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CONSISTENCY CHECKING WITHIN LOCAL SEARCH APPLIED TO THE FREQUENCY ASSIGNMENT WITH POLARIZATION PROBLEM

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Abstract. We present a hybrid approach for the Frequency Assignment Problem with Polarization. This problem, viewed as Max-CSP, is treated as a sequence of decision problems, *CSP* like. The proposed approach combines the Arc-Consistency techniques with a performed *Tabu Search* heuristic. The resulting algorithm gives some high quality solutions and has proved its robustness on instances with approximately a thousand variables and nearly ten thousand constraints.

Keywords. Filtering techniques, consistency checking, *Tabu Search*.

1. INTRODUCTION

The ever-increasing demand for communication, coupled with the limited number available spectra, have made frequency assignment more and more difficult to accomplish effectively. Optimization of this process has therefore become a major issue for network administration and deployment, both civil and military. The Frequency Assignment Problem with Polarization (*FAPP*) can be formalized as a Max-CSP which is known to be *NP-hard*.

Because the data problem to be optimized was very large, we decided to adopt a local search method. From among all the existing algorithms, we chose *Tabu Search*, introduced by Glover in [8]. The specific structure of the *FAPP* constraint network led us to apply local filtering techniques. Consequently, an arc-consistency procedure, AC, was embedded in the *Tabu Search* framework to reduce the search

space. Hence, the consistency checking concept occurred twice (during the resolution): firstly in the filtering pre-processing, and secondly in the kernel of the neighborhood design.

After presenting the physical and formal definitions of the *FAPP* problem in Section 2, we describe in Section 3 our general approach to solve it. Finally, a large number of experimental results are discussed and compared in Section 4, in order to highlight the advantages obtained by combining a constraint programming tool with the local search heuristic.

2. PROBLEM DEFINITION

2.1. PHYSICAL DESCRIPTION

The *FAPP* consists in finding an optimal frequency allocation in hertzian telecommunication networks. The network is composed of a set of sites in which the transmission devices (antennae connected to emitters or receptors) are located. A hertzian connection joins two geographic sites by one or more paths. A path is a uni-directional radio-electric bond, established between antennae at distinct sites, which has a given frequency and polarization.

A frequency resource is therefore a (frequency, polarization) pair in which the components are respectively associated to the carrying frequency of the transmitted signal, and the wave polarization. The polarization is simply either positive or negative. Accordingly, we define the path domain as a set of available resources. This set contains the frequency domain F_i , and the polarization information P_i , which may include a required polarization.

A frequency allocation consists in assigning a (f_i, p_i) pair for each path i which satisfies certain radio-electric compatibility constraints (1, 2, and 3), and minimal distance constraints to avoid interference (4):

1. frequencies equality or not across paths i and j : $f_i = f_j$ or $f_i \neq f_j$;
2. distance between frequencies: $|f_i - f_j| = \varepsilon_{ij}$ or $|f_i - f_j| \neq \varepsilon_{ij}$;
3. polarization equality or not across paths: $p_i = p_j$ or $p_i \neq p_j$;
4. minimal distance between frequencies: $|f_i - f_j| \geq \begin{cases} \gamma_{ij} & \text{if } p_i = p_j \\ \delta_{ij} & \text{if } p_i \neq p_j. \end{cases}$

In the constraints (4), the required distance between frequencies depends on their polarizations: the distance is obviously smaller if the polarizations are different ($\gamma_{ij} \geq \delta_{ij}$).

A feasible solution is an allocation for each path that satisfies the full set of constraints. Unfortunately, most problems do not have any such feasible solution, because the domains are too restrictive or the requirements are too numerous. The operator must therefore search for a “good quality” solution in terms of interference, rather than a feasible one. For this purpose, two constraint classes are introduced:

- CI: strong or imperative constraints (1, 2, and 3 above);
- CEM: constraints of type 4 where progressive relaxation is authorized and controlled by 11 relaxation levels. Level 0 corresponds to no relaxation. Increasing from level k to $k + 1$ involves relaxation of some or all the frequency distances defining each constraint, the maximum level being 10:

$$|f_i - f_j| \geq \begin{cases} \gamma_{ij}^0 \geq \gamma_{ij}^1 \geq \dots \geq \gamma_{ij}^k \geq \dots \geq \gamma_{ij}^{10} & \text{if } p_i = p_j \\ \delta_{ij}^0 \geq \delta_{ij}^1 \geq \dots \geq \delta_{ij}^k \geq \dots \geq \delta_{ij}^{10} & \text{if } p_i \neq p_j. \end{cases}$$

In this context, a feasible solution at level k is an allocation of each path satisfying all the strong constraints CI and all the CEM constraints at level k , noted CEM_k . Such a problem is said to be k -feasible. Assuming that a good quality solution minimizes interference, and that the smaller k is the fewer CEM constraints are relaxed, the hierarchical objective function is dependent on k : first minimizing k , secondly minimizing the number of unsatisfied CEM_{k-1} constraints, and finally minimizing the number of unsatisfied CEM constraints at the levels below $k - 1$.

2.2. FORMAL DEFINITION

From this physical description, modelling the *FAPP* as a Maximal Constraint Satisfaction Problem (Max-CSP), consists in defining the constraint network $\langle \mathcal{X}, \mathcal{D}, \mathcal{C} \rangle$, and the formal criterion to minimize f :

- associating a *variable* x_i to each path: $\mathcal{X} = \{p_i, i = 1, \dots, n\}$;
- associating to x_i a *domain* $D_i = F_i \times P_i$, where F_i is the set of the allowed frequencies for the path x_i and P_i is one of the following sets $\{-1\}$, $\{1\}$, or $\{-1, 1\}$, $\mathcal{D} = \bigcup_{i=1, \dots, n} D_i$;
- adding several imperative constraints CI between two paths x_i and x_j : $f_i = f_j$ or $f_i \neq f_j$, $|f_i - f_j| = \varepsilon_{ij}$ or $|f_i - f_j| \neq \varepsilon_{ij}$, $p_i = p_j$ or $p_i \neq p_j$;
- adding CEM constraints where progressive relaxation is authorized:

$$|f_i - f_j| \geq \begin{cases} \gamma_{ij}^0 \geq \dots \geq \gamma_{ij}^k & \text{if } p_i = p_j \\ \delta_{ij}^0 \geq \dots \geq \delta_{ij}^k & \text{if } p_i \neq p_j \end{cases}$$

where the index k , increasing from 0 to 10, indicates the relaxation level of the CEM constraints.

$\mathcal{C} = \text{CI} \cup \text{CEM}$;

- let $V_{(k)}$ be the set of unsatisfied CEM_k constraints. The hierarchical objective function to be minimized is:

$$f = \left(k, V_{(k-1)}, \sum_{i < k-1} V_{(i)} \right)$$

where k is such that the problem is k -feasible. The formal criterion f is directly related to the hierarchical optimization problem. We must therefore successively minimize k , $V_{(k-1)}$ then $\sum_{i < k-1} V_{(i)}$.

Where necessary, for simplicity, we also define:

- $C_k = CI \cup CEM_k$ the whole constraint set at level k ;
- $CI(i, j) \subset CI$ the imperative constraints involving the paths i and j ;
- $CEM_k(i, j) \subset CEM_k$ the relaxed constraints involving the paths i and j .

3. GENERAL APPROACH FOR SOLVING THE *FAPP*

Among all of the dedicated algorithms to solve frequency assignment problems [1, 4, 11], only heuristic based approaches can produce good quality solutions in reasonable computing time for realistically sized instances. The described algorithm here is also a meta-heuristic one. Although it may seem unworkable to attempt to use a pre-processing filtering operation on an optimisation problem, our *Tabu Search* procedure is reinforced by a consistency technique that reduces the search space and increases the overall efficiency of the neighborhood exploration. Indeed:

- let n be the path number of a *FAPP* instance;
- let S be the non constrained search space: S includes all the configuration vectors $s = ((f_1, p_1), \dots, (f_n, p_n))$ such as $f_i \in F_i$ and $p_i \in P_i$.

The initial size of S , equal to $\prod_{i=1}^n |F_i| \times |P_i|$, is huge, even for the smallest *FAPP* instances ($200 \leq n \leq 3000$ and $19 \leq |F_i| \leq 500$). To cope with this difficulty, we restrict the F_i domains by a filtering process.

The two expected advantages of such a reduction in domain size are: firstly to decrease the time complexity of the neighborhood evaluation, and secondly to avoid exploring wrong areas in the search space. That is why we hybridize our *Tabu Search* algorithm with an Arc-Consistent (AC) filtering process.

Another important feature of this approach, with the aim of reducing computing complexity, is that only the first component k of the objective function f is considered within the move heuristic. Consequently, the optimization process is transformed into a sequence of decision problems verifying the existence of a k -feasible configuration, where k decreases from 11 to k^* (best k found). The search for a lower value of k is started only if a $(k + 1)$ -feasible configuration has been found. Hence, we solve several CSPs rather than one Max-CSP.

To summarize, if AC is the filtering function of the frequency domains, and TABU the exploring function of the search space using the meta-heuristic *Tabu Search*, we have the general pattern described in Algorithm 1.

Algorithm 1. AC-TABU

```

begin
   $k \leftarrow 11$ 
  while AC( $k$ ) = True do
    if TABU( $k$ ) = True then
       $k \leftarrow k^* - 1$ 
end

```

This is a very straightforward iterative procedure. When either $AC(k)$ or $TABU(k)$ fails then $AC-Tabu$ returns $k^* = k + 1$ as the best k^* -feasible solution found. If $AC(k - 1)$ fails and $TABU(k)$ finds a solution, then k is the *optimal* value. Indeed, AC provides the Lower Bound of k , and $TABU$ the Upper one.

On the other hand, if $TABU(k)$ finds a feasible solution, that solution may be k^* -feasible, with $k^* < k$. This means that $TABU$ can jump more than one k -level.

3.1. FILTERING WITH AC

Arc-consistency is a widely studied topic in constraint programming [2,3,12,13]. AC eliminates variable values with no support, since such values cannot lead to a feasible solution.

As $FAPP$ constraints are binary, it is easy to check for every path i ($1 \leq i \leq n$) and for every value couple (f_i, p_i) , if for each neighbor j of i , there is a couple (f_j, p_j) such that $CI(i, j)$ and $CEM_k(i, j)$ are satisfied. This elimination process, $eliminate(F_i, P_i, k)$, is repeated until there is no change in the domain of any variable: in this case $eliminate$ returns *False*, otherwise it returns *True*. For more details about such algorithms, please refer to [2,3,12]. The main features of the following $AC(k)$ procedure are:

- managing a list of updated variable domains, in order to check, at the next iteration, only the variables for which their domains have changed;
- for each f_i and p_i values, and for each constraint $C(i, j)$, the $eliminate(F_i, P_i, k)$ function checks another value in the domain D_i as soon as it has found a pair $f_j \in D_j$ and $p_j \in P_j$ satisfying $C(i, j)$.

Algorithm 2. $AC(k)$

```

begin
  change ← True
  while change do
    change ← False
    for  $i \in [1, n]$  do
      if  $eliminate(F_i, P_i, k)$  then
        if  $F_i = \emptyset$  or  $P_i = \emptyset$  then
          return False
        change ← True
    end for
  end while
end

```

3.2. EXPLORING WITH TABU

Contrary to the random local search, where randomness is extensively used, the meta-heuristic *Tabu Search* is based on the belief that an intelligent search should include more systematic forms of guidance based on adaptive memory and learning.

Designing a *good Tabu Search* requires that its main characteristics should be well defined: the effective search space and therefore the definition of the visited configurations, the neighborhood structure, the move heuristic, and the tabu list management.

Algorithm 3. TABU(k)

```

begin
   $s \leftarrow \vec{0}$                                 % i.e. any path is allocated:  $|s| = 0$ 
   $s^* \leftarrow s$                                % the best configuration found so far
  while stop-criterion = False do
     $s_{\max} \leftarrow \vec{0}$                        % the best neighbor of  $s$ 
    for  $s' \in \mathcal{N}(s)$  do
      if  $\Delta(s, s') > 0 \vee \text{move}(s, s')$  is not tabu then
        if  $\Delta(s, s') > \Delta(s, s_{\max})$  then
           $s_{\max} \leftarrow s'$ 
        if  $s_{\max} \neq \vec{0}$  then
           $s \leftarrow s_{\max}$                        % move( $s, s_{\max}$ )
          if  $|s| > |s^*|$  then
             $s^* \leftarrow s$ 
            if  $|s| = n$  then
              return True
          update tabu list
        else
          stop-criterion  $\leftarrow$  True
    return False
end

```

In our approach, the search space is defined over *partial configurations* expressed by $s = (v_1, v_2, \dots, v_{n_i})$, where $v_i = (f_i, p_i)$ and n_i variables are instantiated (with $n_i = |s|$). To conform with the standard representation by n -ary vectors, we add a new value u (for *uninstantiated*) to each domain D_i , for indicating that x_i is free. Accordingly, the evaluation criterion of a configuration is the number of instantiated variables in s .

This improvement enabled us to work on a *consistent neighborhood* $\mathcal{N}(s)$. At each level k , the visited configurations in $\mathcal{N}(s)$ respect the CI and the CEM $_k$ constraints. In order to build this original neighborhood, a *move*(s, s'), replacing the current configuration s by $s' \in \mathcal{N}(s)$, is achieved in two steps: an instantiation of a free variable (*i.e.* set to u), followed by consistent reparations, which are simply deinstantiations of the conflicting variables with respect to the C_k constraints. Consequently, the selected move must improve the configuration (*i.e.* increase $|s|$). Hence, the evaluation *heuristic* of moves from s to $s' \in \mathcal{N}(s)$, is simply $\Delta(s, s') = |s'| - |s|$. Note that the Δ evaluation is carried out very quickly using efficient incremental techniques, which are now well known [5, 6, 15, 16].

The tabu list is needed to prevent cycling, which notably occurs when we attempt to instantiate the last free variables. To avoid undoing the recent instantiation (x_i, v_i) , we penalize all the conflicting pairs (x_j, v_j) where x_j are the neighbors of x_i in the constraint network, regarding the $CI(i, j)$ and the $CEM_k(i, j)$. In this way, we maintain a table counting the number of times a resource v_l is assigned to a path x_i ($freq[i][l]++$). So, the *tabu tenure* is a dynamic function based on the flip frequencies: $tabu[i][l] = freq[i][l] + iter$, where *iter* is the current iteration number.

The *stop-criterion* is either if a solution is found, or if computing time has elapsed.

With the specifications, we thus present the general TABU algorithm in 3.

3.3. DIVERSIFICATION

The aim of the diversification phase is to enable TABU to escape from attractive zones of the search space. For this purpose, we introduce penalties in the move heuristic. More precisely, each time a configuration s such as $\forall s' \in N(s), |s'| \leq |s|$ is reached, we add a penalty to all the allocated paths having an unallocated neighbor in the constraint graph. This penalty value is then included in the move heuristic during the diversification phase. Hence, the line [1] in Algorithm 3 is replaced by the following code, where $nogood_{min}$ is the threshold value, and *penalty* is the effective penalty value of the move.

```

if ( $\Delta(s, s') > \Delta(s, s_{max})$ )  $\vee$  ( $\Delta(s, s') = \Delta(s, s_{max}) \wedge penalty < nogood_{min}$ ) then
  |  $s_{max} \leftarrow s'$ 
  |  $nogood_{min} \leftarrow penalty$ 

```

4. NUMERICAL EXPERIMENTATION

The algorithms were coded in C programming language. The running tests were carried out on an NT PC station with a Pentium III 600 MHz cpu. The results are presented in four parts. The first describes the reduction of the search space by the filtering step. The second describes the best qualitative results obtained by the hybrid approach *AC-Tabu*. The next section then compares the results produced by our TABU and the hybrid *AC-Tabu* algorithm. This section ends with a comparison between several methods proposed for the ROADEF-2001 challenge. More information about this competition is available on the web site <http://www.prism.uvsq.fr/~vdc/ROADEF/CHALLENGES/2001/>

4.1. CUTTING FREQUENCY DOMAINS

This first table shows some characteristics of the **fapp** instances, their size is given in the second part of their name: up to **3000** variables (paths). These

TABLE 1. Search Space reduction.

| fapp | IDS | FDS | $\Delta\%$ | k_0 | sec. | fapp | IDS | FDS | $\Delta\%$ | k_0 | sec. |
|---------|---------|---------|------------|-------|------|---------|-----------|--------|------------|-------|------|
| 01_0200 | 26 963 | 12 712 | 52.85 | 2 | 0 | 16_0260 | 47 293 | 46 622 | 1.42 | 10 | 0 |
| 02_0250 | 36 618 | 14 759 | 59.69 | 1 | 2 | 17_0300 | 64 034 | 918 | 98.57 | 3 | 0 |
| 03_0300 | 53 536 | 28 212 | 47.30 | 6 | 1 | 18_0350 | 73 016 | 1089 | 98.51 | 7 | 0 |
| 04_0300 | 61 762 | 21 962 | 64.44 | 0 | 4 | 19_0350 | 201 074 | 3414 | 98.30 | 5 | 6 |
| 05_0350 | 79 311 | 54 177 | 31.69 | 7 | 2 | 20_0420 | 87 077 | 1886 | 97.83 | 9 | 0 |
| 06_0500 | 108 024 | 53 034 | 50.91 | 4 | 6 | 21_0500 | 113 594 | 7745 | 93.18 | 3 | 1 |
| 07_0600 | 109 658 | 69 952 | 36.21 | 8 | 2 | 22_1750 | 813 037 | 10 656 | 98.69 | 6 | 25 |
| 08_0700 | 134 020 | 81 933 | 38.87 | 4 | 5 | 23_1800 | 455 735 | 3265 | 99.28 | 8 | 9 |
| 09_0800 | 121 824 | 52 948 | 56.54 | 2 | 6 | 24_2000 | 567 396 | 8328 | 98.53 | 6 | 4 |
| 10_0900 | 197 665 | 122 050 | 38.25 | 5 | 8 | 25_2230 | 610 084 | 18 867 | 96.91 | 2 | 6 |
| 11_1000 | 294 634 | 152 727 | 48.16 | 7 | 15 | 26_2300 | 635 123 | 14 217 | 97.76 | 6 | 4 |
| 12_1500 | 436 967 | 164 613 | 62.33 | 1 | 70 | 27_2550 | 588188 | 93 768 | 84.06 | 4 | 8 |
| 13_2000 | 320 494 | 144 873 | 54.80 | 2 | 21 | 28_2800 | 2 087 947 | 63 597 | 96.95 | 2 | 66 |
| 14_2500 | 774 322 | 320 458 | 58.61 | 3 | 92 | 29_2900 | 1 477 634 | 6435 | 99.56 | 5 | 23 |
| 15_3000 | 515 606 | 306 127 | 40.63 | 4 | 24 | 30_3000 | 1 942 250 | 80 703 | 95.84 | 6 | 103 |

problems contain up to **2 087 947** total domain values, and up to **67 898** binary constraints.

Table 1 gives the *Initial Domain Size* ($IDS = \sum_1^n |D_i^0|$) and the *Filtered* one ($FDS = \sum_1^n |D_i^f|$). Columns k_0 indicate the highest unfeasible level encountered by the *AC-Tabu* procedure, and columns *sec.* give the computing time in seconds for the whole iterative filtering process. This table shows that no more than two minutes are required by *AC-Tabu* to reduce the domains of the largest instances of this benchmark.

Apart from the **fapp16_0260** problem, these benchmarks can be divided into two subsets. The first one (*from 01 to 15*) contains the instances where the domain size is reduced by nearly half. In the second one (*from 17 to 30*), domains are reduced more than 90%.

4.2. HYBRID APPROACH RESULTS

This part begins with the results obtained by the hybrid approach after 1 computing hour. Only one run (with the 0 random seed) was carried out in this experiment.

For each instance, the Table 2 specifies:

- k , the lowest level where a solution is found. This value is underlined when *AC-Tabu* proves its optimality;
- V_{k-1} , the number of unsatisfied CEM constraints at level $k - 1$;
- $\sum V_{k-2}$, the sum of the unsatisfied CEM constraints under the $k - 1$ level;
- t_1 , the elapsed time in seconds required to reach the best value of k ;
- and t_2 , the elapsed time required to obtain the best configuration (considering the three components of the objective function).

TABLE 2. *AC-Tabu* results after 1 computing hour.

| fapp | k | V_{k-1} | $\sum V_{k-2}$ | t_1 | t_2 | fapp | k | V_{k-1} | $\sum V_{k-2}$ | t_1 | t_2 |
|---------|----------|-----------|----------------|-------|-------|---------|-----------|-----------|----------------|-------|-------|
| 01_0200 | 4 | 4 | 203 | 2 | 1839 | 16_0260 | <u>11</u> | 382 | 3864 | 0 | 2976 |
| 02_0250 | <u>2</u> | 16 | 173 | 19 | 19 | 17_0300 | <u>4</u> | 4 | 36 | 1 | 1 |
| 03_0300 | <u>7</u> | 28 | 835 | 30 | 34 | 18_0350 | <u>8</u> | 4 | 55 | 1 | 1 |
| 04_0300 | <u>1</u> | 97 | 0 | 26 | 2595 | 19_0350 | <u>6</u> | 3 | 66 | 16 | 16 |
| 05_0350 | 11 | 1 | 1836 | 1 | 3571 | 20_0420 | <u>10</u> | 6 | 133 | 2 | 2 |
| 06_0500 | <u>5</u> | 30 | 764 | 232 | 233 | 21_0500 | <u>4</u> | 2 | 12 | 2 | 2 |
| 07_0600 | <u>9</u> | 85 | 3039 | 107 | 433 | 22_1750 | <u>7</u> | 22 | 383 | 76 | 76 |
| 08_0700 | <u>5</u> | 67 | 1131 | 21 | 35 | 23_1800 | <u>9</u> | 16 | 189 | 31 | 31 |
| 09_0800 | <u>3</u> | 71 | 707 | 1789 | 1796 | 24_2000 | <u>7</u> | 9 | 91 | 17 | 17 |
| 10_0900 | <u>6</u> | 74 | 1877 | 174 | 372 | 25_2230 | <u>3</u> | 7 | 33 | 19 | 19 |
| 11_1000 | <u>8</u> | 105 | 4278 | 63 | 139 | 26_2300 | <u>7</u> | 8 | 75 | 18 | 18 |
| 12_1500 | 4 | 87 | 2138 | 2873 | 2873 | 27_2550 | <u>5</u> | 9 | 46 | 21 | 21 |
| 13_2000 | 5 | 126 | 3469 | 1020 | 1020 | 28_2800 | <u>3</u> | 38 | 125 | 165 | 203 |
| 14_2500 | 6 | 211 | 6302 | 1875 | 2950 | 29_2900 | <u>6</u> | 25 | 280 | 86 | 86 |
| 15_3000 | <u>5</u> | 198 | 4770 | 2534 | 2807 | 30_3000 | <u>7</u> | 36 | 798 | 268 | 275 |

TABLE 3. Improved results after 3 hours of computing.

| fapp | k | V_{k-1} | $\sum V_{k-2}$ | t_1 | t_2 |
|---------|-----------|-----------|----------------|--------|--------|
| 01_0200 | 4 | 4 | 139 | 2 | 10 194 |
| 05_0350 | 11 | 1 | 1683 | 1 | 5446 |
| 12_1500 | <u>2</u> | 57 | 1263 | 5384 | 5388 |
| 13_2000 | <u>3</u> | 165 | 2253 | 7345 | 7686 |
| 14_2500 | 5 | 168 | 4700 | 10 450 | 10 450 |
| 16_0260 | <u>11</u> | 358 | 3656 | 0 | 8543 |

Regarding the feasibility level, most of the k are proved optimal (25 of the 30 instances). Obviously, the instances for which no k^* is found are those belonging to the first subset described above. Now, we shall see what happens if we give more cpu time to the hybrid algorithm process.

Table 3 shows only the improved solutions reached after 3 computing hours. Unfortunately, no improvement has been obtained for the majority of the instances. Nevertheless, we can see again that it only concerns problems belonging to the first subset of **fapp** instances. Elsewhere, two new optimal values of k are proved (for the 12_1500 and the 13_2000 problems).

4.3. TABU *versus* AC-TABU

This section compares the behaviors of TABU alone and the hybrid *AC-Tabu* algorithm. These algorithms were executed 8 times with seed values varying from 0 to 7, and one hour of computing time.

Table 4 focuses on the k -feasibility level and gives the best (k_{\min}) and the worst (k_{\max}) solutions found by the TABU and hybrid *AC-Tabu* processes.

TABLE 4. Best and worst results for the level of feasibility.

| fapp | AC-Tabu | | TABU | | fapp | AC-Tabu | | TABU | |
|---------|------------|------------|------------|------------|---------|------------|------------|------------|------------|
| | k_{\min} | k_{\max} | k_{\min} | k_{\max} | | k_{\min} | k_{\max} | k_{\min} | k_{\max} |
| 01_0200 | 4 | 4 | 4 | 4 | 16_0260 | 11 | 11 | 11 | 11 |
| 02_0250 | 2 | 2 | 2 | 3 | 17_0300 | 4 | 4 | 4 | 4 |
| 03_0300 | 7 | 7 | 7 | 7 | 18_0350 | 8 | 8 | 8 | 8 |
| 04_0300 | 1 | 1 | 1 | 11 | 19_0350 | 6 | 6 | 6 | 9 |
| 05_0350 | 11 | 11 | 11 | 11 | 20_0420 | 10 | 10 | 10 | 10 |
| 06_0500 | 5 | 5 | 5 | 6 | 21_0500 | 4 | 4 | 4 | 5 |
| 07_0600 | 9 | 10 | 9 | 10 | 22_1750 | 7 | 7 | 7 | 11 |
| 08_0700 | 5 | 6 | 5 | 6 | 23_1800 | 9 | 9 | 9 | 11 |
| 09_0800 | 3 | 4 | 3 | 8 | 24_2000 | 7 | 7 | 7 | 11 |
| 10_0900 | 6 | 7 | 6 | 11 | 25_2230 | 3 | 3 | 10 | 11 |
| 11_1000 | 8 | 9 | 8 | 11 | 26_2300 | 7 | 7 | 7 | 10 |
| 12_1500 | 4 | 7 | 6 | 8 | 27_2550 | 5 | 5 | 11 | 11 |
| 13_2000 | 5 | 6 | 6 | 6 | 28_2800 | 3 | 3 | 9 | 11 |
| 14_2500 | 6 | 6 | 7 | 11 | 29_2900 | 6 | 6 | 8 | 12 |
| 15_3000 | 5 | 6 | 7 | 11 | 30_3000 | 7 | 7 | 11 | 11 |

The gap between k_{\min} and k_{\max} is higher for TABU, than for *AC-Tabu*, notably on the second subset of the benchmark. In conclusion, the behavior of *AC-Tabu* is more stable than that of TABU alone.

Note that the results of **fapp12_1500**, **fapp13_2000** and **fapp14_2500** do not contradict those from Table 3: it simply means that *AC-Tabu* finds a better solution in 3 hours with 0 random seed than in 8×1 hour with different random seeds.

4.4. COMPARISON WITH OTHERS METHODS

We will now present the results obtained during the ROADEF-2001 challenge, which took place in FRANCORO III, in the city of Québec (Canada).

The three compared algorithms (apart from ours) were developed by Caseau (MH+PPC), the Bisailon team (Tabu), and the Michelin junior team (LNS+PPC).

The (MH+PPC) algorithm combines some constraint propagation with meta-heuristics. More precisely, the algorithm increases the level, and at each one it shaves by strong consistency, shuffles by a Large Neighborhood Search (LNS) [14], which gives a non-feasible solution, and finally tries to obtain a feasible solution by a Limited Discrepancy Search (LDS) [10], such as a Variable Neighborhood local search (VNS) [9].

The second approach (Tabu) [7] is a local search approach based on *Tabu Search* that includes some original features, including a specialized neighborhood, heuristics to determine critical variables and values, different diversification techniques, an auto-adaptative mechanism to set the tabu list and finally, a pre-processing operation based on consistency techniques inherited from constraint programming. It is used to treat three different problems:

- Constraint Satisfaction Problem to find a solution at level k ;
- the problem which satisfies all the constraints at level $k + 1$, and as much as possible at level k ;
- the problem which satisfies all the constraints at level $k + 1$, and minimizes the objective function.

The Michelon junior team solves the *FAPP* by using a meta-heuristic based on a Large Neighborhood Search [14] and Constraint Propagation. Also, they relax certain constraints to obtain a Maximal Cover Tree. Indeed, their algorithm works in three phases: first it computes a lower bound of k , then it searches for a solution at this level k , and finally it improves this solution with respect to the two last points of the objective function. So as to compare several methods, in terms of quality, we have just given the three values of the objective function: k (underlined when the method proves optimality), the unsatisfied constraint number at level $k - 1$, and the sum of the unsatisfied constraints at the levels below $k - 1$.

The *AC-Tabu* column gives the results obtained by our method during the challenge. Note that they have been improved since the competition. Indeed, we have polished the code, and experiments are made on different computers. More details about the challenge, benchmarks and all the results can be found on the web site <http://www.prism.uvsq.fr/~vdc/ROADEF/CHALLENGES/2001/>

The main test condition is a computing time limited to one hour on a Pentium III, 500 Mhz, 128 Mo.

Regarding only the level k , MH+PPC found the optimal k 26 times, Tabu 27 times, LNS+PPC 27 times, and *AC-Tabu* 28 times. It seems that there is a problem with the LNS+PPC method, because in 4 instances, it could not find a solution ($k = 12$) and in several others it found no solution better than 11.

Regarding the two other components of the objective function, Tabu without AC gave the best results 27 times, although it could not prove optimality. In fact, the time spent on the PPC techniques in the other methods, was spent to improve the full objective function.

This method comparison reveals that the most difficult instances of the benchmark were 12_1500, 13_2000 and 14_2500.

To conclude, Table 5 highlights the effectiveness of approximate methods combined with some arc-consistent processes on very large instances.

TABLE 5. Comparison with three other methods.

| fapp | MH+PPC | | | Tabu | | | LNS+PPC | | | AC-Tabu | | |
|---------|-----------|-----------|----------------|------|-----------|----------------|-----------|-----------|----------------|-----------|-----------|----------------|
| | k | V_{k-1} | $\sum V_{k-2}$ | k | V_{k-1} | $\sum V_{k-2}$ | k | V_{k-1} | $\sum V_{k-2}$ | k | V_{k-1} | $\sum V_{k-2}$ |
| 01_0200 | <u>4</u> | 6 | 279 | 4 | 4 | 56 | <u>4</u> | 5 | 210 | 4 | 14 | 233 |
| 02_0250 | <u>2</u> | 18 | 248 | 2 | 7 | 86 | 11 | 1 | 435 | <u>2</u> | 20 | 195 |
| 03_0300 | <u>7</u> | 27 | 1076 | 7 | 10 | 341 | 11 | 1 | 1211 | <u>7</u> | 32 | 892 |
| 04_0300 | <u>1</u> | 164 | 0 | 1 | 31 | 0 | 2 | 1 | 282 | <u>1</u> | 184 | 0 |
| 05_0350 | <u>11</u> | 892 | 12364 | 11 | 1 | 372 | 11 | 97 | 3459 | 11 | 364 | 5694 |
| 06_0500 | <u>5</u> | 53 | 1029 | 5 | 12 | 246 | 6 | 1 | 1086 | <u>5</u> | 31 | 811 |
| 07_0600 | <u>9</u> | 132 | 4419 | 9 | 22 | 714 | 12 | - | - | <u>9</u> | 106 | 3375 |
| 08_0700 | <u>5</u> | 53 | 1359 | 5 | 16 | 266 | 11 | 3 | 2144 | <u>5</u> | 73 | 1225 |
| 09_0800 | <u>3</u> | 63 | 937 | 3 | 28 | 195 | 4 | 2 | 999 | <u>3</u> | 104 | 846 |
| 10_0900 | <u>6</u> | 82 | 2365 | 6 | 18 | 475 | 11 | 4 | 3661 | <u>6</u> | 103 | 2003 |
| 11_1000 | <u>8</u> | 119 | 5206 | 8 | 8 | 1015 | 11 | 8 | 6146 | <u>8</u> | 119 | 4191 |
| 12_1500 | <u>7</u> | 180 | 6538 | 3 | 83 | 1698 | 11 | 647 | 13797 | <u>2</u> | 62 | 1310 |
| 13_2000 | 7 | 229 | 7503 | 3 | 49 | 2003 | 11 | 671 | 15145 | 5 | 132 | 3645 |
| 14_2500 | 8 | 18 | 10661 | 4 | 35 | 3485 | 11 | 1209 | 24751 | 5 | 217 | 5045 |
| 15_3000 | 7 | 333 | 9988 | 5 | 15 | 1569 | 11 | 1060 | 22898 | <u>5</u> | 192 | 4727 |
| 16_0260 | <u>11</u> | 572 | 5779 | 11 | 5 | 56 | <u>11</u> | 590 | 5968 | <u>11</u> | 514 | 5189 |
| 17_0300 | <u>4</u> | 4 | 36 | 4 | 4 | 34 | <u>4</u> | 4 | 36 | <u>4</u> | 4 | 36 |
| 18_0350 | <u>8</u> | 4 | 55 | 8 | 4 | 55 | <u>8</u> | 4 | 57 | <u>8</u> | 4 | 59 |
| 19_0350 | <u>6</u> | 3 | 79 | 6 | 2 | 51 | <u>6</u> | 2 | 60 | <u>6</u> | 3 | 70 |
| 20_0420 | <u>10</u> | 6 | 145 | 10 | 5 | 97 | <u>10</u> | 5 | 106 | <u>10</u> | 7 | 142 |
| 21_0500 | <u>4</u> | 2 | 12 | 4 | 2 | 10 | <u>4</u> | 2 | 12 | <u>4</u> | 2 | 12 |
| 22_1750 | <u>7</u> | 16 | 356 | 7 | 15 | 187 | <u>7</u> | 15 | 292 | <u>7</u> | 25 | 503 |
| 23_1800 | <u>9</u> | 17 | 197 | 9 | 16 | 187 | 12 | - | - | <u>9</u> | 17 | 197 |
| 24_2000 | <u>7</u> | 7 | 90 | 7 | 6 | 71 | <u>7</u> | 6 | 77 | <u>7</u> | 9 | 91 |
| 25_2230 | <u>3</u> | 7 | 33 | 3 | 7 | 32 | <u>3</u> | 7 | 34 | <u>3</u> | 7 | 33 |
| 26_2300 | <u>7</u> | 10 | 81 | 7 | 9 | 74 | <u>7</u> | 9 | 75 | <u>7</u> | 10 | 86 |
| 27_2550 | <u>5</u> | 7 | 46 | 11 | 4 | 64 | <u>5</u> | 4 | 22 | <u>5</u> | 11 | 54 |
| 28_2800 | <u>3</u> | 32 | 129 | 3 | 13 | 32 | <u>3</u> | 14 | 72 | <u>3</u> | 42 | 142 |
| 29_2900 | <u>6</u> | 28 | 351 | 6 | 25 | 239 | 12 | - | - | <u>6</u> | 25 | 310 |
| 30_3000 | <u>7</u> | 17 | 602 | 11 | 1166 | 12029 | 12 | - | - | <u>7</u> | 48 | 1045 |

5. CONCLUSION

In this paper we have presented a hybrid approach to solve the Frequency Assignment Problem with Polarization. *AC-Tabu* combines some arc-consistent techniques and an original *Tabu Search* process, with a large neighborhood exploration by means of maintaining consistency. It produces very good results on large-sized *FAPP* instances.

TABU alone proves its own worth. Indeed, it provides most of the best results but at the same time, some of the worst ones. There are two main reasons that justify the filtering step: the behavior regularity of the hybrid search algorithm, and the proven optimality of the k -feasibility level, for most of the *FAPP* instances.

Some improvements to this approach are still to be expected. Two perspectives for future work can be identified. Firstly, making AC faster by studying the most recent AC algorithms [3] and adapting to the *FAPP* constraint semantic. Secondly, considering the two other objective function components V_{k-1} and $\sum_{i < k-1} V_i$ in the TABU procedure.

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