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Observer based Model Predictive Control
of the Water based Painting Drying
using a Humidity Profile Soft Sensor
and a Temperature Measurement

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Outline

Objective: provide a **real-time controller** for **processes modeled by Partial Differential Equations:**

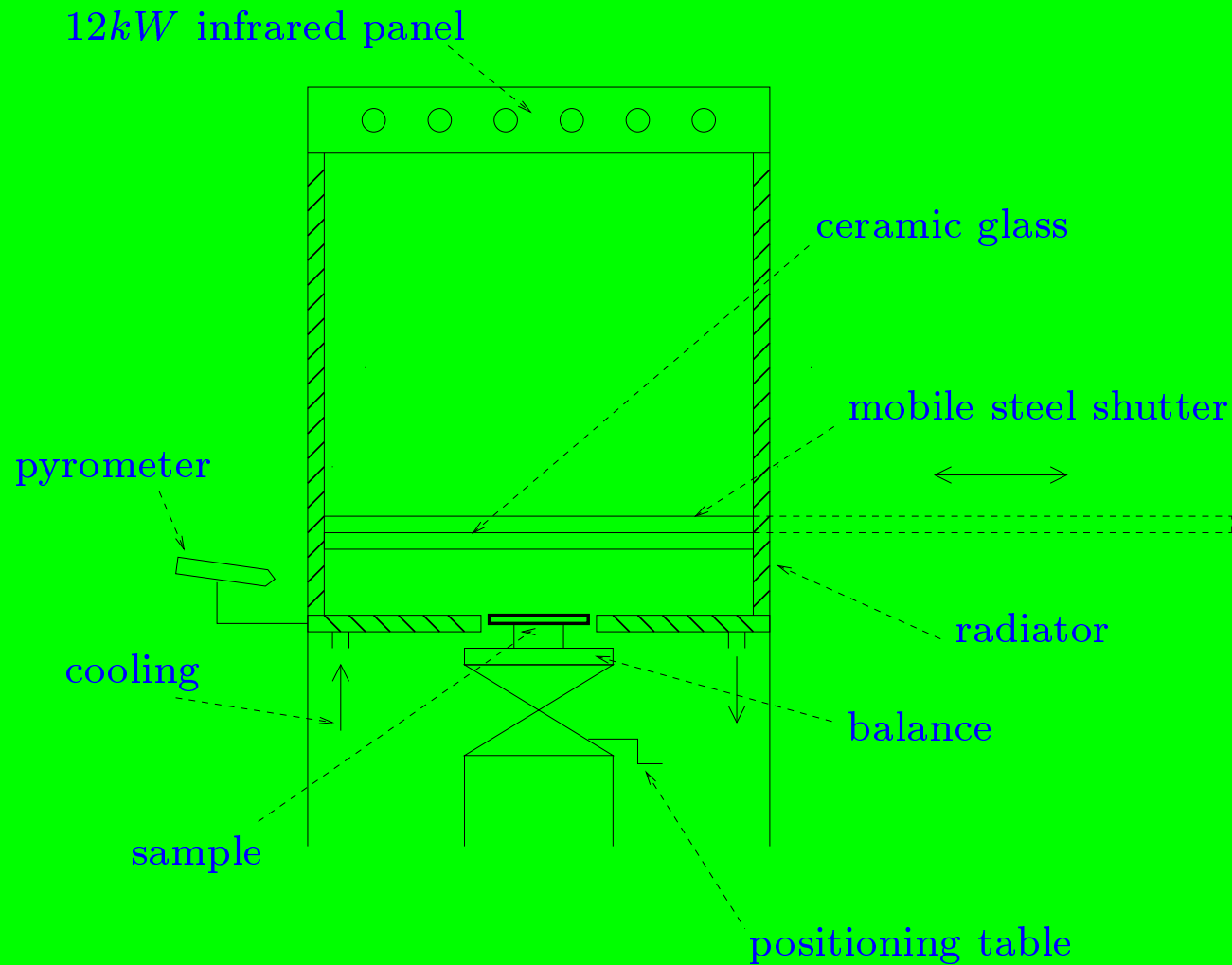
- in order to solve any **optimization task** (i.e. : optimize productivity, decrease processing time, ...) ,
- while including any **process limitations** (i.e. : actuators saturation, safety conditions, limiting operating conditions, ...).

Solution: a **model based predictive control (MBPC)** approach.

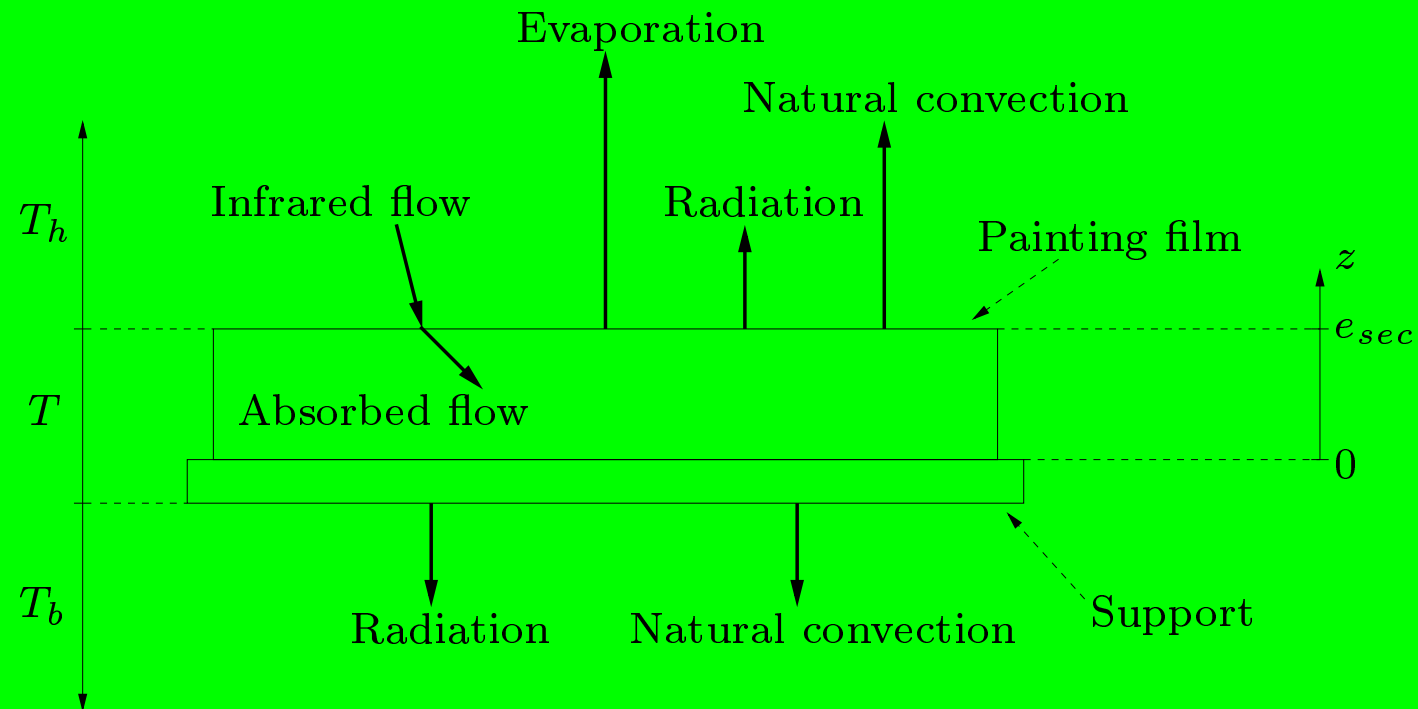
↪ **Necessary to reduce the time required by the on-line controller** since we have to:

- solve an on-line constrained optimization problem,
- handle a nonlinear dynamic PDE model,
- **estimate unavailable measurements with a software sensor.**

Infrared drying process of painting film [Blanc et al., 1998]



Thermal flows



Energy balance [Blanc et al., 1997]

$$\begin{cases} (\rho_p C_p(\bar{\chi}, T)e_p + \rho_s C_s e_s) \frac{\partial T}{\partial t} = - \sum_{i=1}^5 P_i + P_{abs}, \quad t > 0 \\ T(0) = T_0 \end{cases} \quad (1)$$

with the losses P_i :

$$\begin{cases} P_1 = h_c(T - T_b), \quad P_2 = \sigma_2(T^4 - T_b^4) \\ P_3 = h_c(T - T_h), \quad P_4 = \sigma_4(T^4 - T_h^4) \\ P_5 = l_v(T)\dot{m}(\bar{\chi}, T) \end{cases} \quad (2)$$

and the absorbed **infrared flow**:

$$P_{abs} = \alpha_{ir}(\bar{\chi})\varphi_{ir}(t) \quad (3)$$

Mass balance [Blanc et al., 1997]

$$\left\{ \begin{array}{l} \frac{\partial \chi}{\partial t} = \frac{\partial}{\partial z} \left[D_{eff}(\chi, T) \frac{\partial \chi}{\partial z} \right] \text{ for } 0 < z < e_{sec}, t > 0 \end{array} \right.$$

$$\text{with } D_{eff}(\chi, T) = \frac{D_0 \exp\left(\frac{-a}{\chi}\right) \exp\left(\frac{-E_a}{RT}\right)}{(1+\phi\chi)^2}$$

with the boundary conditions :

$$\left\{ \begin{array}{l} -D_{eff}(\chi, T) \frac{\partial \chi}{\partial z} = \frac{\dot{m}(\bar{\chi}, T)}{\rho} \text{ for } z = e_{sec}, t > 0 \end{array} \right. \quad (4)$$

$$\frac{\partial \chi}{\partial z} = 0 \text{ for } z = 0, t > 0$$

with the initial condition:

$$\left\{ \begin{array}{l} \chi(z, 0) = x_i(z) \text{ for } 0 \leq z \leq e_{sec} \end{array} \right.$$

Nonlinear dynamic PDE model (\mathcal{S}_{NL})

$$\left\{ \begin{array}{l} \frac{\partial x_m}{\partial t} = F_d(x_m) \text{ for } 0 < z < e_{sec}, t > 0 \\ F_b(x_m, u) = 0 \text{ sur for } z \in \{0, e_{sec}\}, t > 0 \\ y_m = Cx_m \text{ sur for } z = 0, t > 0 \\ x_m(z, 0) = x_i(z) \end{array} \right. \quad (5)$$

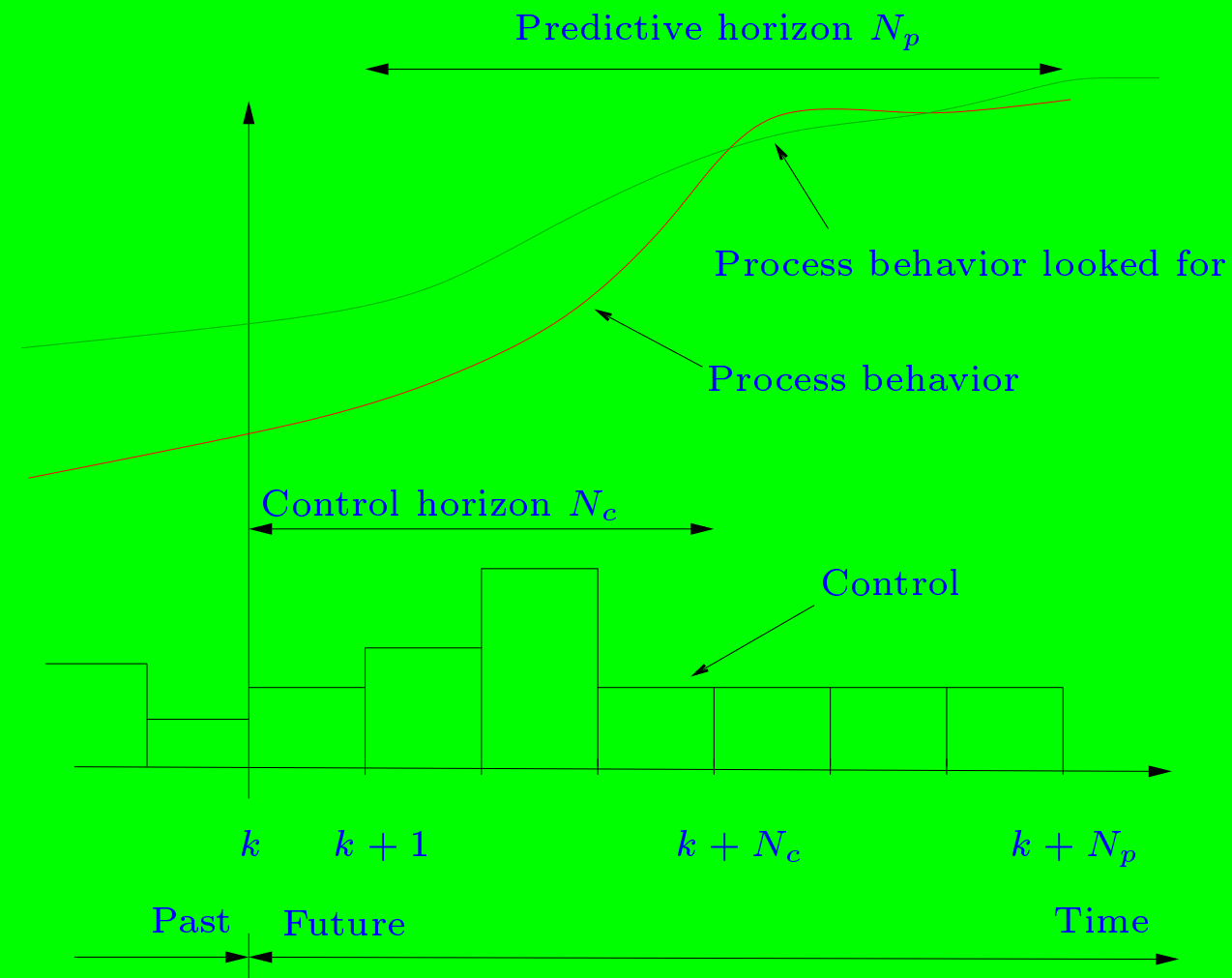
the **state vector** is $x_m(z, t) = (T(t), \chi(z, t))^T$, the **manipulated variable** $u(t)$ is the infrared flow $\varphi_{ir}(t)$. The **output variable vector** is $y_m(t)$.

Key issue for the drying process:

Software estimation of unmeasured state variables

- **Definition:** Observers are algorithms used to estimate variables which are not measured or can not be measured.
- **1. Observer design:** two informations about the process are needed:
 - its theoretical behaviour: a **model**,
 - its experimental behaviour: the **on-line available measurements**.
- **2. Experimental validation of the observer:** real dynamic measurements are first needed. Here, which **new additional devices** can we use to get the humidity profile inside the low thickness painting sample ?
- **3. Use of the observer:** the estimated process humidity profile $\hat{\chi}_p$ is then used by the controller.

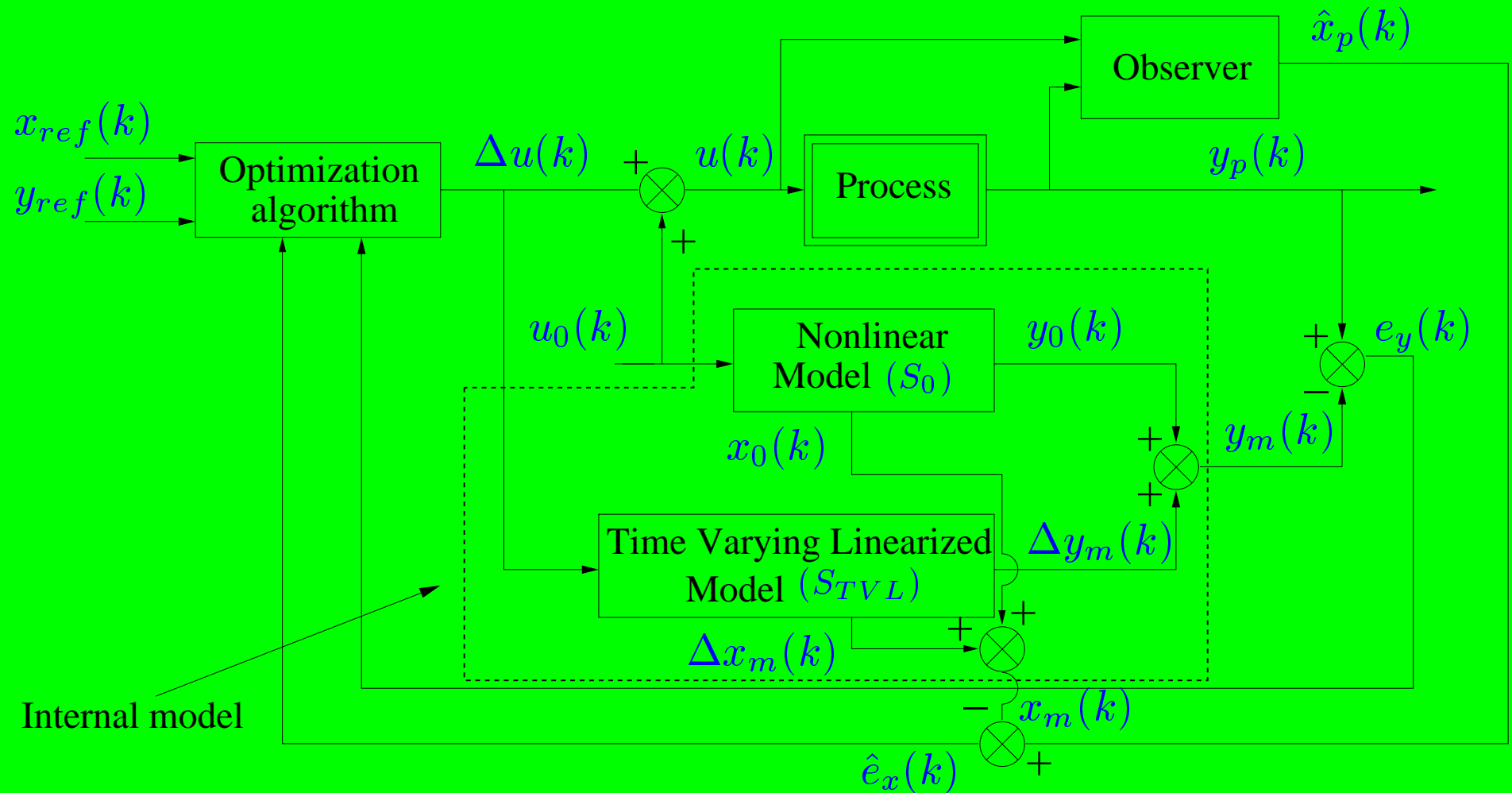
MPC framework: the paradigm [Qin & Badgwell, 2003]



MPC framework: main features

- **Key idea: forecast future process behavior: a model is needed.**
 - ★ Historically, models obtained from **experimental step response** were used: simple but local,
 - ★ Now, use of **first principle models**: complex but more global.
 - **On-line decision level: need to solve an on-line constrained optimization problem** over the future window, based on the model:
 - ★ Need to **reduce the on-line computational burden**,
 - ★ Need to **handle constraints**.
 - In the closed loop structure, **proof of stability** are needed.
- ⇒ **Our approach**: explicitly handle a **nonlinear dynamic PDE model** in a MPC strategy, and **reduce the on-line computational burden**.

Control structure



Initial constrained optimization problem

$$\left\{ \begin{array}{l}
 \min_{\Delta \tilde{u}} J(\Delta \tilde{u}) = \frac{1}{N_p} \sum_{j=k+1}^{j=k+N_p} g(y_{ref}(j), x_{ref}(j), \Delta y_m(j), \Delta x_m(j), e_y(k), \hat{e}_x(k)) \\
 \Delta \tilde{u} = [\Delta u(k) \dots \Delta u(k + N_c - 1)]^T \\
 (\mathcal{S}_{TVL}) \left\{ \begin{array}{l}
 \frac{\partial \Delta x_m}{\partial t} = \frac{\partial F_d}{\partial x_m|_{(S_0)}} \Delta x_m \text{ for } 0 < z < e_{sec}, t > 0 \\
 \frac{\partial F_b}{\partial x_m|_{(S_0)}} \Delta x_m + \frac{\partial F_b}{\partial u|_{(S_0)}} \Delta u = 0 \text{ for } z \in \{0, e_{sec}\}, t > 0 \\
 \Delta y_m = C \Delta x_m \text{ for } 0 \leq z \leq e_{sec}, t > 0 \\
 \Delta x_m(z, 0) = 0 \text{ with for } 0 \leq z \leq e_{sec} \\
 \Delta u_{min} \leq \Delta u(j) \leq \Delta u_{max} \\
 \Delta(\Delta u_{min}) \leq \Delta u(j) - \Delta u(j - 1) \leq \Delta(\Delta u_{max}) \\
 c_i(y_{ref}(j), x_{ref}(j), \Delta y_m(j), \Delta x_m(j), e_y(k), \hat{e}_x(k)) \leq 0 \quad (i \in \mathcal{I})
 \end{array} \right.
 \end{array} \right. \quad (6)$$

Constraints on the manipulated variable:

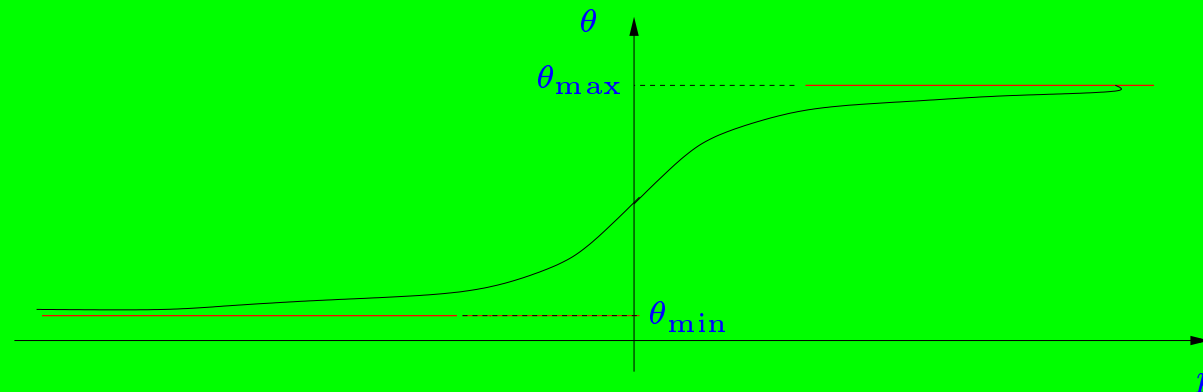
a transformation method [Fletcher, 1987]

From the **constrained variable** θ :

$$\theta_{\min} \leq \theta \leq \theta_{\max} \quad (7)$$

an **unconstrained variable** p is defined such that:

$$\theta = f(p) = \frac{1}{2}(\theta_{\max} + \theta_{\min}) + \frac{1}{2}(\theta_{\max} - \theta_{\min}) \tanh(p) \quad (8)$$



Constraints on measured and estimated variables:

external penalty method [Fletcher, 1987]

$$\min_{\theta} J_{tot}(\theta) = J(\theta) + w H(c_i(\theta)) \quad (i \in \mathcal{I})$$

$$\theta \in \mathbb{R}^{\theta}, w > 0$$

$$H(c_i(\theta)) = \sum_{i \in \mathcal{I}} h_i(c_i(\theta))$$

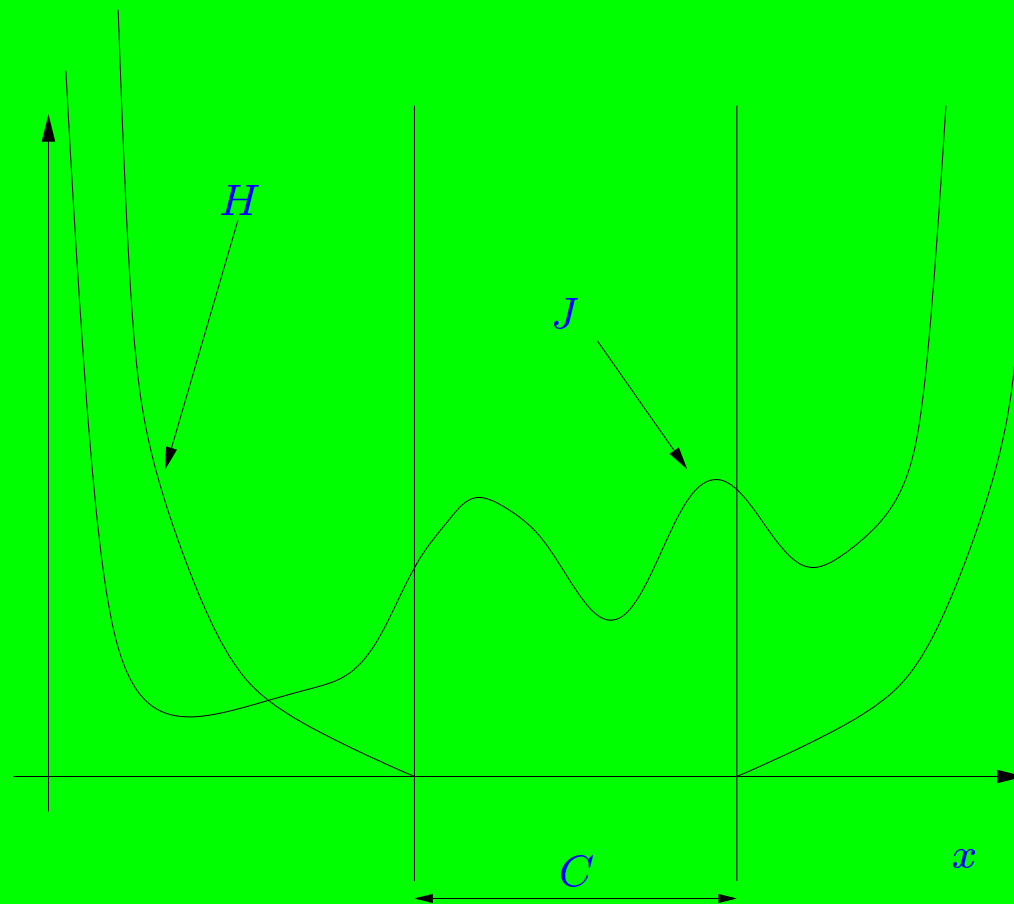
here h_i is any C^1 function (at least) such that:

- $h_i(c_i(\theta)) = 0$ if $c_i(\theta) \leq 0$ (i.e. when the constraint is satisfied or saturated)
- and $h_i(c_i(\theta)) > 0$ if $c_i(\theta) > 0$ (i.e. when the constraint is not satisfied)

$$\text{here: } h_i(c_i(\theta)) = \max^2(0, c_i(\theta))$$

Constraints on measured and estimated variables:

external penalty method [Fletcher, 1987]



Final unconstrained optimization problem

$$\min_{\Delta \tilde{p}} J_{tot}(\Delta \tilde{p}) = \frac{1}{N_p} \sum_{j=k+1}^{j=k+N_p} g(y_{ref}(j), x_{ref}(j), \Delta y_m(j), \Delta x_m(j), e_y(k), \hat{e}_x(k))$$

$$\Delta \tilde{p} = [\Delta p(k) \dots \Delta p(k + N_c - 1)]^T$$

based on the transformation method: $u \rightleftharpoons p, \Delta u \rightleftharpoons \Delta p$

$$(\mathcal{S}_{TVL}) \left\{ \begin{array}{l} \frac{\partial \Delta x_m}{\partial t} = \frac{\partial F_d}{\partial x_m|_{(S_0)}} \Delta x_m \text{ for } 0 < z < e_{sec}, t > 0 \\ \frac{\partial F_b}{\partial x_m|_{(S_0)}} \Delta x_m + \frac{\partial F_b}{\partial u|_{(S_0)}} \Delta u = 0 \text{ for } z \in \{0, e_{sec}\}, t > 0 \\ \Delta y_m = C \Delta x_m \text{ for } 0 \leq z \leq e_{sec}, t > 0 \\ \Delta x_m(z, 0) = 0 \text{ with for } 0 \leq z \leq e_{sec} \end{array} \right.$$

(10)

⇒ Optimization problem solved by the Levenberg-Marquardt's algorithm.

Simulation results:

optimization problem tackled for the drying process

Minimize the drying time, hence maximize the drying rate $\dot{m}(\bar{\chi}, T)$:

$$\max_{\varphi_{ir}} J(\varphi_{ir}) = \frac{1}{N_p} \sum_{j=k+1}^{j=k+N_p} [\dot{m}(\bar{\chi}(j), T(j))]^2$$

Accounting for the infrared flow limitations:

$$\text{magnitude: } 0 \text{ W.m}^{-2} \leq \varphi_{ir}(j) \leq 12000 \text{ W.m}^{-2} \quad (11)$$

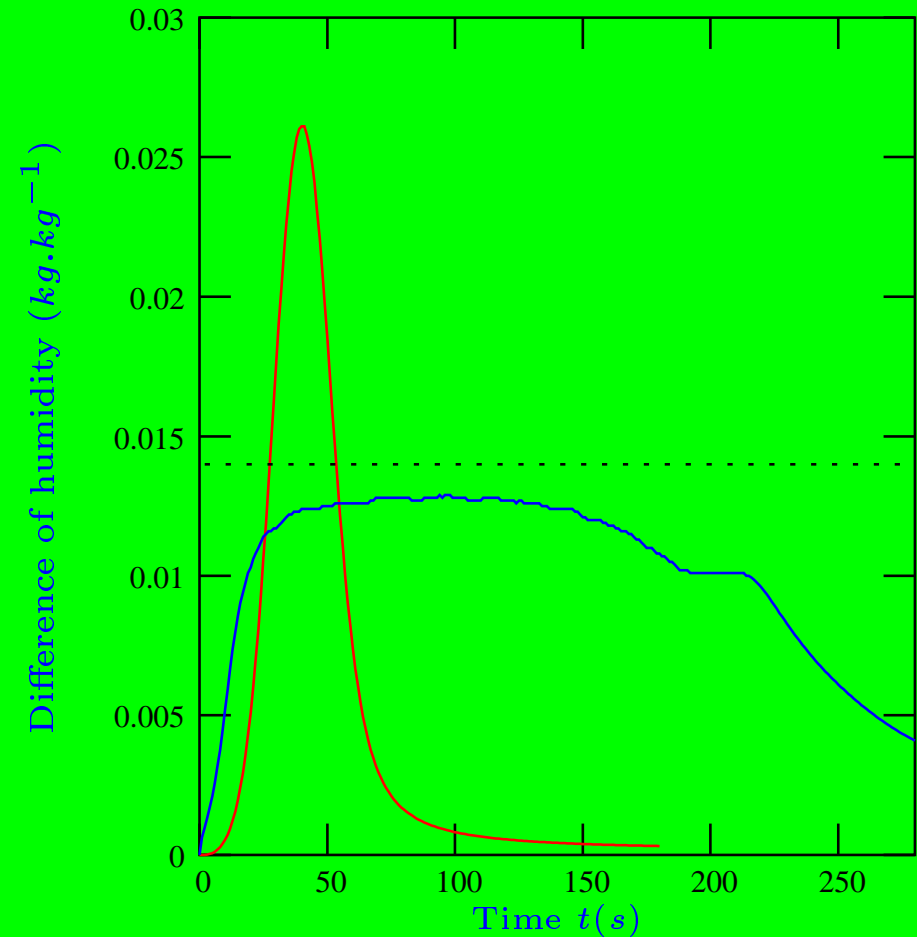
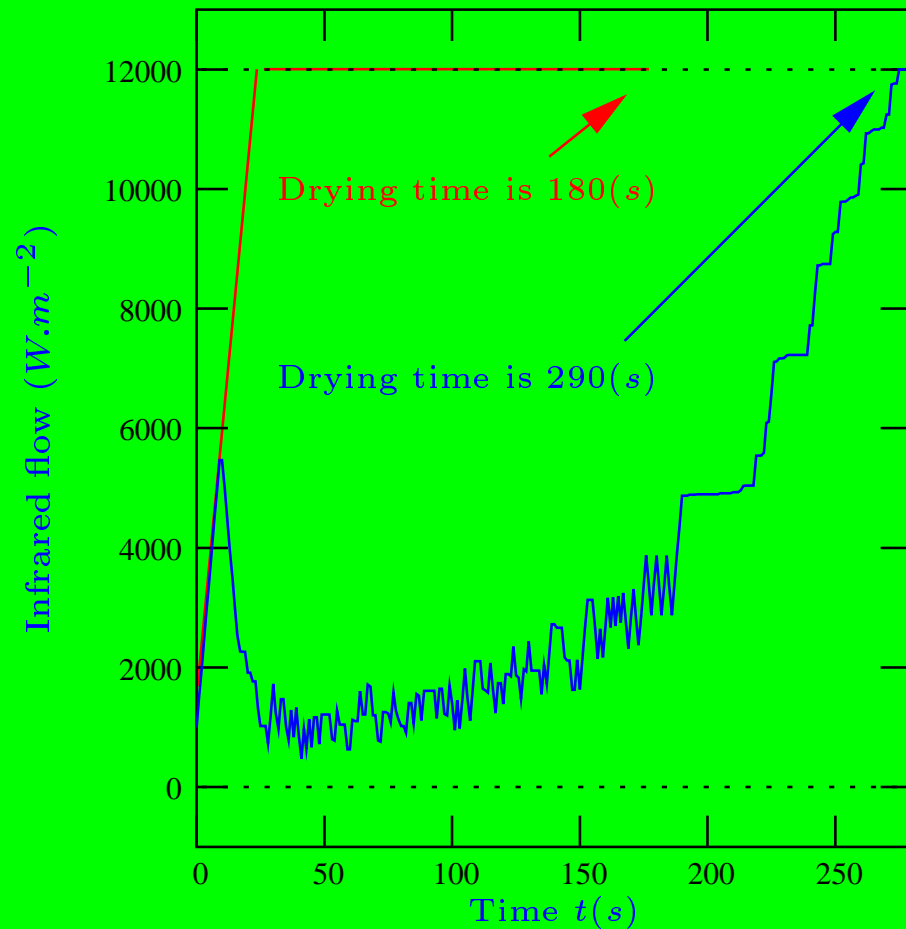
$$\text{velocity: } -500 \text{ W.m}^{-2} \leq \varphi_{ir}(j) - \varphi_{ir}(j-1) \leq +500 \text{ W.m}^{-2}$$

Accounting for a state constraint:

$$\chi(0.8e_{sec}, j) - \chi(0.9e_{sec}, j) \leq 0.014 \text{ kg.kg}^{-1}$$

Stop the procedure when the sample is dry: $\chi(z, j) \leq 0.01 \forall z$

Simulation results



Minimization of the drying time without (red)/with (blue) the state constraint.

Conclusions

- A real-time model based predictive control for processes modeled by PDE has been shown:
 - to solve any **optimization task**,
 - while including any **process limitations**.
- For the drying process:
 - influence of the **estimated difference of humidity** over drying time optimization has been shown,
 - **any specified constraints** based on input, measured outputs and estimated unmeasured variables may be **explicitly accounted for**.

Perspectives: Experimental observer validation

Which new additional devices can we use to measure the humidity profile inside the low thickness painting sample ?

Real time aspect: variation model

A model **linearization** about (\mathcal{S}_0) (u_0, x_0, y_0) , with variations s.t. :

$$\begin{cases} u(k) = u_0(k) + \Delta u(k) \\ x_m(k) = x_0(k) + \Delta x_m(k) \\ y_m(k) = y_0(k) + \Delta y_m(k) \end{cases} \quad (12)$$

leads to (\mathcal{S}_{TVL}) :

$$\begin{cases} \frac{\partial \Delta x_m}{\partial t} = \frac{\partial F_d}{\partial x_m | (\mathcal{S}_0)} \Delta x_m \text{ for } 0 < z < e_{sec}, t > 0 \\ \frac{\partial F_b}{\partial x_m | (\mathcal{S}_0)} \Delta x_m + \frac{\partial F_b}{\partial u | (\mathcal{S}_0)} \Delta u = 0 \text{ for } z \in \{0, e_{sec}\}, t > 0 \\ \Delta y_m = C \Delta x_m \text{ for } 0 \leq z \leq e_{sec}, t > 0 \\ \Delta x_m(z, 0) = 0 \text{ for } 0 \leq z \leq e_{sec} \end{cases} \quad (13)$$

Internal model control structure

