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# A Nonlinear Distributed Parameter Process Control : an Internal Linearized Model Control Approach

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## ABSTRACT

This paper deals with the nonlinear distributed parameter process control. The aim is to realize a predictive type control that minimize an optimization criterion under some constraints. The well-known internal model control (IMC) structure is used to make the predictive control strategy where the internal model is a time-varying linearized model around a tracking trajectory. This nonlinear distributed parameter system control is developed and presented through a practical distributed control application : an infrared drying process control.

## 1.Introduction

This work concerns the distributed parameter system control governed by nonlinear partial differential parabolic type equations. There are few works directly concerning the nonlinear distributed parameter systems. The existed ones deal with structurally interested cases for one partial derivative equation but not for a set of equations. Notably there are the works of Ibragimov & Shabat [7] based on the inverse scattering problem developed by Miura [9] or Magri's one [8], where an exact linearization by geometrical transformation is proposed. Moreover, the order of this transformation has to be fixed there by a non trivial manner. The most extended approach in the nonlinear systems study (distributed or lumped parameter systems) is the linearization approach, or else the finite dimensional approximation [11] [5].

In this work, we give an intermediate alternative in the sense that the linear model is time-varying and its parameters depend on the nonlinear model at each time. It allows a larger validity domain and so a most extended initial nonlinear system control. However, a control synthesis, like explicit control law for example, becomes more difficult. So, we use the IMC structure in its indirect version that bring to a predictive control approach by minimization on a receding horizon. The framework is developed through an application example, but the results can be seen in the generic sense since the assumptions used aren't very strong, the main ones being the open loop stability and the controllability assumptions.

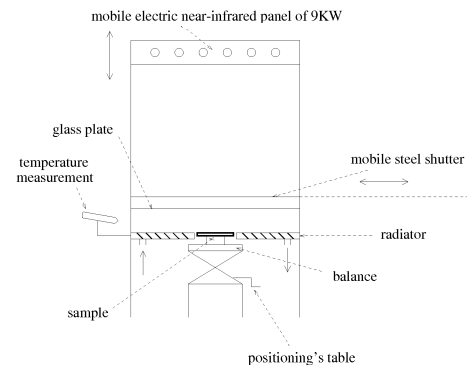
This application example is an infrared drying plant. It's a physically stable process because the system always tends to its minimum humidity. The control

aim is to track a given drying profile corresponding to a given dried material quality.

In the first section that follows, we present the drying plant and its control problem. The model is then described and the final nonlinear distributed control system stated. Then, the time-varying linear model around a given profile is determined and its validity limits studied. The next section deals with control strategy. We first recall the IMC structure and bring out its main properties, advantages and difficulties. The tracking control problem, stated in term of constrained minimization, is then exposed. The last section deals with simulation results on the optimal profile tracking improving.

## 2.Process and control problem

The drying process, presented behind, allows to dry a painting film coated on a car iron sample by supply of infrared flow :



This painting coat, of low thickness (order of 0.1mm), is characterised by its temperature supposed uniform  $T$  and by its dry basis humidity  $\chi$ , assumed to vary-

ing only according to the thickness  $z$  (by definition, the dry basis humidity is the ratio of the water mass on the dry mass of the sample). The drying under infrared results in a migration of the water contained in the painting film to the radiated surface as well as a vaporization of this water at this surface. The humidity is deduced from the weighing follow-up of the iron+painting film system during the drying; this measurement allowing us to accede only to the painting film mean humidity  $\bar{\chi}$ . Finally, the system can be represented by the state variables  $\chi(z, t)$  and  $T(t)$  cause of the uniform assumption on temperature [10].

### Mass balance

A diffusional drying model, written in a landmark linked to the solid matrix representing the paint, allows to obtain the following equations :

- for  $0 < z < e_{sec}$  :

$$\chi_t = \frac{\partial}{\partial z} [D_{eff}(\chi, T) \chi_z]$$

with the effective diffusion coefficient :

$$D_{eff}(\chi, T) = \frac{D_0 \exp(\frac{-a}{\chi}) \exp(\frac{-E_a}{RT})}{(1+\phi\chi)^2}$$

where  $D_0, a, E_a, R$  and  $\phi$  are positive constants,

- at  $z=0$  :

$$\chi_z = 0$$

- at  $z = e_{sec}$ , the leaving flow is linked to the drying velocity  $\dot{m}(\chi, T)$  by the relation :

$$-D_{eff}(\chi, T) \chi_z = \frac{\dot{m}(\chi, T)}{\rho}$$

with the drying velocity given in the annexe.

### Energical balance

Energical balance, taking into account the different losses  $P_i$  as well as the absorbed infrared power, is given by the following equations :

- 

$$(\rho C_p(\bar{\chi}, T)e + \rho_s C_s e_s) T_t = - \sum_{i=1}^5 P_i + P_{abs}$$

where  $\rho C_p(\bar{\chi}, T)e$  and  $\rho_s C_s e_s$  are respectively the surface thermal capacity of the paint and its support and where the different losses due to the convection as well as the radiation by the two faces have for expression :

$$\begin{cases} P_1 = h_c(T - T_{air}) \\ P_2 = 0.96\sigma(T^4 - T_{air}^4) \\ P_3 = h_c(T - T_{sup}) \\ P_4 = \alpha_s\sigma(T^4 - T_{sup}^4) \end{cases}$$

and where the vaporization energy  $P_5$  and the infrared energy  $P_{abs}$ , both depend on the temperature  $T$  and the humidity :

$$\begin{cases} P_5 = l_v(T) \dot{m}(\chi, T) \\ P_{abs} = \alpha_{ir}(\bar{\chi}) \varphi_{ir} \end{cases}$$

The remaining coefficients expressions are given in the annexe.

### Nonlinear distributed control system

According to the spatial uniform property assumption on the temperature, the control problem becomes a distributed control problem one, since the control, ie the infrared flow  $\varphi_{ir}(t)$ , acts instantaneously at the boundary ( $z = e_{sec}$ ) and over the painted film. So, the system can be summarized as follows :

$$(S_1) \left\{ \begin{array}{l} \chi_t = F_1(\chi_{zz}, \chi_z, \chi, T) \text{ for } 0 < z < e_{sec} \\ T_t = F_2(\bar{\chi}, \chi(e_{sec}, t), T) + F_3(\bar{\chi}, T) \varphi_{ir}(t) \text{ for all } z \\ F_4(\chi_z, \chi, T) = 0 \text{ at } z = e_{sec} \\ \chi_z = 0 \text{ at } z = 0 \\ \text{with the initial conditions :} \\ \chi(z, 0) = \chi_0 \\ T(0) = T_0 \end{array} \right.$$

The control problem now is to find the power  $\varphi_{ir}$  such that the painting film mean humidity  $\bar{\chi}$  measurement tracks a desired profile. This desired profile is chosen to avoid bubbling and fissures phenomena. This process control problem can be schematized in input-output external representation as follows :



where the mean humidity  $\bar{\chi}$  is :

$$\bar{\chi}(t) = \frac{1}{e_{sec}} \int_0^{e_{sec}} \chi(\zeta, t) d\zeta$$

From a practical point of view, this control problem can be restated as : for a given humidity profile (for any given starting infrared power, there is a given humidity profile), find a control profile such that the final drying profile moves to a desired one. This can be reached by moving the representation model around any initial profile. This is a linearization problem which follows.

### 3. Time-varying linearized model

For a more accuracy presentation, let set the state  $\underline{\omega}^T = [\chi(z, t) \ T(t)]$ , the control  $u(t) = \varphi_{ir}(t)$  and the measured output  $y(t) = \bar{\chi}(t)$ . Then, the nonlinear control problem becomes :

$$\begin{cases} \dot{\underline{\omega}}(z, t) = \mathcal{A}(\underline{\omega}(z, t)) + \mathcal{B}(\underline{\omega}(z, t)) u(t) \\ y(t) = \mathcal{C}(\underline{\omega}(z, t)) \end{cases}$$

where the nonlinear operators  $\mathcal{A}, \mathcal{B}, \mathcal{C}$  depend on the  $F_1, F_2, F_3, F_4$  in  $(\mathcal{S}_1)$  :

$$\begin{cases} \mathcal{A}(\underline{\omega}(z, t)) = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \\ \mathcal{B}(\underline{\omega}(z, t)) = \begin{bmatrix} 0 \\ F_3 \end{bmatrix} \\ \mathcal{C}(\underline{\omega}(z, t)) = \frac{1}{e_{sec}} \int_0^{e_{sec}} \chi(\zeta, t) d\zeta \end{cases}$$

with :

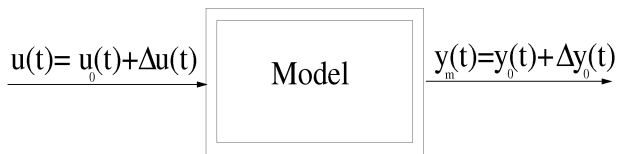
$$\begin{cases} D(\mathcal{A}) = D_1(\mathcal{A}) \cap D_2(\mathcal{A}) \cap D_3(\mathcal{A}) \\ \Omega = (0, e_{sec}) \\ D_1(\mathcal{A}) = \{\chi, \chi_z, \chi_{zz}, T \in L_2(\Omega)\} \\ D_2(\mathcal{A}) = \{\chi_z, \chi, T \mid F_4(\chi_z, \chi, T) = 0 \text{ at } z = e_{sec}\} \\ D_3(\mathcal{A}) = \{\chi_z \mid \chi_z = 0 \text{ at } z = 0\} \end{cases}$$

We assume that system  $(\mathcal{S}_1)$  is regular in the sense of the solution according to [2] or [4] for example.

A given profile can be represented by some control  $u_0(t)$  and  $y_0(t)$  according to a state vector profile  $\underline{\omega}_0(t)$  :

$$\begin{cases} \dot{\underline{\omega}}_0(z, t) = \mathcal{A}(\underline{\omega}_0) + \mathcal{B}(\underline{\omega}_0) u_0(t) \\ y_0(t) = \mathcal{C}(\underline{\omega}_0) \end{cases}$$

A variation model around this profile can be get by local linearization :



The control problem now is to find the control variation  $\Delta u$  such that the output tracks a known reference profile  $y_{ref}(t)$ .

A variation model around the profiles  $y_0(t), u_0(t)$  and  $\underline{\omega}_0(t)$  can be get by local linearization if  $y_{ref}(t)$  is in the neighbourhood of  $y_0(t)$ . This variation model around these trajectory is a time-varying one :

$$(\mathcal{S}_2) \begin{cases} \Delta \dot{\underline{\omega}}(z, t) = A(t) \Delta \underline{\omega}(z, t) + B(t) \Delta u(t) \\ \Delta y_0(t) = C \Delta \underline{\omega}(z, t) \\ \text{with the initial conditions :} \\ \Delta \underline{\omega}(z, 0) = 0 \end{cases}$$

Where  $A, B$  are time varying linear operators and  $C$  is a constant bounded operator with :

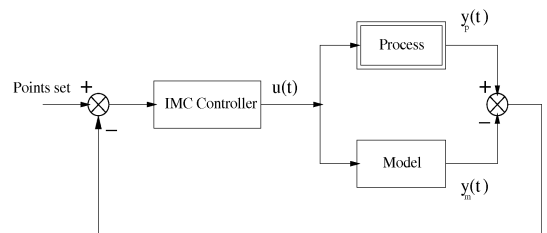
$$\begin{cases} A(t) = \begin{bmatrix} \nabla_{\chi} F_1(\underline{\omega}_0(t)) & \nabla_T F_1(\underline{\omega}_0(t)) \\ \nabla_{\chi} (F_2 + F_3 u_0)(\underline{\omega}_0(t)) & \nabla_T (F_2 + F_3 u_0)(\underline{\omega}_0(t)) \end{bmatrix} \\ B(t) = \begin{bmatrix} 0 \\ F_3(\underline{\omega}_0(t)) \end{bmatrix} \\ C \Delta \underline{\omega} = \frac{1}{e_{sec}} \int_0^{e_{sec}} \Delta \chi(\zeta, t) d\zeta \end{cases}$$

$$\begin{cases} D(A(t)) = D_1(A(t)) \cap D_2(A(t)) \cap D_3(A(t)) \\ D_1(A(t)) = \{\Delta \underline{\omega} \in L_2(\Omega) \mid \Delta \underline{\omega}, \Delta \underline{\omega}_z \text{ are a.c.}, \Delta \underline{\omega}_{zz} \in L_2(\Omega)\} \\ D_2(A(t)) = \{\Delta \underline{\omega}_z(0, t) = 0\} \\ D_3(A(t)) = \{\nabla_{\chi} F_4(\underline{\omega}_0) \Delta \chi(e_{sec}, t) + \nabla_T F_4(\underline{\omega}_0) \Delta T = 0\} \end{cases}$$

For the regularity property for the time-varying evolution system on can see [1] for example. Now this final time-varying control problem can be achieved by the following IMC strategy.

### 4. Internal linearized model control

The principle schema of IMC is described as follows :

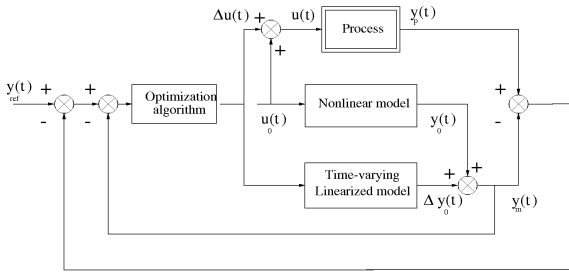


The objective of the IMC structure is to find a controller such that the process output tracks some reference points set in spite of some modelling error and added disturbances. With a good modelling, this structure allows to make a total disturbance rejection but has poor closed-loop performances. The IMC controller can be an explicite control law for linear time invariant systems even for distributed system [6]. But for nonlinear system and time-varying system, a more faisable approach is the indirect one.

The features are :

- to fixe a reference behaviour by possibly a variable points set,
- to compute over a prediction horizon, a control sequence which optimizes (minimizes) a tracking criterion,
- to use the model over this prediction horizon to predict the process output,
- the computation is restarted at each sampling to make a receding horizon over desired time.

The resulting structure, including optimization and according to the preceding local linearization, becomes :



### Optimization algorithm

The optimization concerns now the minimization problem for the tracking criterion over a given horizon. The discrete mathematical formulation using a simple quadratic criterion is :

$$\begin{cases} \min J(\Delta \underline{u}) = \sum_{j=k+1}^{j=k+N_p} \varepsilon(j)^T \varepsilon(j) \\ \Delta \underline{u} = (\Delta u_k \dots \Delta u_{k+N_c-1})^T \\ u_{min} \leq u_o + \Delta u_i \leq u_{max} \quad (i = 1 \text{ to } N_c) \\ \text{with } \varepsilon(j) = y_{ref}(jT_e) - y_p(jT_e) \end{cases}$$

where  $T_e$  is the time sampling period,  $N_c$  and  $N_p$  are respectively the control horizon and the prediction horizon.  $\Delta u_i$  denotes the time-varying linearized model control for  $t = iT_e$ .

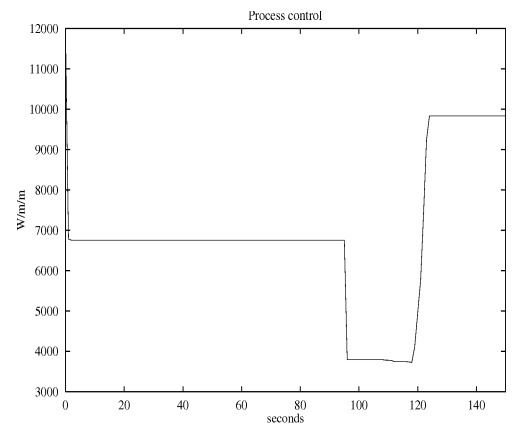
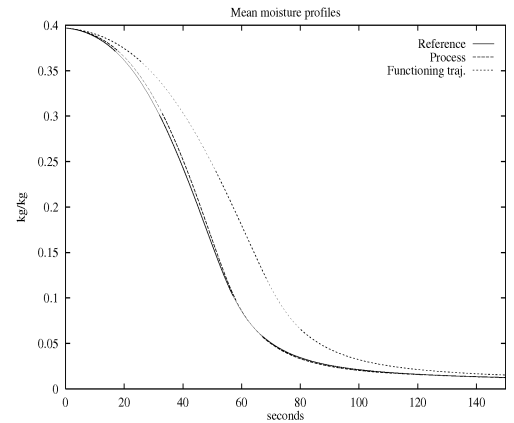
To solve this constrained optimization problem, we choose an interior penalty method associated with the Levenberg-Marquardt method [3]. The computation is made at each sampling period to get the optimal control vector  $\Delta \underline{u}$ , but only the first is applied to both time-varying linearized model and process (nonlinear model).

## 5.Simulation results

The nonlinear partial differential system and the time-varying linearized one are both solved by semi-finite difference technique (i.e finite difference only in spatial dimension variable). The resulting ordinary differential system is solved by using a predictor corrector method in the ACSL software package. The final control profile is found by the preceding nonlinear programming algorithm. The initial desired profile, as well as the linearized model, are chosen according to a final quality result. The conditions, according to the set of physical relations in the annexe, are :

$$\begin{cases} T_{air} = 52^\circ C \\ T_{sup} = 20^\circ C \\ T_0 = 26^\circ C \\ \chi_{air} = 20\% \\ \chi_0 = 0.397 \end{cases}$$

We find the following results for  $N_p = 4$ ,  $N_c = 1$ ,  $T_e = 1s$  :



As it can be seen, these results show a satisfying behaviour for the process simulated output by the control strategy. We can improve the control variable evolution by adding some regularity constraints without more difficulty. Remaining problem are more practical such as the horizons and sample periode determination. As for the computational time, it can be improved using over numerical method.

## 6. Conclusions and perspectives

In this work, we have given an efficient approach for the control of a class of nonlinear distributed parameter systems. It concerns nonlinear partial differential parabolic type equations with a tracking control problem. We have presented an internal linearized model control structure with predictive control approach using a local time-varying linear model around the tracking trajectory. As showed by the simulation results, this approach is efficient. The framework has been developed through an infrared drying plant example. The perspectives are a generalization study for theoretical limits and accurate characterization.

## 7. Annexe

The drying velocity  $\dot{m}(\chi, T)$  expression is :

$$\dot{m}(\chi, T) = C_7 \frac{1}{T_{moy}} [a_w(\chi) P_{v_{sat}}(T) - P_{v_{air}}]$$

where :

$$T_{moy} = \frac{T + T_{air}}{2}$$

The activity  $a_w(\chi)$  is solution of the equation :

$$\frac{a_w(\chi)}{\chi} = A_1 a_w^2(\chi) + A_2 a_w(\chi) + A_3$$

The saturated vapour pressure  $P_{v_{sat}}(T)$  is given in millibar by the expression :

$$\begin{aligned} \log_{10} P_{v_{sat}}(T) &= C_0 \left(1 - \frac{T_1}{T}\right) - C_1 \log_{10} \frac{T}{T_1} \\ &+ C_2 (1 - 10^{-C_3(T/T_1 - 1)}) \\ &+ C_4 (10^{C_5(1 - T_1/T)} - 1) + C_6 \end{aligned}$$

with  $T_1 = 273.16K$ , the  $C_i$  being positive constants.

The different following coefficients depend on the temperature  $T$  in Celsius degrees :

$$\left\{ \begin{aligned} l_v(T) &= [a_5 T^5 + a_4 T^4 + a_3 T^3 + a_2 T^2 + a_1 T + a_0] * 10^3 \\ \alpha_{ir}(\bar{\chi}) &= \alpha_p(\bar{\chi})(1 - \rho_p) + \alpha_s(1 - \rho_p)(1 - \alpha_p(\bar{\chi})) \\ &+ \alpha_p(\bar{\chi})(1 - \alpha_s)(1 - \rho_p)(1 - \alpha_p(\bar{\chi})) \\ \alpha_p(\bar{\chi}) &= 1 - [aa_4 \left(\frac{\bar{\chi}}{\bar{\chi}_0}\right)^3 + aa_3 \left(\frac{\bar{\chi}}{\bar{\chi}_0}\right)^2 + aa_2 \frac{\bar{\chi}}{\bar{\chi}_0} + aa_1] \frac{1}{1 - \rho_p} \\ C_p(\bar{\chi}, T) &= [oT + b + \bar{\chi}\{c_{p_3} T^3 + c_{p_2} T^2 + c_{p_1} T + c_{p_0}\}] * 10^3 \end{aligned} \right.$$

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