

**This document must be cited according to its final version  
which is published in a conference proceeding as:  
P. Dufour, Y. Touré, D.J. Michaud, P.S. Dhurjati,  
"Optimal trajectory determination and tracking  
of an autoclave curing process: A model based approach",  
Proceedings of the 5<sup>th</sup> European Control Conference (ECC),  
Paper F1033-6,  
Karlsruhe, Germany, august 31-september 3, 1999.**

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**Optimal trajectory determination and tracking**

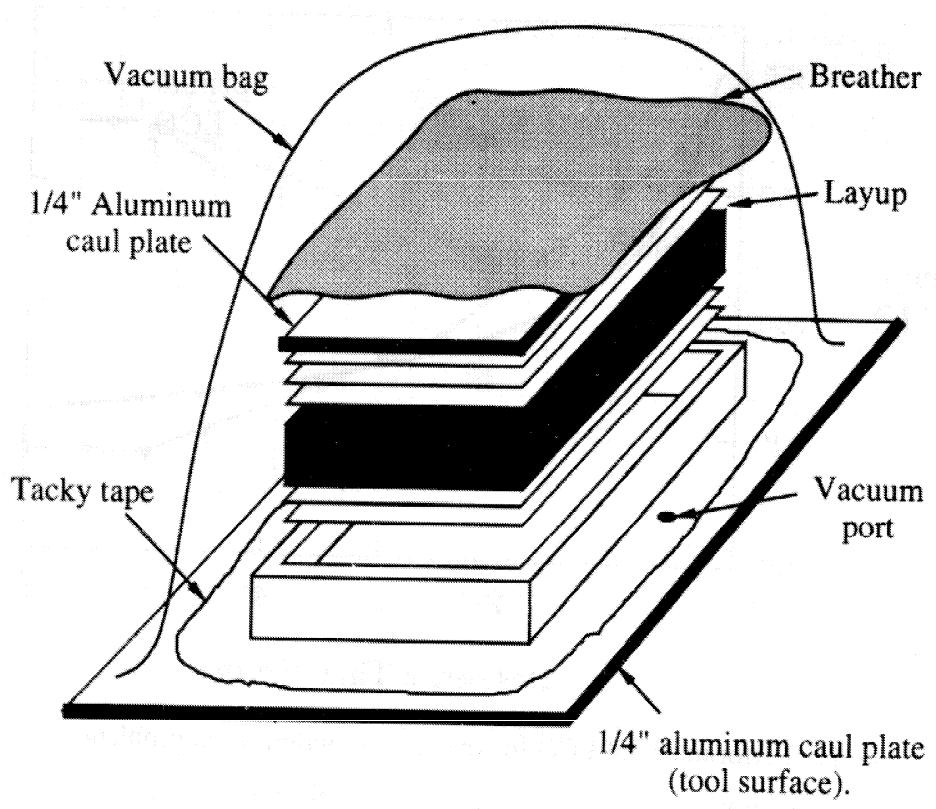
**of an autoclave curing process:**

**a model based approach**

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# Autoclave curing process



## Problem statement

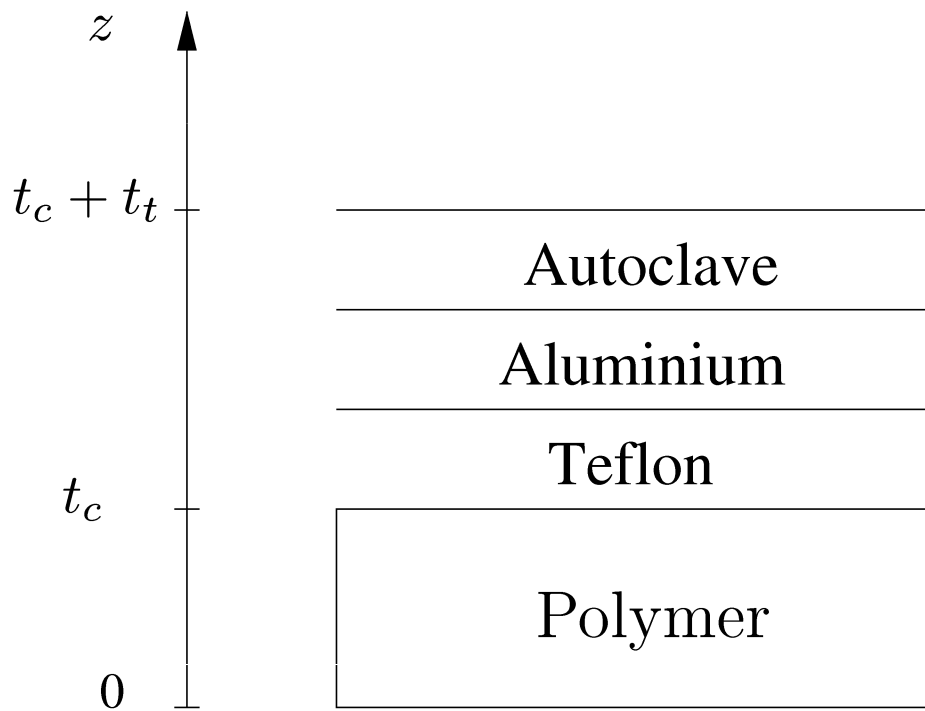
Find the optimal boundary control trajectory in order to:

- complete the cure in minimum time,
- limit temperature gradients (functional constraints),
- ensure an inside out cure (optimal behaviour).

## Existent approaches

- Pontryagin's maximum principle [Nagendra and Loos, 1988].
- Expert control [LeClair and Abrams, 1988].
- Heuristic approach with a predictive strategy [Pillai and al, 1996].

## Spatial domains



## Nonlinear DPS with boundary control

$$\rho_t c_{p_t} \dot{T} = k_t \frac{\partial^2 T}{\partial z^2} \quad (\text{Center and top parts})$$

$$\rho_c c_{p_c} \dot{T} = k_c \frac{\partial^2 T}{\partial z^2} + \rho_c \Delta H_r A e^{-\frac{\Delta E_a}{RT}} \alpha^m (1 - \alpha)^n$$

$$\dot{\alpha} = A e^{-\frac{\Delta E_a}{RT}} \alpha^m (1 - \alpha)^n$$

(Center part)

with the initial conditions :

$$(\mathcal{S}_{NL}) \left\{ \begin{array}{l} T(z, 0) = T_0 \quad (\text{Center and top parts}) \\ \alpha(z, 0) = 0 \quad (\text{Center part}) \end{array} \right.$$

with the two boundary conditions :

$$T(t_c + t_t, t) = T_a(t)$$

$$\frac{\partial T}{\partial z}(0, t) = 0$$

**Model based Predictive Control strategy :**

**constrained optimization problem.**

$$(\mathcal{P}_c) \left\{ \begin{array}{l}
 \tilde{p} = (T_a(k) \dots T_a(k + N_c - 1))^T \\
 \min_{\tilde{p}} J(\tilde{p}) = \frac{1}{2} \sum_{j=k+1}^{j=k+N_p} \frac{1}{\|\dot{\alpha}(0, j)\|^2} \\
 c_i(\tilde{p}) \leq 0 \quad (i \in I = \{1 \rightarrow nco\}) \quad \text{ex: } T(t_c, t) - T(0, t) \leq \Delta T(t) \\
 N_c : \text{control horizon,} \\
 N_p : \text{prediction horizon,} \\
 nco : \text{number of constraints.}
 \end{array} \right.$$



## Unconstrained resolution method

↪ Transformation law :

Searching for the **constrained variable**  $T_a$  such that :

$$T_{a.min} \leq T_a \leq T_{a.max} \quad (1)$$

becomes searching for the **unconstrained variable**  $\theta_a$  such that :

$$T_a = \frac{T_{a.max} + T_{a.min}}{2} + \frac{T_{a.max} - T_{a.min}}{2} * \tanh(\theta_a) \quad (2)$$

↪ Exterior penalty method :

From the constrained problem :

$$\min_x J(x) \text{ with : } c_i(x) \leq 0 \quad (i \in I = \{1 \rightarrow nco\}) \quad (3)$$

we introduce an **exterior penalty term**  $J_{ext}$  :

$$J_{ext}(c_i(x)) = \sum_{i=1}^{i=nco} \max^2(0, c_i(x)) \quad (4)$$

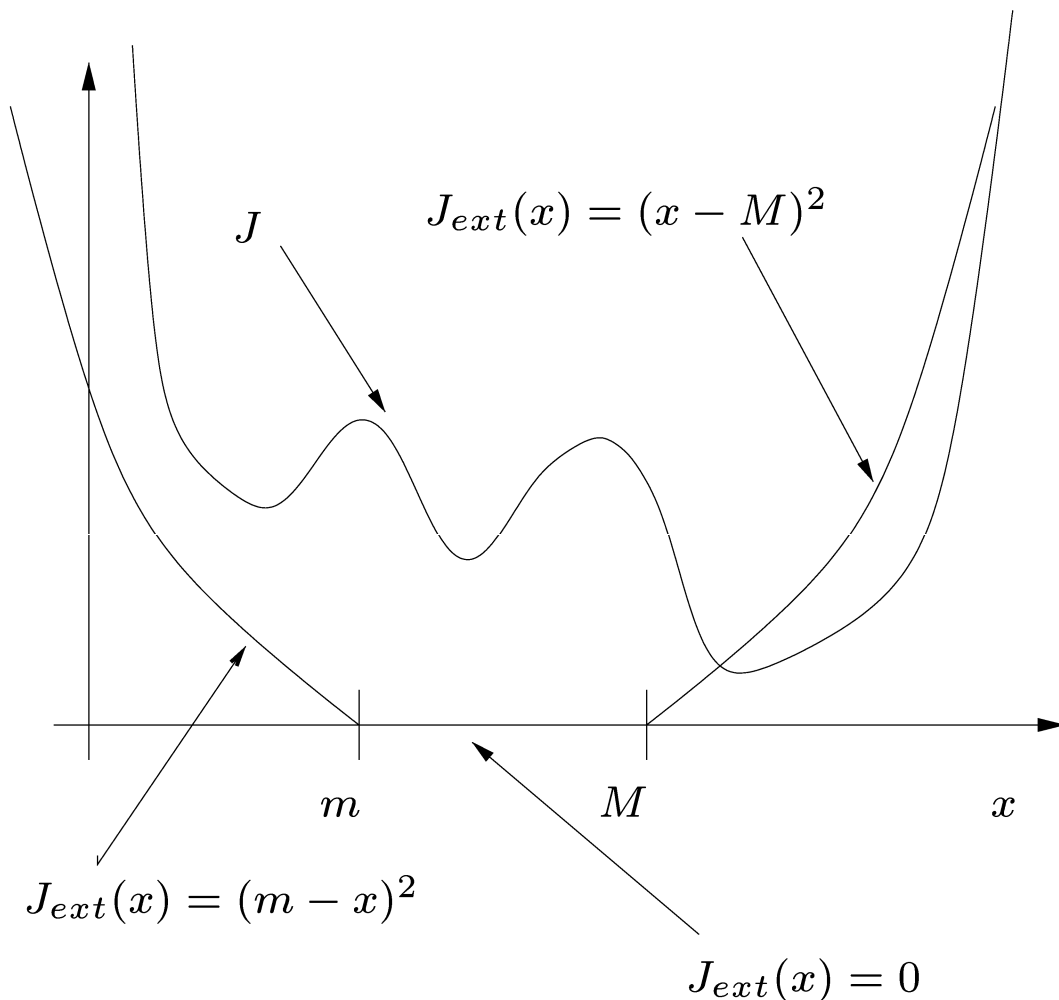
## Exterior penalty term example

From :

$$m \leq x \leq M \tag{5}$$

we get :

$$(C) \begin{cases} c_1(x) = m - x \\ c_2(x) = x - M \end{cases}$$



**Resolution :**

**unconstrained optimization problem:**

$$(\mathcal{P}_u) \begin{cases} \tilde{p} = (\theta_a(k) \dots \theta_a(k + N_c - 1))^T \\ \min_{\tilde{p}} J_{tot}(\tilde{p}) = J(\tilde{p}) + p_{ext} J_{ext}(c_i(\tilde{p})) \end{cases}$$

## Numerical resolution

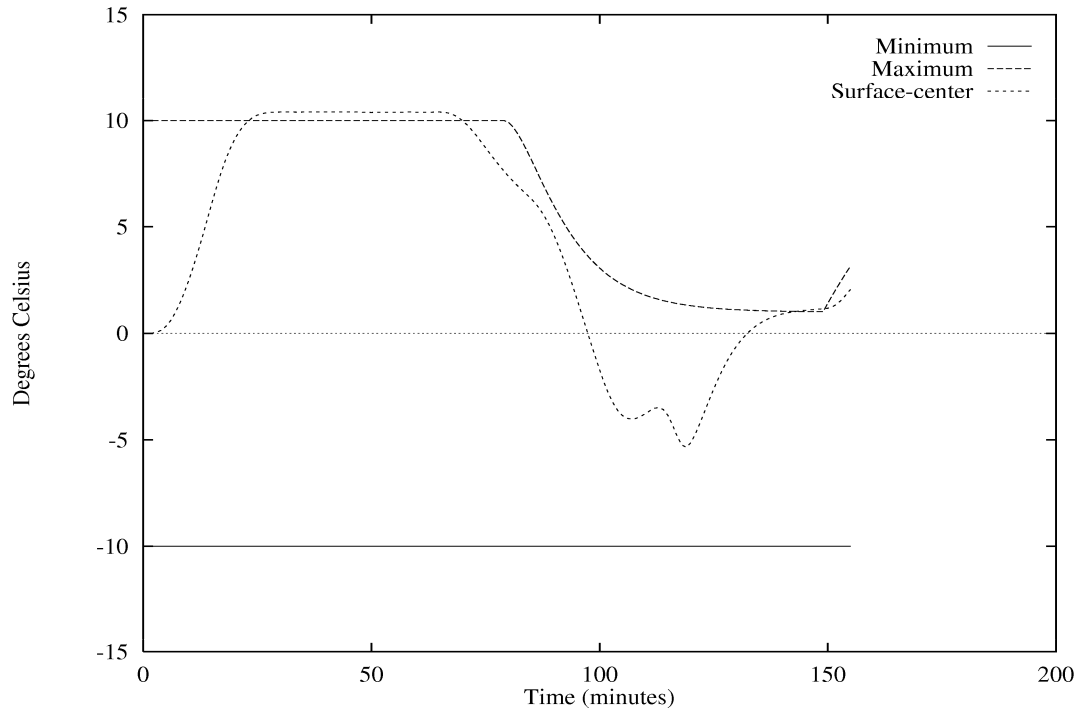
Determination of the sequence  $\tilde{p}$  at each sampling period  $k$ :

- Resolution of the model (NL PDE) and the sensitivities model (NL PDE) over the receding horizon for each  $\tilde{p}^i$ .
- Levenberg-Marquardt's algorithm :

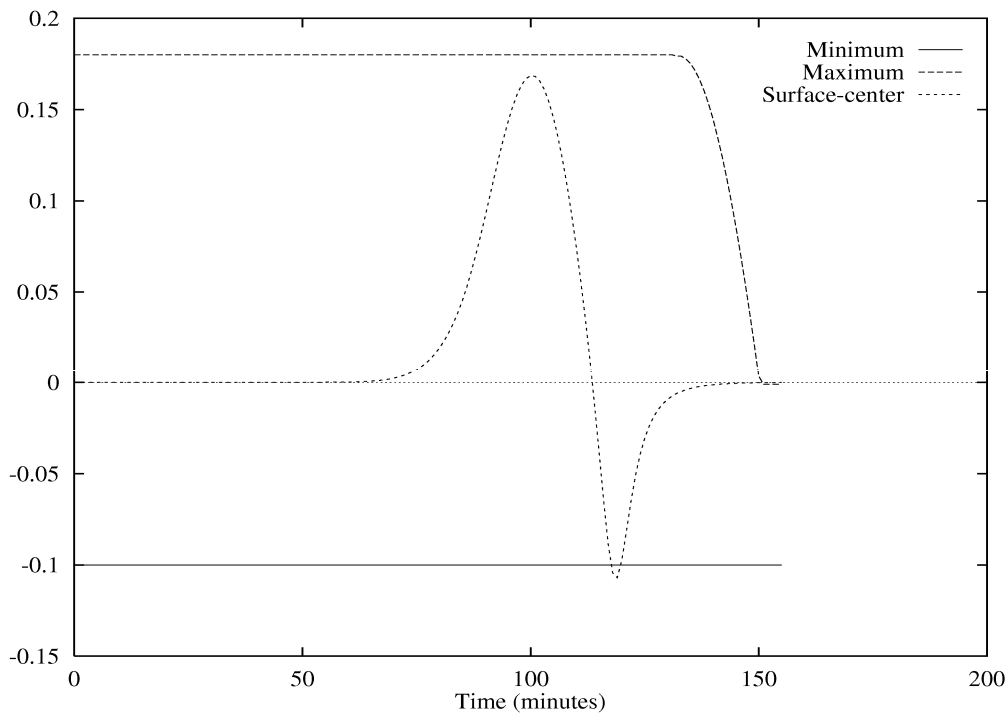
$$\tilde{p}^{i+1} = \tilde{p}^i - (\nabla^2 J_{tot}^i + \lambda I)^{-1} \nabla J_{tot}^i \quad (6)$$

- End of calculation when :
  - a possible optimal solution is found,
  - the external constraints are weekly violated.

# Results with $N_c = 1, N_p = 25$

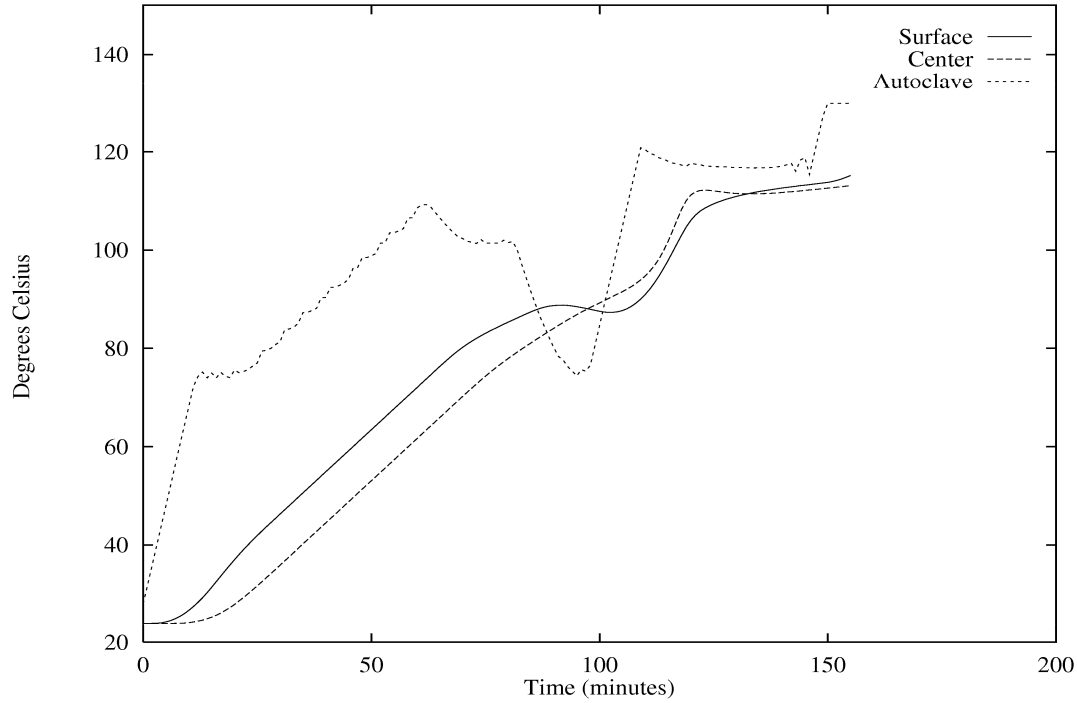


## Temperature gradients

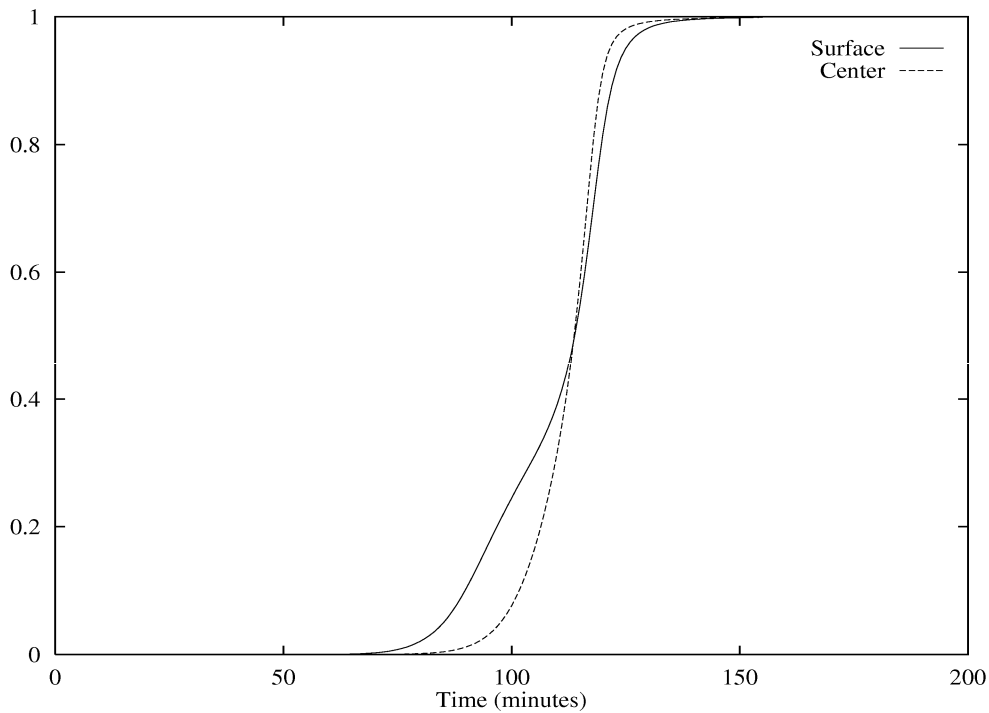


## Extent of cure gradient

# Results with $N_c = 1, N_p = 25$

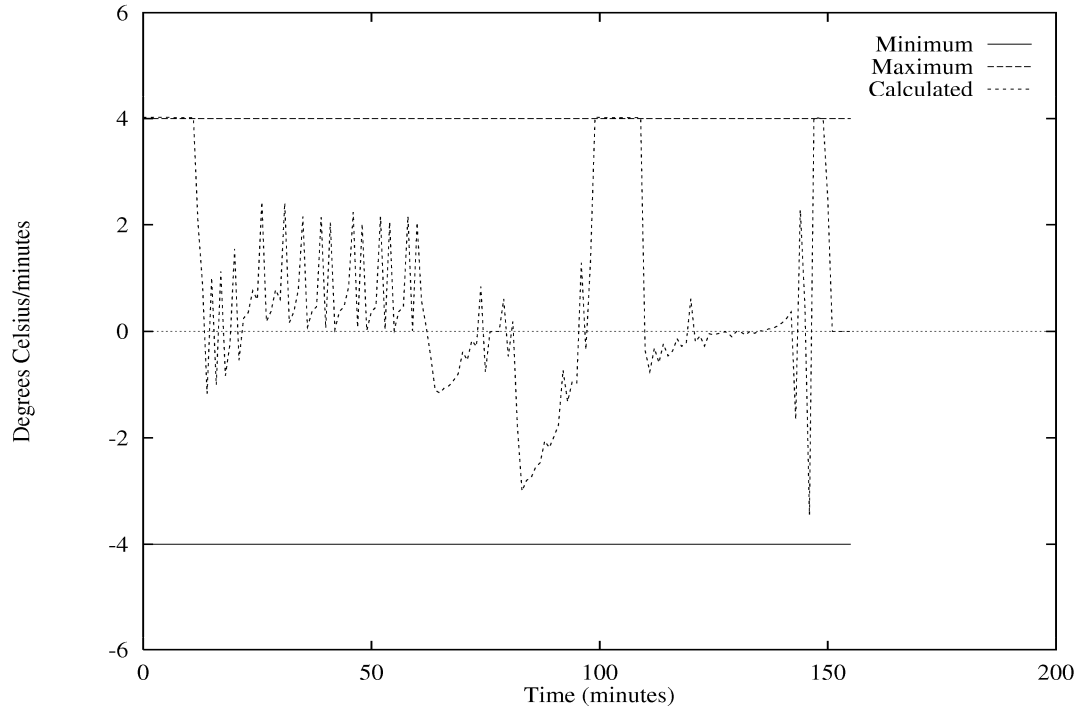


## Temperatures

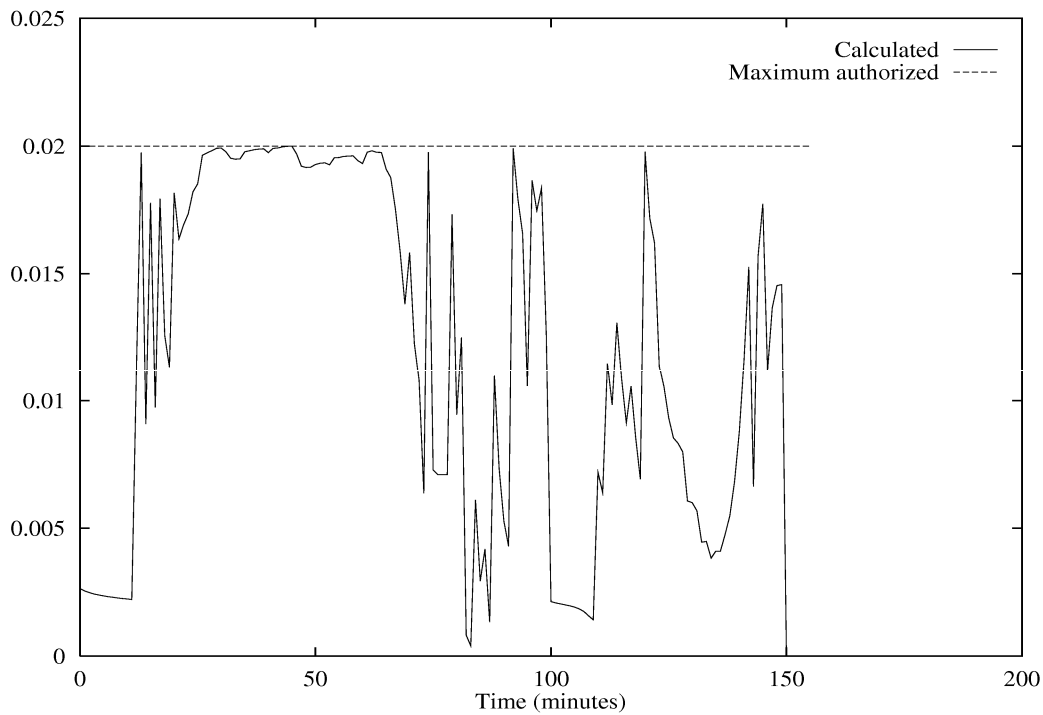


## Extent of cure

# Results with $N_c = 1, N_p = 25$

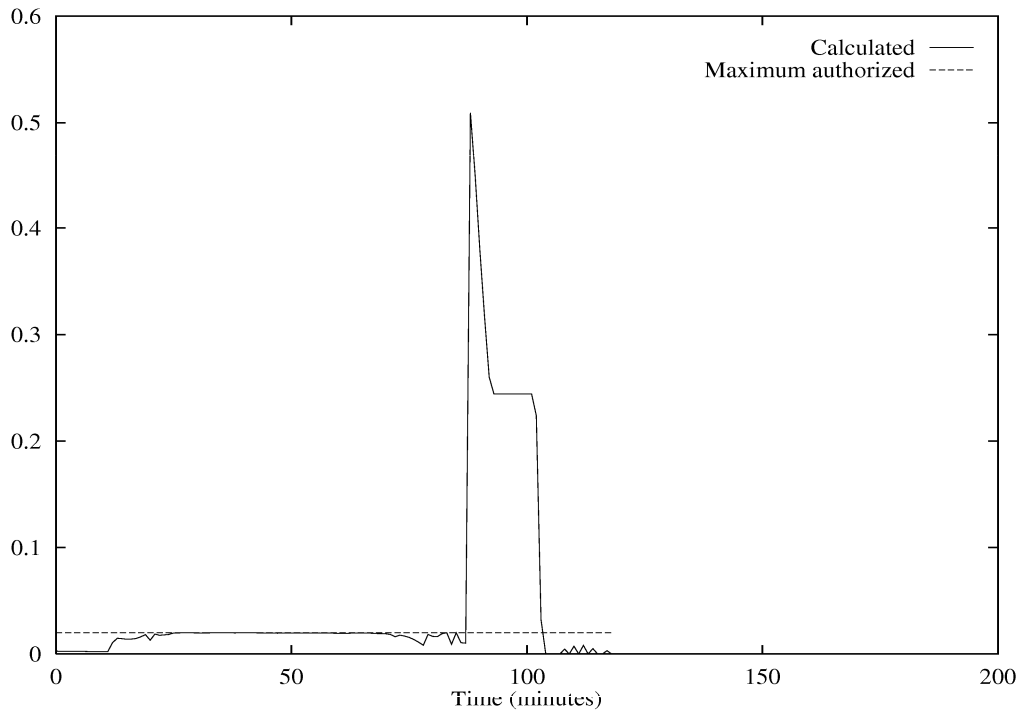


## Control velocity



## Algorithm output condition

**Results with  $N_c = 1, N_p = 5$**



Algorithm output condition



## Conclusions

1. Method **easily implementable** to many process.
2. Difficulties to define the constraints : multidisciplinary approach (**accurated model**).

## Perspectives

**Trajectory tracking** problem :

- **feasibility** problem,
- **theoretical stability** of the closed loop system,
- **real time** control.