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Optimal trajectory determination and tracking

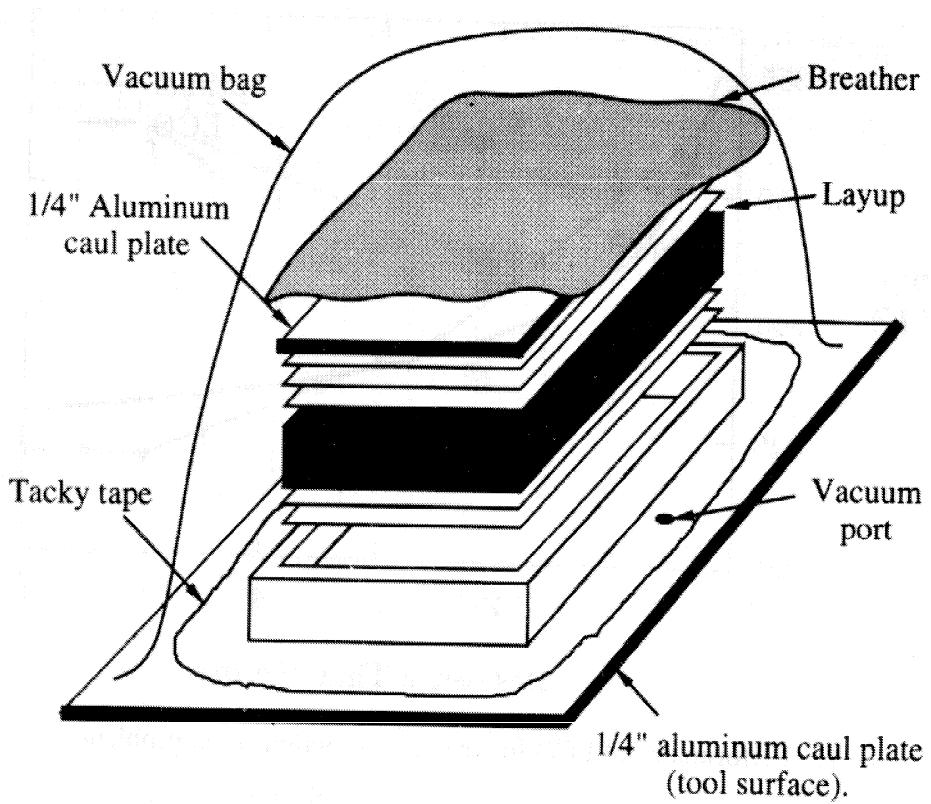
of an autoclave curing process:

a model based approach

presented by : P. DUFOUR (LAGEP)

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Autoclave curing process



Problem statement

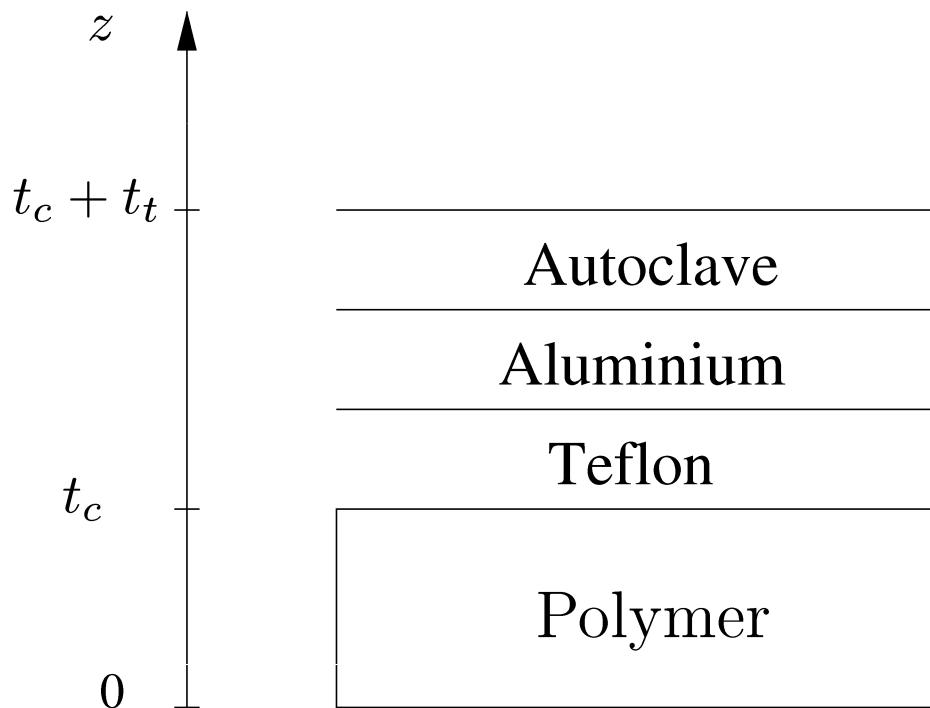
Find the optimal boundary control trajectory in order to :

- complete the cure in minimum time,
- limit temperature gradients (functional constraints),
- ensure an inside out cure (optimal behaviour).

Existential approaches

- Pontryagin's maximum principle [Nagendra and Loos, 1988].
- Expert control [LeClair and Abrams, 1988].
- Heuristic approach with a predictive strategy [Pillai and al, 1996].

Spatial domains



Nonlinear DPS with boundary control

$$(S_{NL}) \left\{ \begin{array}{l} \rho_t c_{p_t} \dot{T} = k_t \frac{\partial^2 T}{\partial z^2} \quad (\text{Center and top parts}) \\ \rho_c c_{p_c} \dot{T} = k_c \frac{\partial^2 T}{\partial z^2} + \rho_c \Delta H_r A e^{-\frac{\Delta E_a}{RT}} \alpha^m (1-\alpha)^n \\ \dot{\alpha} = A e^{-\frac{\Delta E_a}{RT}} \alpha^m (1-\alpha)^n \end{array} \right\} \quad (\text{Center part})$$

with the initial conditions :

$$T(z, 0) = T_0 \quad (\text{Center and top parts})$$

$$\alpha(z, 0) = 0 \quad (\text{Center part})$$

with the two boundary conditions :

$$T(t_c + t_t, t) = T_a(t)$$

$$\frac{\partial T}{\partial z}(0, t) = 0$$

Model based Predictive Control strategy :

constrained optimization problem.

$$\left\{ \begin{array}{l} \tilde{p} = (T_a(k) \dots T_a(k + N_c - 1))^T \\ \min_{\tilde{p}} J(\tilde{p}) = \frac{1}{2} \sum_{j=k+1}^{j=k+N_p} \frac{1}{||\dot{\alpha}(0,j)||^2} \\ c_i(\tilde{p}) \leq 0 \quad (i \in I = \{1 \rightarrow nco\}) \quad \text{ex: } T(t_c, t) - T(0, t) \leq \Delta T(t) \\ N_c : \text{control horizon,} \\ N_p : \text{prediction horizon,} \\ nco : \text{number of constraints.} \end{array} \right.$$

Unconstrained resolution method

↪ Transformation law :

Searching for the **constrained variable** T_a such that :

$$T_{a.\min} \leq T_a \leq T_{a.\max} \quad (1)$$

becomes searching for the **unconstrained variable** θ_a such that :

$$T_a = \frac{T_{a.\max} + T_{a.\min}}{2} + \frac{T_{a.\max} - T_{a.\min}}{2} * \tanh(\theta_a) \quad (2)$$

↪ Exterior penalty method :

From the constrained problem :

$$\min_x J(x) \text{ with : } c_i(x) \leq 0 \quad (i \in I = \{1 \rightarrow nco\}) \quad (3)$$

we introduce an **exterior penalty term** J_{ext} :

$$J_{ext}(c_i(x)) = \sum_{i=1}^{i=nco} \max^2(0, c_i(x)) \quad (4)$$

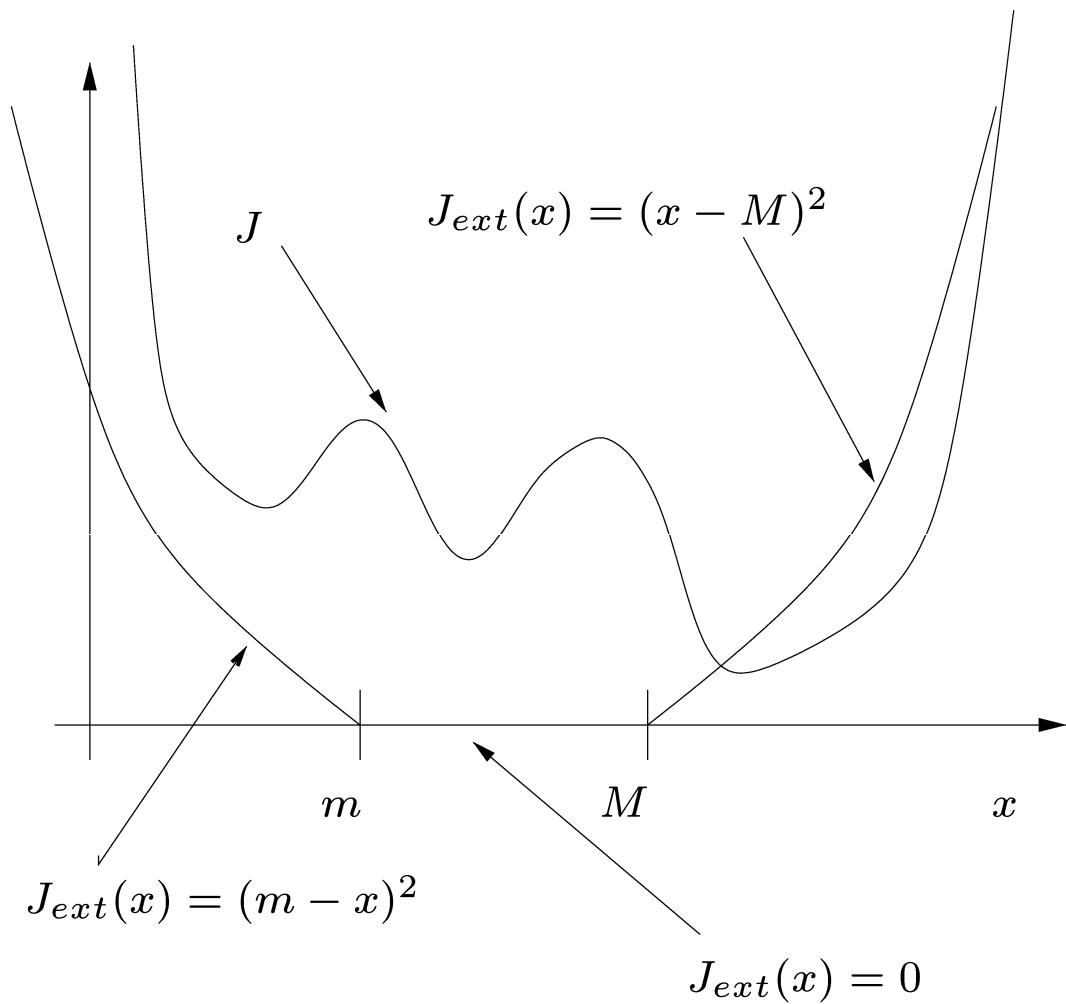
Exterior penalty term example

From :

$$m \leq x \leq M \quad (5)$$

we get :

$$(C) \begin{cases} c_1(x) = m - x \\ c_2(x) = x - M \end{cases}$$



Resolution :

unconstrained optimization problem:

$$(\mathcal{P}_u) \left\{ \begin{array}{l} \tilde{p} = (\theta_a(k) \dots \theta_a(k + N_c - 1))^T \\ \min_{\tilde{p}} J_{tot}(\tilde{p}) = J(\tilde{p}) + p_{ext} J_{ext}(c_i(\tilde{p})) \end{array} \right.$$

Numerical resolution

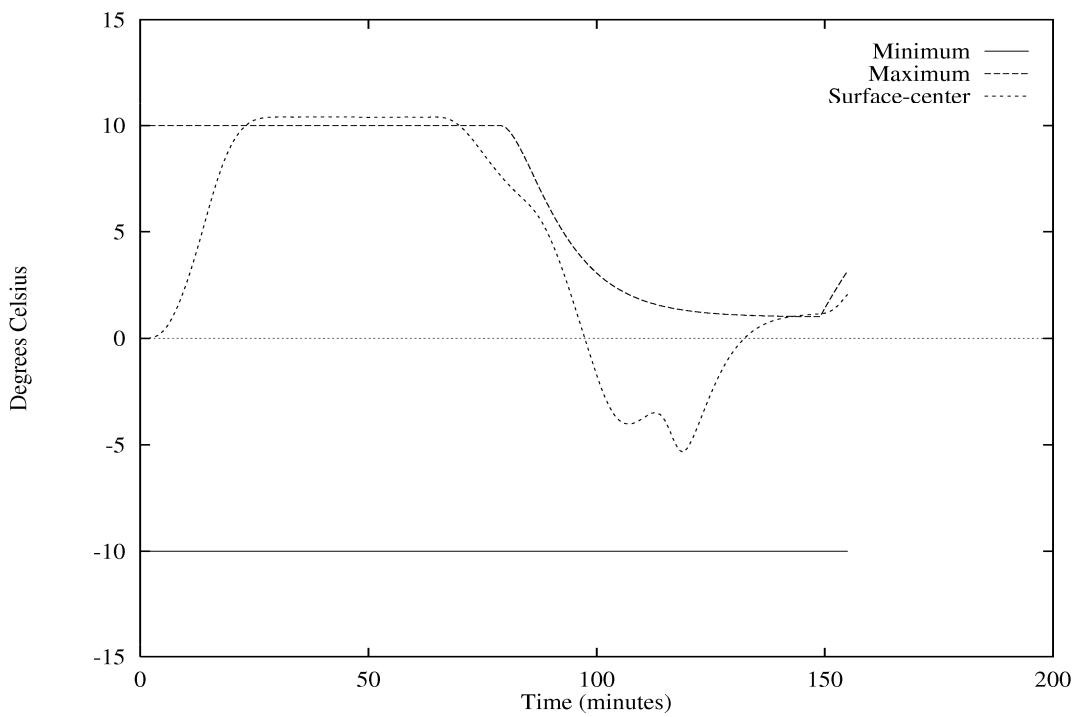
Determination of the sequence \tilde{p} at each sampling period k :

- Resolution of the model (NL PDE) and the sensitivities model (NL PDE) over the receding horizon for each \tilde{p}^i .
- Levenberg-Marquardt's algorithm :

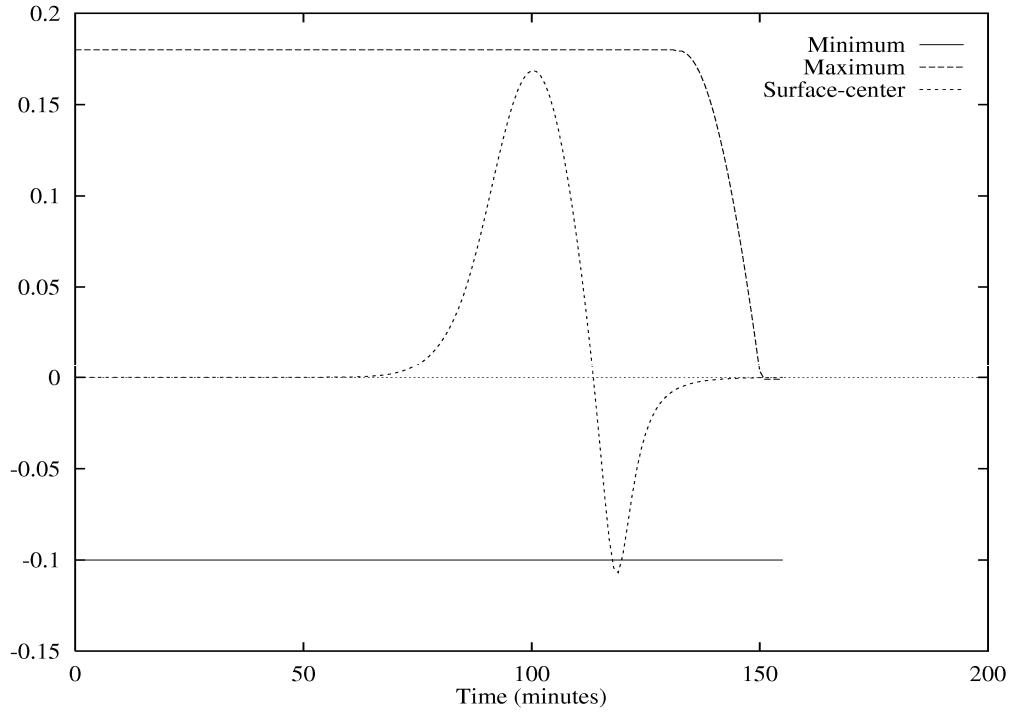
$$\tilde{p}^{i+1} = \tilde{p}^i - (\nabla^2 J_{tot}^i + \lambda I)^{-1} \nabla J_{tot}^i \quad (6)$$

- End of calculation when :
 - a possible optimal solution is found,
 - the external constraints are weekly violated.

Results with $N_c = 1, N_p = 25$

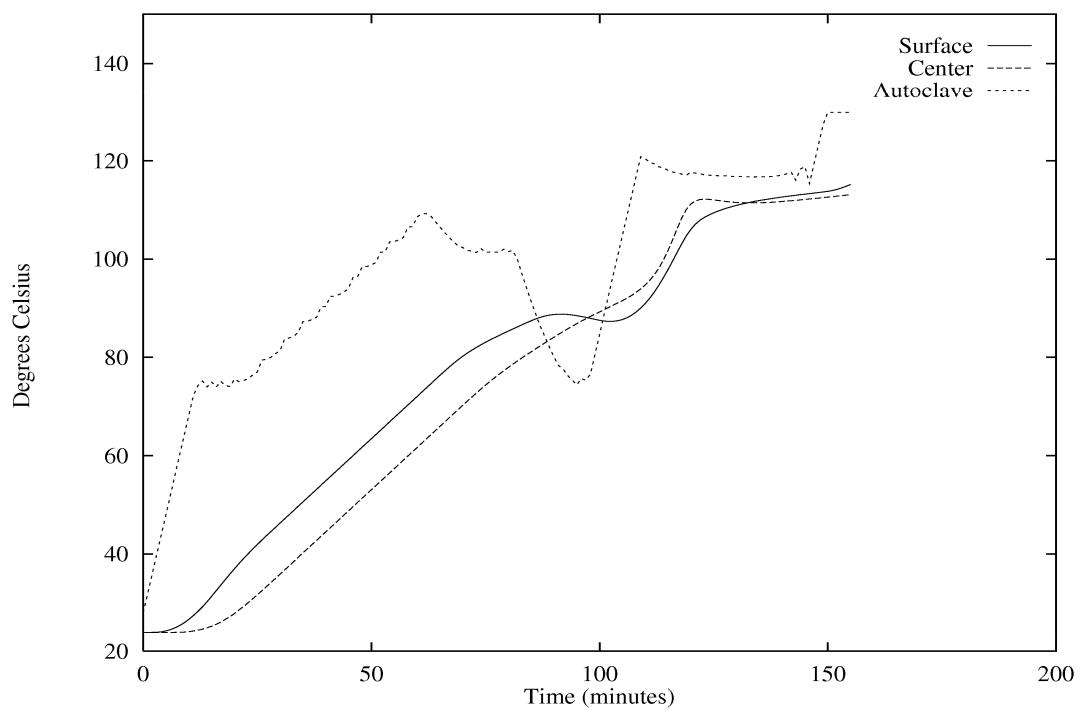


Temperature gradients

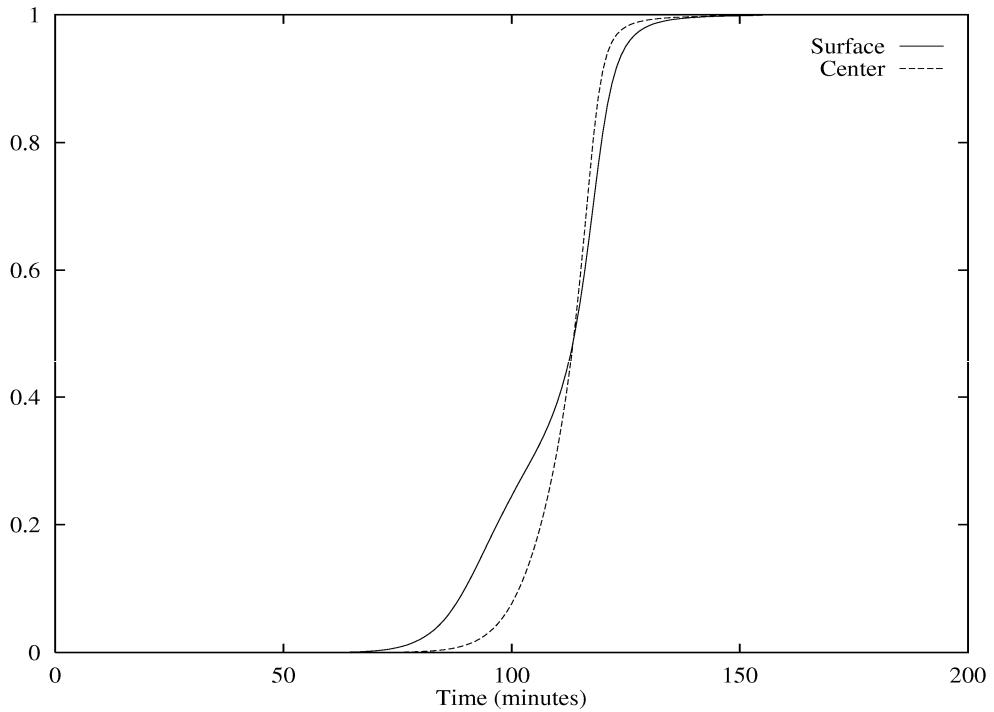


Extent of cure gradient

Results with $N_c = 1, N_p = 25$

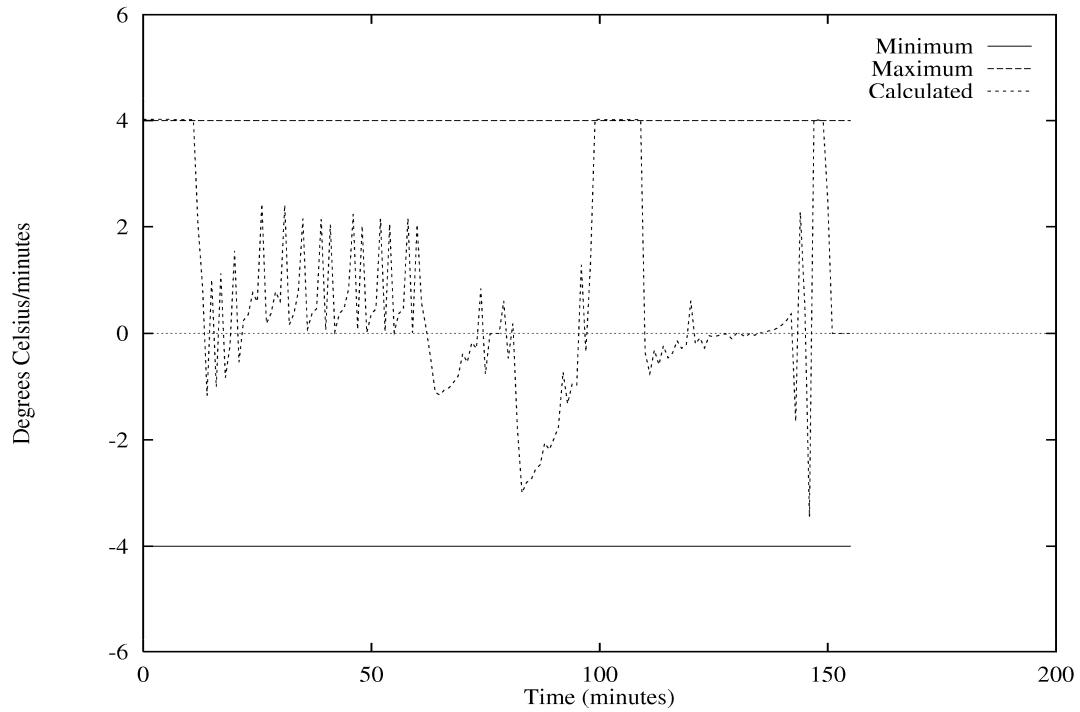


Temperatures

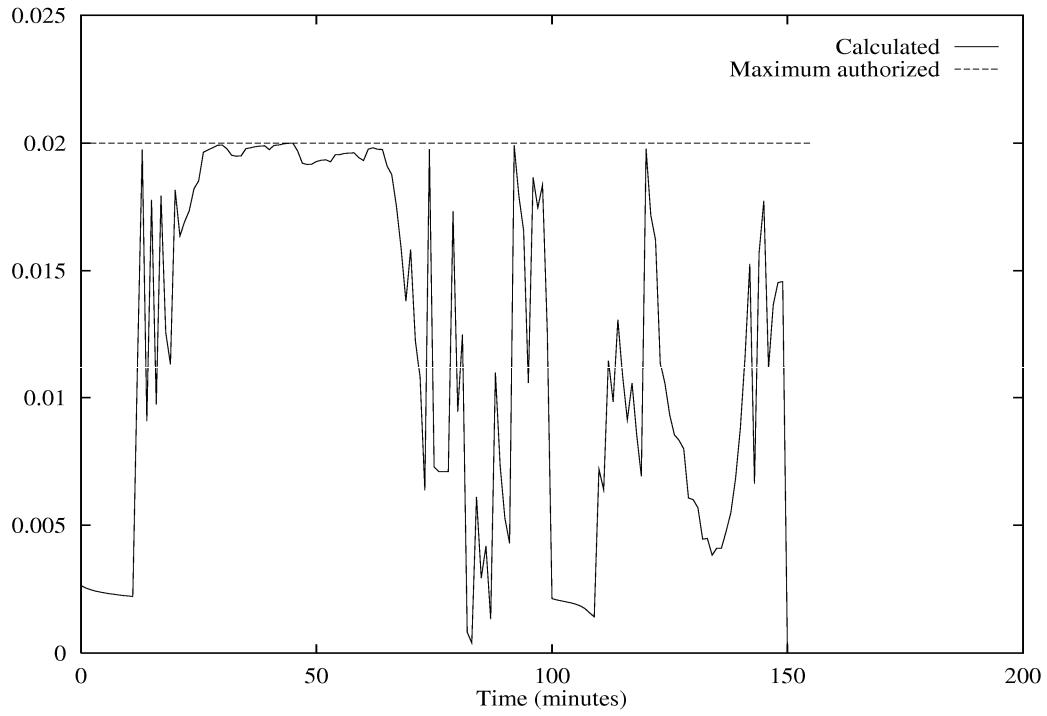


Extent of cure

Results with $N_c = 1, N_p = 25$

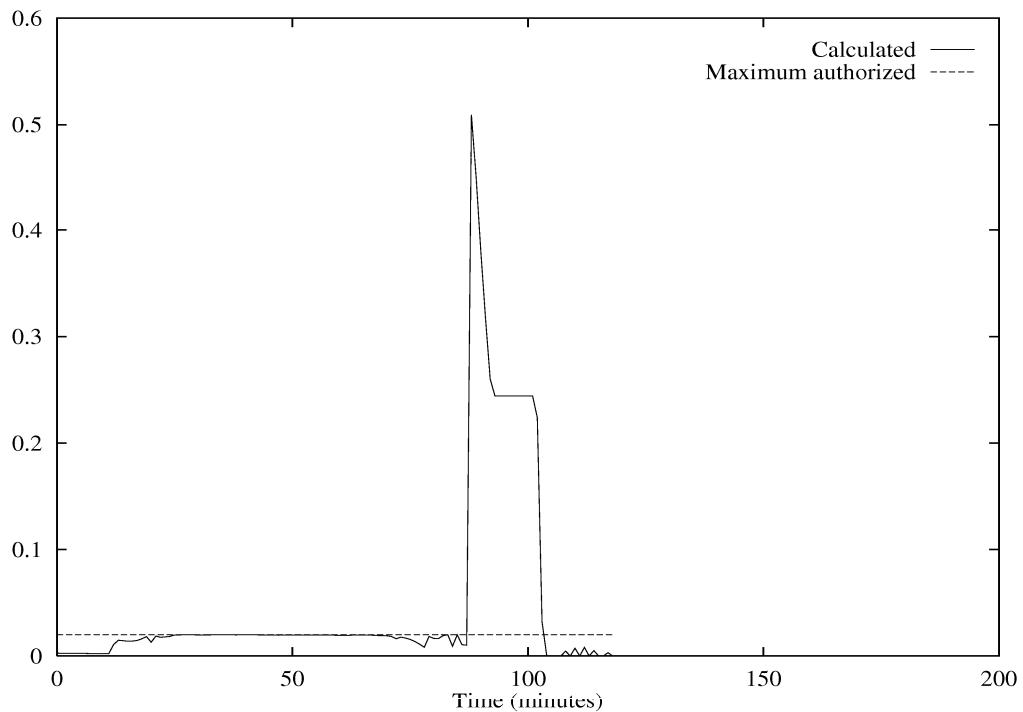


Control velocity



Algorithm output condition

Results with $N_c = 1, N_p = 5$



Algorithm output condition

Conclusions

1. Method **easily implementable** to many process.
2. Difficulties to define the constraints : multidisciplinary approach (**accurated model**).

Perspectives

Trajectory tracking problem :

- **feasibility** problem,
- **theoretical stability** of the closed loop system,
- **real time** control.