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A nonlinear distributed parameter process control :

an internal linearized model control approach

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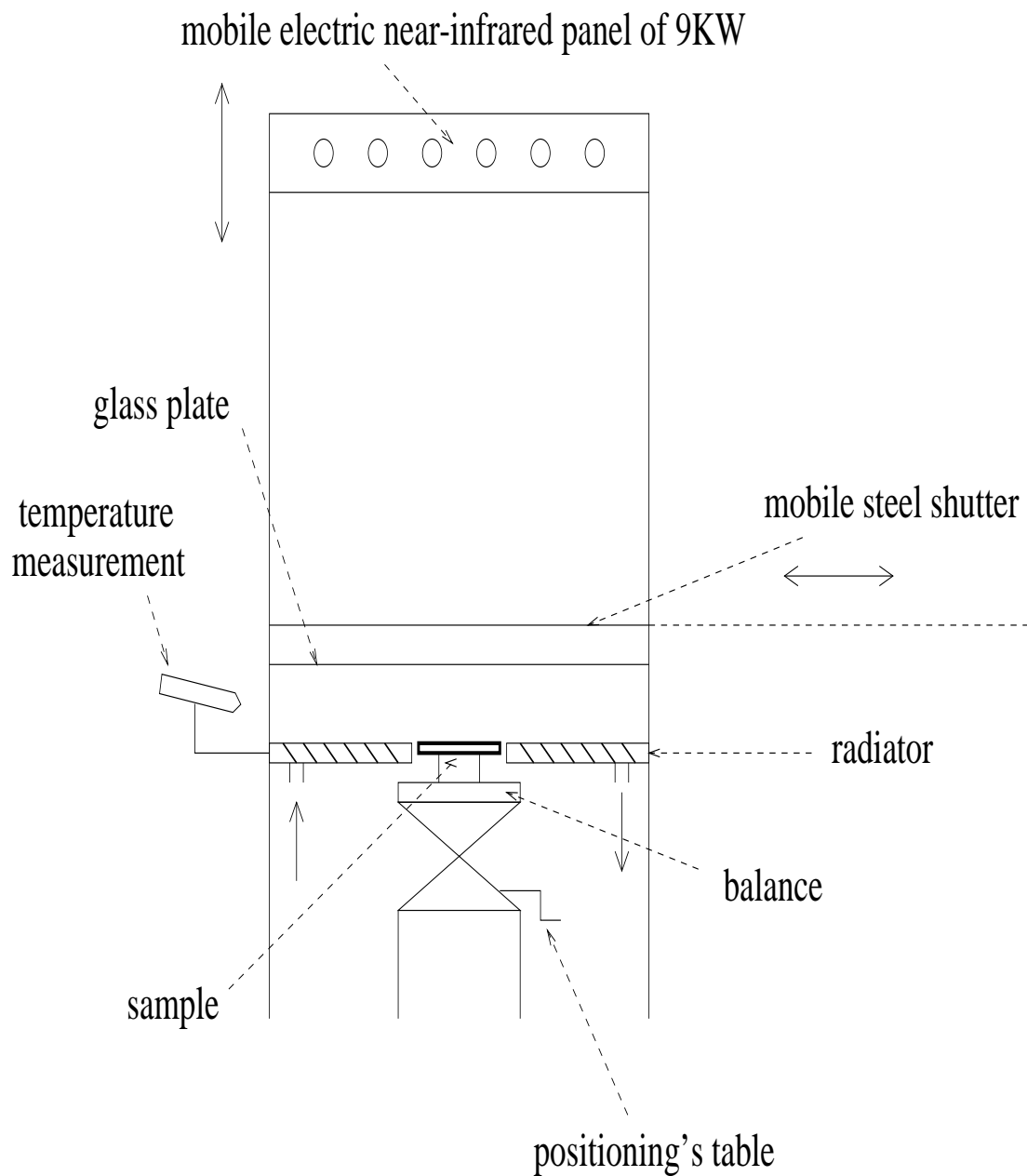
Plan

1. The problem statement.
2. Application example : a drying process.
3. Nonlinear DPS control approaches.
4. Time-varying linearized model.
5. Internal model control structure.
6. Predictive control : optimization approach.
7. Simulation results.
8. Conclusions, perspectives.

Introduction

1. Nonlinear distributed parameter process control.
2. Internal time-varying linearized model control structure.
3. Predictive type control with constrained optimization.

Drying process



Nonlinear DPS model

Main hypothesis : the temperature is uniform according to the thickness.

- for $0 < z < e_{sec}$:

$$\chi_t = \frac{\partial}{\partial z} [D_{eff}(\chi, T) \chi_z]$$

- at $z=0$:

$$\chi_z = 0$$

- at $z = e_{sec}$:

$$\chi_z = \frac{-\dot{m}(\chi, T)}{\rho D_{eff}(\chi, T)}$$

with the effective diffusion coefficient :

$$D_{eff}(\chi, T) = \frac{D_0 \exp\left(\frac{-a}{\chi}\right) \exp\left(\frac{-Ea}{RT}\right)}{(1+\phi\chi)^2}$$

$$(\rho C_p(\chi, T)e + \rho_s C_s e_s) T_t = - \sum_{i=1}^5 P_i + P_{abs}$$

with the different losses expression P_i :

$$\left\{ \begin{array}{l} P_1 = h_c(T - T_{air}) \\ P_2 = \sigma' (T^4 - T_{air}^4) \\ P_3 = h_c(T - T_{sup}) \\ P_4 = \alpha_s \sigma (T^4 - T_{sup}^4) \\ P_5 = l_v(T) \dot{m}(\chi, T) \end{array} \right.$$

and the infrared energy P_{abs} expression :

$$P_{abs} = \alpha_{ir}(\chi) \varphi_{ir}(t)$$

where h_c , σ' , α_s , σ are positive constants, $\varphi_{ir}(t)$ is the control variable.

Nonlinear system reformulation

$$\begin{cases} \underline{\dot{\omega}}(z, t) = A(\underline{\omega}(z, t)) + B(\underline{\omega}(z, t)) u(t) \\ y(t) = C(\underline{\omega}(z, t)) = \bar{\chi}(t) = \frac{1}{e_{sec}} \int_0^{e_{sec}} \chi(\zeta, t) d\zeta \end{cases}$$

with :

$$(S_1) \left\{ \begin{array}{l} \underline{\omega}(z, t) = [\chi \ T]^T \\ \chi_t = F_1(\chi_{zz}, \chi_z, \chi, T) \text{ for } 0 < z < e_{sec} \\ T_t = F_2(\bar{\chi}, \chi(e_{sec}, t), T) + F_3(\bar{\chi}, T) \varphi_{ir}(t) \text{ for all } z \\ \text{with the boundary conditions :} \\ F_4(\chi_z, \chi, T) = 0 \text{ at } z = e_{sec} \\ \chi_z = 0 \text{ at } z = 0 \\ \text{with the initial conditions :} \\ \underline{\omega}(z, 0) = [\chi_0 \ T_0]^T \end{array} \right.$$

Nonlinear DPS control approaches

- Transformation law : [Ibragimov-Shabat], [Magri]
 - advantage : exact transformation law,
 - disadvantages :
 - * monovariaible case,
 - * the choice of order of the transformation [Magri],
 - * applicability for physical process not proved.
- Finite dimensionnal approximation (collocation or finite differences) :
 - advantage : simplicity,
 - disadvantage : the state and control dimension may becomes too large.
- Keep the infinite dimension nonlinear system and make a local linearization.

Time-varying linearized model

Let :

$$\left\{ \begin{array}{l} u(t) = u_0(t) + \Delta u(t) \\ y_m(t) = y_0(t) + \Delta y_0(t) \\ \chi(t) = x_0(t) + \Delta x(t) \\ T(t) = \theta_0(t) + \Delta \theta(t) \\ \Delta \underline{w} = [\Delta x \ \Delta \theta]^T \end{array} \right.$$

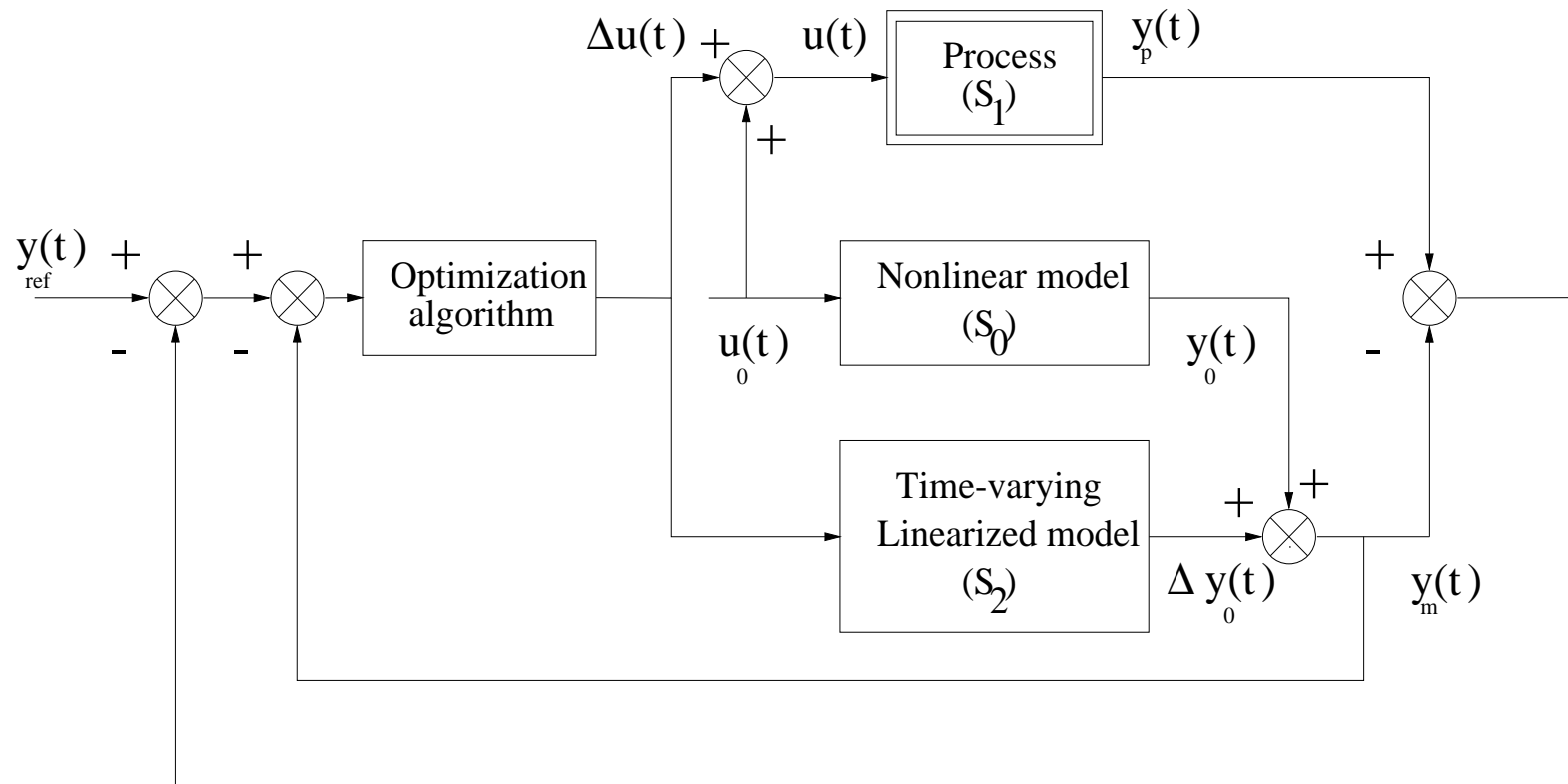
and :

$$(S_0) \left\{ \begin{array}{l} \dot{\underline{\omega}}_0(z, t) = A(\underline{\omega}_0(z, t)) + B(\underline{\omega}_0(z, t)) u_0(t) \\ y_0(t) = C(\underline{\omega}_0(z, t)) \end{array} \right.$$

then the time-varying linearized model is :

$$(S_2) \left\{ \begin{array}{l} \Delta \dot{\underline{\omega}}(z, t) = A_2(t) \Delta \underline{w}(z, t) + B_2(t) \Delta u(t) \\ \Delta y_0(t) = C_2 \Delta \underline{w}(z, t) \end{array} \right.$$

Internal time-varying linearized model control



Predictive control : optimization problem

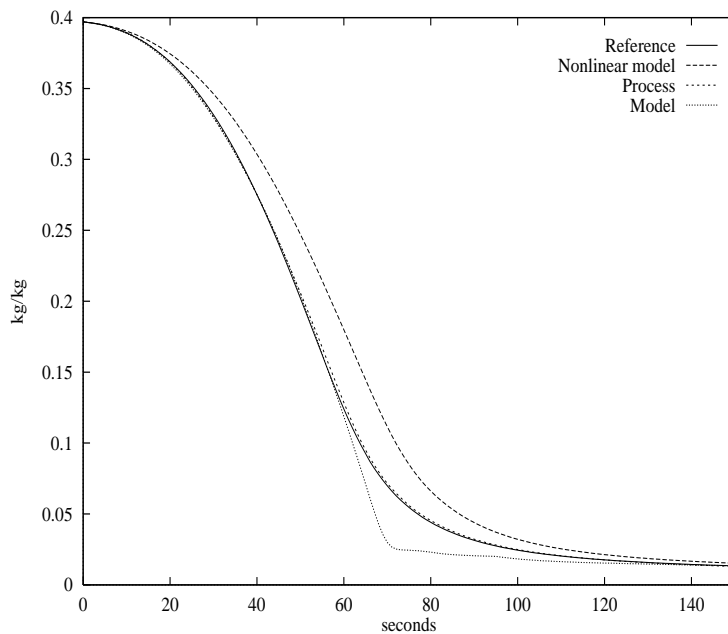
$$\left\{ \begin{array}{l}
 \min J(\Delta \underline{u}) = \frac{1}{2} \sum_{j=k+1}^{j=k+N_p} \varepsilon^2(j) \\
 \\
 \Delta \underline{u} = (\Delta u_k \dots \Delta u_{k+N_c-1})^T \\
 \\
 \text{with } \varepsilon(j) = y_{ref}(jT_e) - y_p(jT_e) \\
 \\
 u_{min} \leq u_o + \Delta u_{k+i-1} \leq u_{max} \quad (i = 1 \text{ to } N_c) \\
 \\
 \dot{u}_{min} \leq \frac{\Delta u_{k+i-1} - \Delta u_{k+i-2}}{T_e} \leq \dot{u}_{max} \quad (i = 1 \text{ to } N_c)
 \end{array} \right.$$

Simulation conditions

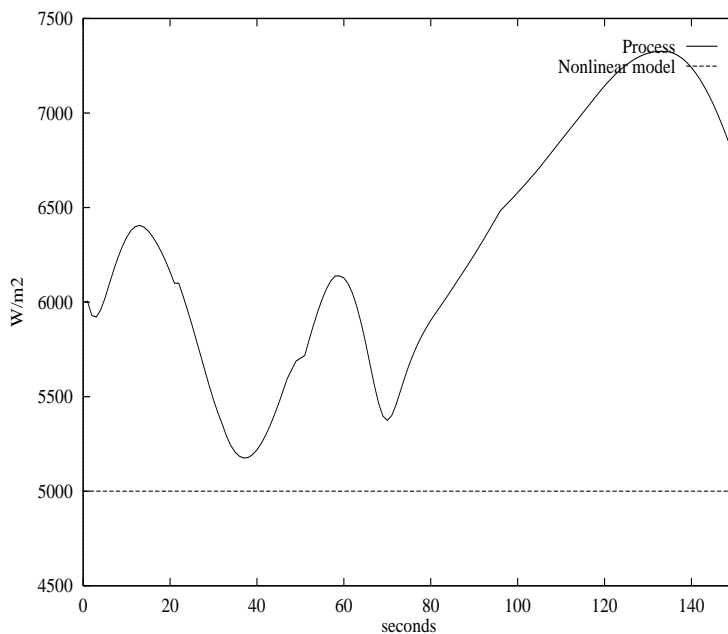
$$\left\{ \begin{array}{l} T_{air} = 52^{\circ}C \\ T_{sup} = 20^{\circ}C \\ \chi_{air} = 20\% \\ T_0 = 26^{\circ}C \\ \chi_0 = 0.397 \\ u_o = 5kW m^{-2} \\ N_c = 1 \\ T_e = 1 \end{array} \right.$$

with the following constraints :

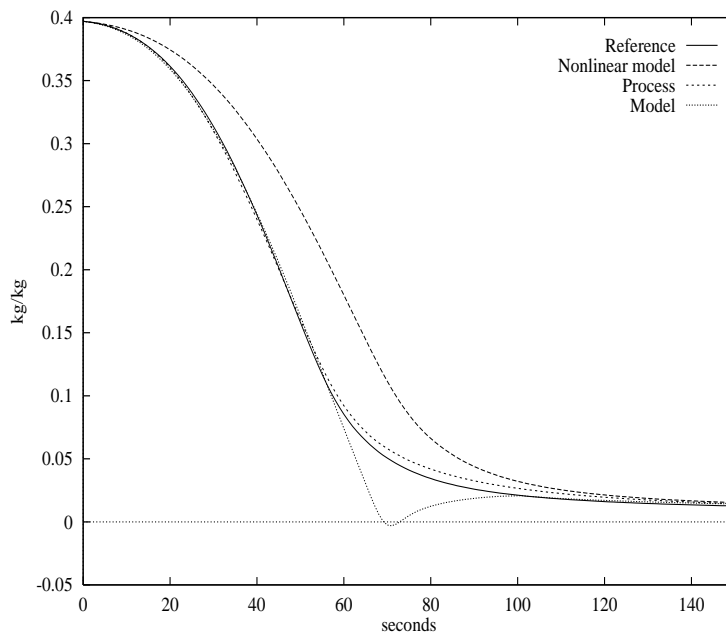
$$\left\{ \begin{array}{l} 0 \leq u \leq 12kW m^{-2} \\ -500W m^{-2} s^{-1} \leq u_i - u_{i-1} \leq 500W m^{-2} s^{-1} \end{array} \right.$$



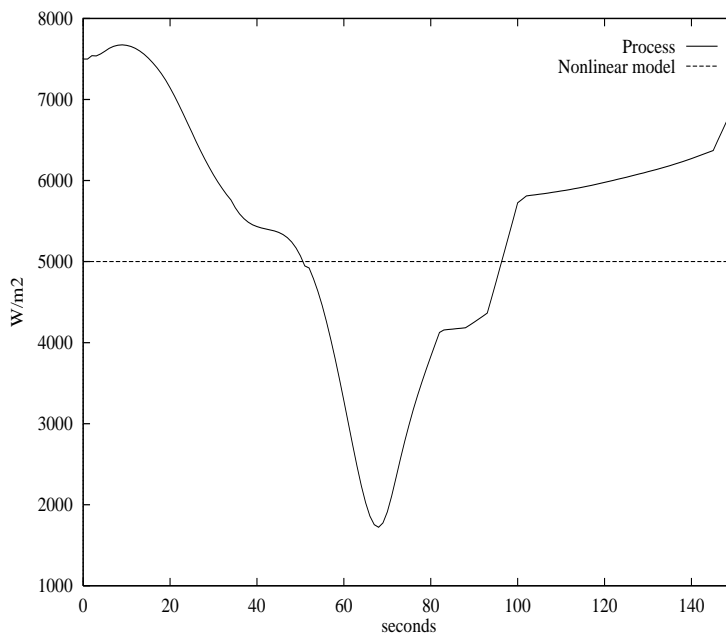
- Mean humidity profiles, $N_p = 4s$ -



- Infrared flow profiles, $N_p = 4s$ -



- Mean humidity profiles, $N_p = 10s$ -



- Infrared flow profiles, $N_p = 10s$ -

Conclusions

- Trajectory tracking,
- linearized model validity domain,
- optimal choice of N_p .

Perspectives

- Theoretical study : stability conditions,
- in progress : application to other processes.