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## Multivariable Control of a

Catalytic Reverse Flow Reactor:

## Comparison between LQR and MPC Approaches

Co-authors: Dufour P. (speaker), Edouard D. & Hammouri H.

LAGEP University Claude Bernard Lyon 1-UMR CNRS 5007, France

# Outline

- Catalytic reverse flow reactor (RFR): description and control issues
- Model description
- Observer synthesis
- Control objective: Compare LQR and MPC performances
  - Provide **real-time multivariable controllers** for a RFR such that:
    - \* concentration of **pollutant released** at the inlet stay **below a threshold**,
    - \* limiting operating conditions are accounted for,
    - \* optimize the use of the control actions.
  - LQR design and simulation results
  - MPC design and simulation results
- Conclusions

## Principle scheme for the catalytic RFR



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## Monolith section

Cordierite monoliths of square cross sections (1\*1 mm) channels:



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- Monolith in the core region is catalytically active and is inert in both end sections.
- Through **periodic flow reversal**, **heat released by reaction** is first trapped in the packing and then used to **heat up the feed**. Therefore, high temperatures exist in catalyst bed whereas the inlet and outlet gas stream (VOC) have ambient temperature.
- In the core region, an **electric heater** maintains ignition temperature, while the temperature in the catalytic layer is decreased by **fresh air dilution**.
- A blower located downstream of the RFR keeps aspiration of the pollutant at a **constant flow rate**.
- The packed layer is **adiabatic**, **except** in the core region where **heat loss** is inevitable due to both the installation for air dilution and the high temperature in this region.
- Highest temperature is located at the inlet of catalytic monoliths.

## Various behaviours of the RFR: control issues

According to the **operating conditions**, **various behaviours** are possible:

- If feed of pollutant concentration is too rich: release of heat due to the reaction produces thermal overheating that deteriorates the catalysts. Temperature inside the reactor has therefore to be maintained under 600K [Ramdani et al., 2001].
- If feed of pollutant concentration is too lean: low heat released during the reaction leads to the extinction of the reactor.
   Temperature inside the reactor has therefore to be maintained over 450K [Ramdani et al., 2001].
- If the **feed of pollutant concentration** is such that the reaction temperature falls **within an envelope** outlined by the two previous boundary temperatures: **autothermal operation** is reached and **no control action** would be required.

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## Overview of RFR control (1)

- Various ideas have been proposed to provide efficient means of **temperature control**:
  - [Nieken et al., 1995] to suppress temperature run-away when rich feed: cold gas injection, hot gas withdrawing or heat recovery through internal heat exchangers;
  - [Cunill et al., 1997] to prevent extinction for lean feed: hot gas supply or internal heating.
- [Budman et al., 1996] was the first complete study on RFR control: a parametric study allows characterizing the use of 2 RFR MVs: coolant flow rate and cycle time. 2 SISO controllers (MV=coolant flow rate) assume that the temperature and the concentration at the RFR inlet were constant input disturbances: use of PID and a feed forward controller which is not usable during transient conditions.

## Overview of RFR control (2)

- [Keith, 2003]: to avoid extinction and overheating, a simple switching control law strategy **tunes the switching time**.
- [Dufour et al., 2003]: **SISO MPC** optimizes the consumption of electrical power accounting for the **maximum amount of concentration pollutant released** at the outlet. Limitation was concerned with the **impossibilities to control overheating** and to handle **stochastic variations** of the inlet concentration.
- [Dufour et al., 2004]: **MIMO MPC** optimizes the consumption of electrical power and the dilution rate while **maintaining the hot spot temperature inside the prescribed enveloppe**. Limitation was concerned with the difficulties to handle **stochastic variations** of the inlet concentration. **Real time** control aspect becomes **critical**.

- Motivation: the model is used in the optimal model based control framework and the observer synthesis. The current periodic model used [Ramdani et al., 2001] need a set of 800 DAE, and the sampling time is 10s: accuracy and time spent for model calculation are critical.
- The new model considered for control purpose is obtained from a countercurrent pseudo-homogeneous model [Edouard, Hammouri, 2004]):
  - Assume that the **kinetic reaction can be neglected** under strong mass transfer limitation,
  - Accounts for **periodic frequency correction**,
  - 1 nonlinear parabolic PDE, 2 algebraic equations, and nonlinear boundary conditions,
  - Model more accurate and faster to solve than [Dufour et al., 2004]).



Left: main geometrical characteristics of the RFR. Right: the countercurrent model.

#### Notation: some elements [Edouard & Hammouri, 2004]

- x = Normalized space variable [-]
- $T_s =$ Solid temperature [K]

 $T_{g1}, T_{g2} =$ Upstream and downstream gas temperature in the monolith [K]

- $T_0$  = ambient temperature and feed temperature [K]
- $\omega = \text{VOC} \text{ mass fraction [-]}$
- $\theta$  = Period of flow reversal [s]
- $\alpha$  = fraction of feed flow rate [-]:
  - if  $\alpha = 1$ : no dilution
  - if  $\alpha = 0$ : maximum dilution
- $Q_j = \text{External power supply } [W]$

 $\varphi(x) = \text{Characteristic function of the catalytic monolith [-]:}$ 

- $\varphi(x) = 0$  in the inert monoliths  $(x < \xi_{\theta})$
- $\varphi(x) = 1$  in the catalytic monoliths  $(x \ge \xi_{\theta})$

N' = Number of transfer units for heat loss [-]

Normalizing some variables, we obtain the nonlinear PDE model:

$$\tau \frac{\partial T_s(x,t)}{\partial t} = \left(\frac{1}{P_{ax_{\theta}}} + \frac{1+\alpha^2}{2P_{\theta}}\right) \frac{\partial^2 T_s(x,t)}{\partial x^2} + \frac{1-\alpha}{2} \frac{\partial T_s(x,t)}{\partial x} + P_{\theta} \Delta T_{ad}(t) \psi(x,t)$$
$$\alpha \frac{\partial \omega_1(x,t)}{\partial x} + P_{\theta} \omega_1(x,t) = 0, \quad -\frac{\partial \omega_2(x,t)}{\partial x} + P_{\theta} \omega_2(x,t) = 0$$
(1)

with:

$$P_{\theta} = \frac{ha_{c}H}{2\rho_{0}u_{v0}c_{pmg}} (1 - \frac{\theta}{2\tau}), \quad \frac{1}{P_{ax_{\theta}}} = \frac{2\lambda_{ax}^{s}}{(H\rho_{0}u_{v0}c_{pmg})(1 - \frac{\theta}{2\tau})}$$
$$\tau = \frac{(1 - \varepsilon)\rho_{s}c_{ps}H}{2\rho_{0}u_{v0}c_{pmg}}, \quad x = \frac{z}{H/2}$$
$$\Delta T_{ad}(t) = \frac{\Delta H}{Mc_{pmg}}\omega_{10}(t), \quad \psi(x, t) = \varphi(x)\frac{\omega_{1}(x, t) + \omega_{2}(x, t)}{2\omega_{10}(t)}$$

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The boundary condition at x = 0 is:

$$\begin{split} \omega_1(x,t) &= \omega_{10}(t) \\ T_{g1}(x,t) &= T_s(x,t) - \frac{\alpha}{P_\theta} \frac{\partial T_s(x,t)}{\partial x} = T_0 \end{split}$$

The boundary condition at x = 1 is:

$$\begin{cases} (1+N')(T_{g2}(x,t)-T_0) = \alpha(T_{g1}(x,t)-T_0) + \frac{Q_j}{S\rho_0 u_{v0}C_{pmg}} \\ T_{g1}(x,t) = T_s(x,t) - \frac{\alpha}{P_{\theta}} \frac{\partial T_s(x,t)}{\partial x} \\ T_{g2}(x,t) = T_s(x,t) + \frac{1}{P_{\theta}} \frac{\partial T_s(x,t)}{\partial x} \\ \alpha \omega_1(x,t) = \omega_2(x,t) \end{cases}$$

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Initial conditions are at t = 0:

$$T_{g1}(x,t) = T_{g2}(x,t) = T_s(x,t) = T_0$$

The first term in the right hand side of (1) involves an effective axial heat conductivity given by:

$$\left(\frac{1}{P_{ax_{\theta}}} + \frac{1+\alpha^2}{2P_{\theta}} = \frac{\lambda_{eff}}{\rho_0 u_{v0} c_{pmg} H/2(1-\frac{\theta}{2\tau})} \\ \lambda_{eff} = \lambda_{ax}^s + \frac{1+\alpha^2}{2} \frac{(\rho_0 u_{v0} c_{pmg})^2}{ha_c} \right)$$
(2)

When there is no dilution ( $\alpha = 1$ ),  $\lambda_{eff}$  reduces to the well-known estimate of [Vortmeyer & Schaefer, 1974] as used by Nieken [Nieken et al., 1995]. **This model experimentally matches the process behaviour during open-loop control and identification** [Edouard & Hammouri, 2004].

#### Observer [Edouard & Hammouri, 2004]

An observer has to be designed since:

- The **input disturbance** (inlet VOC concentration) has a more realistic stochastic behaviour than in the previous RFR control studies, and **has** a large influence over the process behavior,
- The LQR formulation needs the temperature profile,
- The MPC needs the input disturbance.

The input disturbance and the temperature profile in the RFR are estimated on-line using a high gain observer based on three temperatures measurements [Edouard & Hammouri, 2004].

## LQR formulation

Since it seems natural to avoid heating and cooling at the same time, 2 cost functions are considered (based on the linearized model):

- When the **pollutant concentration is too low**, the **LQR tunes**  $\Delta Q_j$  such that the temperature is kept over a 450K:

$$\int_{0}^{\infty} (p_1^0(\Delta X(t))^2 + p_2^0(\Delta Q_j(t))^2) dt$$

- When the **pollutant concentration is too high**, the **LQR tunes**  $\Delta \alpha$  such that the temperature is kept below 600K:

$$\int\limits_{-\infty}^{\infty} (p_1^1(\Delta X(t))^2 + p_2^1(\Delta \alpha(t))^2) dt$$



## LQR formulation

- In both cases, the control obtained from the minimization of these cost functions is a linear feedback of the form  $\Delta Q_j(t) = -F^0 \Delta X$  or  $\Delta \alpha(t) = -F^1 \Delta X$  with  $\Delta X = X_e(t) - Z^i$  where  $(i = 0, 1), X_e(t)$  is the estimate state given by the observer and each steady state is  $Z^i$ .
- The gains of the feedback matrix are obtained from the solution of the classical algebraic Riccati equation.

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## MPC formulation: model linearizations

To decrease the on-line computational burden, the model is linearized about the nominal nonlinear model with variations s.t.:

$$Q_j(k) = Q_{j,0}(k) + \Delta Q_j(k)$$
$$\alpha(k) = \alpha_0(k) + \Delta \alpha(k)$$
$$X(k) = X_0(k) + \Delta X(k)$$

The linearized model used in the MPC is:

 $\Delta \dot{X}(t) = A(\alpha_0, Q_0, X_0) \Delta X(t) + B_1(\alpha_0, Q_0, X_0) \Delta \alpha(t) + B_2(\alpha_0, Q_0, X_0) \Delta Q_j(t)$ 

### MPC formulation [Dufour & Touré, 2004]

#### The MPC formulation is:

$$\begin{split} &\min_{\Delta Q_j(k),\Delta\alpha(k)} J(\Delta Q_j(k),\Delta\alpha(k)) = (w_Q[\frac{Q_j(k)-Q_{j,\min}}{Q_{j,\max}-Q_{j,\min}}]^2 + w_\alpha[\frac{\alpha(k)-\alpha_{\max}}{\alpha_{\max}-\alpha_{\min}}]^2) \\ &\text{with constraints on MVs magnitude:} \\ &Q_{j,\min} = 0W \leq Q_j(j) = Q_{j,0}(j) + \Delta Q_j(j) \leq Q_{j,\max} = 3000W \\ &\Delta\alpha_{\min} = 0 < \alpha(j) = \alpha_0(j) + \Delta\alpha(j) \leq \Delta\alpha_{\max} = 0.95 \\ &\text{with 2 constraints } c_i \text{ on the estimated state:} \\ &c_1(X(j),Q_j(j),\alpha(j)) = T_{s,\min} - T_s(x_{101},j) \leq 0 \quad (j \in \{k + hp_1^i, \ k + hp_1^f\}) \\ &c_2(X,Q_j(j),\alpha(j)) = T_s(x_{101},j) - T_{s,\max} \leq 0 \quad (j \in \{k + hp_2^i, \ k + hp_2^f\}) \end{split}$$

### MPC formulation [Dufour & Touré, 2004]

- k=actual discrete time index, j=discrete time index.
- $w_Q$  and  $w_\alpha$  are positive weights.
- In order to decrease the computational burden, the **tuning of the control horizon is one**. Even if it is true that this tuning reduces strongly the degrees of freedom to solve the optimization problem, it **allows decreasing the computational time**, which is an important issue here.
- Tuning parameters are the horizons:  $hp_1^i$  and  $hp_1^f$  (resp.  $hp_2^i$  and  $hp_2^f$ ) are the initial and future prediction horizons describing the lower (resp. upper) temperature constraint.

### MPC: Tuning of the horizons

It is clear that the tuning of the controller parameters directly influences the constrained optimization problem and therefore closed-loop control results, since:

- This constrained optimization problem uses the estimation of the input disturbance (the adiabatic temperature rise ΔT<sub>ad</sub>) which has an evident impact over the closed-loop performances (especially the constraints satisfaction).
- The **non-miminum phase behavior** of the process has to be handled.

Regarding the **impact of the input disturbance** over the optimization task and the constraints satisfaction required, some **guidelines for the tuning** of the controller parameters are first given.

Non-miminum phase behavior

Time evolution of 2 distributed sensitivities of  $T_s$  w.r.t.  $\alpha$ :



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## MPC: Tuning of the initial horizon prediction

- The cooling action creates a non-minimum phase behaviour in the temperature involved in output constraints. Therefore, initial prediction horizon has to be tuned sufficiently large such that this behaviour is not accounted for into the controller, especially when constraints are saturated.
  - Conclusion: a sufficiently large initial prediction horizon is required.
- If some constraints are currently saturated, they may no more be satisfied in the very close future due to possible strong variations of ΔT<sub>ad</sub>.

Conclusion: the smallest initial prediction horizon is required.

## MPC: Tuning of the final horizon prediction

• The **dynamic of the temperature** involved in the output constraints is relatively **low** and has to be accounted for into the constrained optimization problem.

Conclusion: a sufficiently large final prediction horizon is required.

Since ΔT<sub>ad</sub> is used in the prediction framework, is strongly stochastic and has a large impact over closed-loop performances, large prediction horizons will forecast an uncertain future.
 Conclusion: the smallest final prediction horizon is required.

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## Conclusions

- In spite of the **input disturbance**,  $T_s$  can be **maintained** by both controllers **inside the specified temperature envelope**.
- Concerning the optimization performances, LQR > MPC since it requires less heating action while treating more gas: (\$\vec{Q}\_j\$ = 83.4\$W\$, \$\vec{\alpha}\$ = 0.894\$) for LQR, (\$\vec{Q}\_j\$ = 274.6\$W\$, \$\vec{\alpha}\$ = 0.849\$) for MPC (\$0 ≤ t ≤ 4500\$s\$). MPC overevaluates the real control needs since the stochastic variations of the input disturbance.
- With the non minimum phase behaviour of the process, the **horizons tuning is uneasy for the MPC**.
- A switching control structure is used for the LQR: therefore, heating and cooling action are initially decoupled. The drawback is that it introduces a severe nonlinearity for stability analysis whereas MPC is more suitable than the LQR for general MIMO control problems.

# Notation

$$\begin{split} a_c &= \text{Specific solid-fluid surface area } [m^{-1}] \\ c_{ps} &= \text{Solid heat capacity } [J.kg^{-1}.K^{-1}] \\ c_{pmg} &= \text{Fluid heat capacity } [J.kg^{-1}.K^{-1}] \\ H &= \text{Total length of monolith } [m] \\ h &= \text{Solid-fluid heat transfer coefficient } [W.m^{-2}.K^{-1}] \\ k_D &= \text{Solid-fluid mass transfer coefficient } [m.s^{-1}] \\ M &= \text{VOC molecular weight } [kg.mol^{-1}] \\ N' &= \text{Number of transfer units for heat loss } [-] \\ P &= \text{Peclet number for solid-fluid heat transfer } [-] \\ P_{ax} &= \text{Axial Peclet number for heat conduction } [-] \\ P_{\theta} &= \text{P corrected for the finite frequency } [-] \\ Q_j &= \text{External power supply } [W] \\ S &= \text{Total cross-section of the monolith } [m^2] \\ T_{g1}, T_{g2} &= \text{Gas temperature in the upstream, downstream monolith } [K] \end{split}$$

# Notation

 $T_0$  = Inlet and external temperature [K]

$$T_{max}$$
 = Maximum solid temperature in the RFR [K]

 $T_s$  or T = Solid temperature [K]

t = Time [s]

 $u_{v0}$  = Superficial gas velocity in the reference state  $[m.s^{-1}]$ 

- $x={\rm Reduced}$ abscissa,2z/H[-]
- z = Abscissa [m]
- $\alpha$  = fraction of feed flow rate [-]
- $\Delta H = \text{Reaction enthalpy } [J.mol^{-1}]$
- $\Delta T_{ad} = \text{Adiabatic temperature rise [K]}$
- $\varepsilon$  = fraction of open frontal area [-]

$$\varphi(x) = ext{Characteristic function of the catalytic monolith [-]}$$

$$\psi(x)=arphi(x)(\omega_1+\omega_2)/(2\omega_{10})$$
[-]

 $\rho = \text{Fluid density } [kg.m^{-3}]$ 

# Notation

- $\rho_0 = \text{Gas density in the reference state } [kg.m^{-3}]$
- $\rho_s = \text{Solid density } [kg.m^{-3}]$
- $\tau$  = Heat storage time constant [s]
- $\theta$  = Period of flow reversal [s]

 $\omega_{1su}, \omega_{2su} =$  VOC mass fraction of solid phase in the up/downstream monoliths [-]

 $\omega_1,\omega_2={
m VOC}\ {
m mass}\ {
m fraction}\ {
m in}\ {
m the}\ {
m up/downstream}\ {
m monoliths}\ [-]$ 

$$\omega_{10} = \text{VOC}$$
 mass fraction in the feed [-]

- $\xi = {
  m Reduced \ abscissa \ of \ the \ boundary \ between \ inert/catalytic \ monoliths}$
- $\xi_{\theta} = \xi$  corrected for the finite frequency [-]

X =State vector

 $X_e = \text{Estimated state vector}$ 

y = Filtered temperature measurements [K]

 $\Omega$  = Calibration parameter of the observer [-]

## Gains in the LQR: algebraic Riccati equation

$$\begin{split} F^{0} &= (p_{2}^{0})^{-1} (B_{2}^{0})^{T} S^{0} \\ S^{0} A(\alpha^{0}) &+ (A(\alpha^{0}))^{T} S^{0} + C^{T} C - S^{0} B_{2}^{0} (p_{2}^{0})^{-1} (B_{2}^{0})^{T} S^{0} = 0 \\ F^{1} &= (p_{2}^{1})^{-1} (B_{1}^{1})^{T} S^{1} \\ S^{1} A(\alpha^{1}) &+ (A(\alpha^{1}))^{T} S^{1} + C^{T} C - S^{1} B_{1}^{1} (p_{2}^{1})^{-1} (B_{1}^{1})^{T} S^{1} = 0 \end{split}$$

where  $C = [C_1 \dots C_{100} \ C_{101} \dots C_{200}]$  with  $[C_1 \dots C_{100}] = [0 \dots 0]$  and  $[C_{101} \dots C_{200}] = [1 \ 0 \dots 0].$ 

(3)

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a transformation method [Fletcher, 1987]

From the **constrainted variable**  $\theta$ :

 $\theta_{\min} \le \theta \le \theta_{\max}$ 

an **unconstrained variable** p is defined such that:

$$\theta = f(p) = \frac{1}{2}(\theta_{\max} + \theta_{\min}) + \frac{1}{2}(\theta_{\max} - \theta_{\min})\tanh(p)$$



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