. The Ryggfonn project: Avalanche data from the 1983 – 1989 winters. Technical report, Norwegian Geotechnical Institute, 1990. [3](#)
- [OFDI04] T. Özgökmen, P.J. Fischer, J. Duan, and T. Iliescu. Entrainment in bottom gravity currents over complex topography from three-dimensional nonhydrostatic simulations. *Geophys. Res. Lett.*, 31:L13212, 2004. [5](#)
- [OFDI06] T. Özgökmen, P.F. Fischer, J. Duan, and T. Iliescu. Three-dimensional turbulent bottom density currents from a high-order nonhydrostatic spectral element method. *J. Phys. Ocean.*, 34:2006–2026, 2006. [5](#)
- [OIF+07] T. Özgökmen, T. Iliescu, P.F. Fischer, A. Srinivasan, and J. Duan. Large eddy simulation of stratified mixing in two-dimensional dam-break problem in a rectangular enclosed domain. *Ocean Modelling*, 16:106–140, 2007. [5](#)
- [Ope07] OpenCFD. *OpenFOAM. The Open Source CFD Toolbox. User*

- Guide*, August 2007. [12](#)
- [PNBNF04] M. Primus, F. Naaim-Bouvet, M. Naaim, and T. Faug. Physical modeling of the interaction between mounds or deflecting dams and powder snow avalanches. *Cold Regions Science and Technology*, 39:257–267, 2004. [6](#), [12](#), [22](#)
- [Pri03] M. Primus. *Modélisation physique des interactions entre avalanches de neige poudreuse et dispositifs de protection*. Mémoire du stage de DEA mmge, Cemagref, 2003. [6](#), [12](#), [22](#)
- [Raj02a] J. Rajchenbach. Development of grain avalanches. *Phys. Rev. Lett.*, 89:074301, 2002. [4](#)
- [Raj02b] J. Rajchenbach. Dynamics of grain avalanches. *Phys. Rev. Lett.*, 88:014301, 2002. [4](#)
- [Raj05] J. Rajchenbach. Rheology of dense granular materials: steady, uniform flow and the avalanche regime. *J. Phys.: Condens. Matter*, 17:S2731–S2742, 2005. [4](#)
- [Rap95] F. Rapin. *Université Européenne d’été sur les Risques Naturels. Session 1992: Neige et avalanches*, chapter French theory for the snow avalanches with aerosol, pages 149–154. Editions du CEMAGREF, 1995. [20](#)
- [Ray83] L. Rayleigh. Investigation of the character of the equilibrium of an incompressible heavy fluid of variable density. *Proceedings of the London Mathematical Society*, 14:170–177, 1883. [14](#)
- [Rey83] O. Reynolds. An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels. *Phil. Trans. Roy. Soc.*, 174:935–982, 1883. [11](#)
- [RH04] M. Rastello and E.J. Hopfinger. Sediment-entraining suspension clouds: a model of powder-snow avalanches. *J. Fluid. Mech.*, 509:181–206, 2004. [3](#), [4](#), [14](#)
- [Rus02] H. Rusche. *Computational Fluid Dynamics of Dispersed Two-Phase Flows at High Phase Fractions*. PhD thesis, University of London and Imperial College, 2002. [12](#)
- [SH89] S.B. Savage and K. Hutter. The motion of a finite mass of granular material down a rough incline. *J. Fluid Mech.*, 199:177–215, 1989. [3](#), [5](#)
- [SH91] S.B. Savage and K. Hutter. The dynamics of avalanches of granular materials from initiation to runout, I: Analysis. *Acta Mechanica*, 86:201–233, 1991. [3](#)
- [SS03] J. Schumacher and K.R. Sreenivasan. Geometric features of the mixing of passive scalars at high schmidt numbers. *Phys. Rev. Lett.*, 91:174501, 2003. [11](#)
- [Tay50] G. Taylor. The instability of liquid surfaces when accelerated in a direction perpendicular to their planes. *Proceedings of the Royal Society of London. Series A*, 201:192–196, 1950. [14](#)
- [TK96] I. Toumi and A. Kumbaro. An approximate linearized Riemann

- solver for a two-fluid model. *J. Comput. Physics*, 124:286–300, 1996. [7](#)
- [TKP99] I. Toumi, A. Kumbaro, and H. Paillère. Approximate Riemann solvers and flux vector splitting schemes for two-phase flow. Technical report, CEA Saclay, 1999. [7](#)
- [TM08] B. Turnbull and J.N. McElwaine. Experiments on the non-Boussinesq flow of self-igniting suspension currents on a steep open slope. *J. Geophys. Res.*, 113:F01003, 2008. [6](#)
- [Voe55] A. Voellmy. Über die zerstörungskraft von lawinen. *Schweizerische Bauzeitung*, 73:159–162, 212–217, 246–249, 280–285, 1955. [2](#)
- [YXS02] P.K. Yeung, S. Xu, and K.R. Sreenivasan. Schmidt number effects on turbulent transport with uniform mean scalar gradient. *Phys. Fluids*, 14:4178–4191, 2002. [11](#)