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vol. B, pp. 1147-1154,
Beijing, PRC, august 27-30, 2002.**

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<http://www.researcherid.com/rid/C-3926-2008>**

**Model Predictive Control for
an Infrared Drying Process
of a Water Based Epoxy Amine Painting**

Co-authors: Dufour P.¹, Blanc D.², Laurent P.¹, Touré Y.³

¹ LAGEP, University Claude Bernard Lyon 1-CNRS, France

² LAEPSI, INSA Lyon, France

³ LVR, University of Orléans, France

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Outline

Objective: provide a *real-time control strategy* for complex processes (here, a drying process):

- for *trajectory tracking*,
- to solve any *optimization task* (i.e. with performance specifications and process limitations).

↪ **Proposed solution:**

- new *model based predictive control* (MBPC) approach.

Difficulties:

- solve a *on-line constrained optimization* problem,
- handle a *nonlinear dynamic PDE model*.

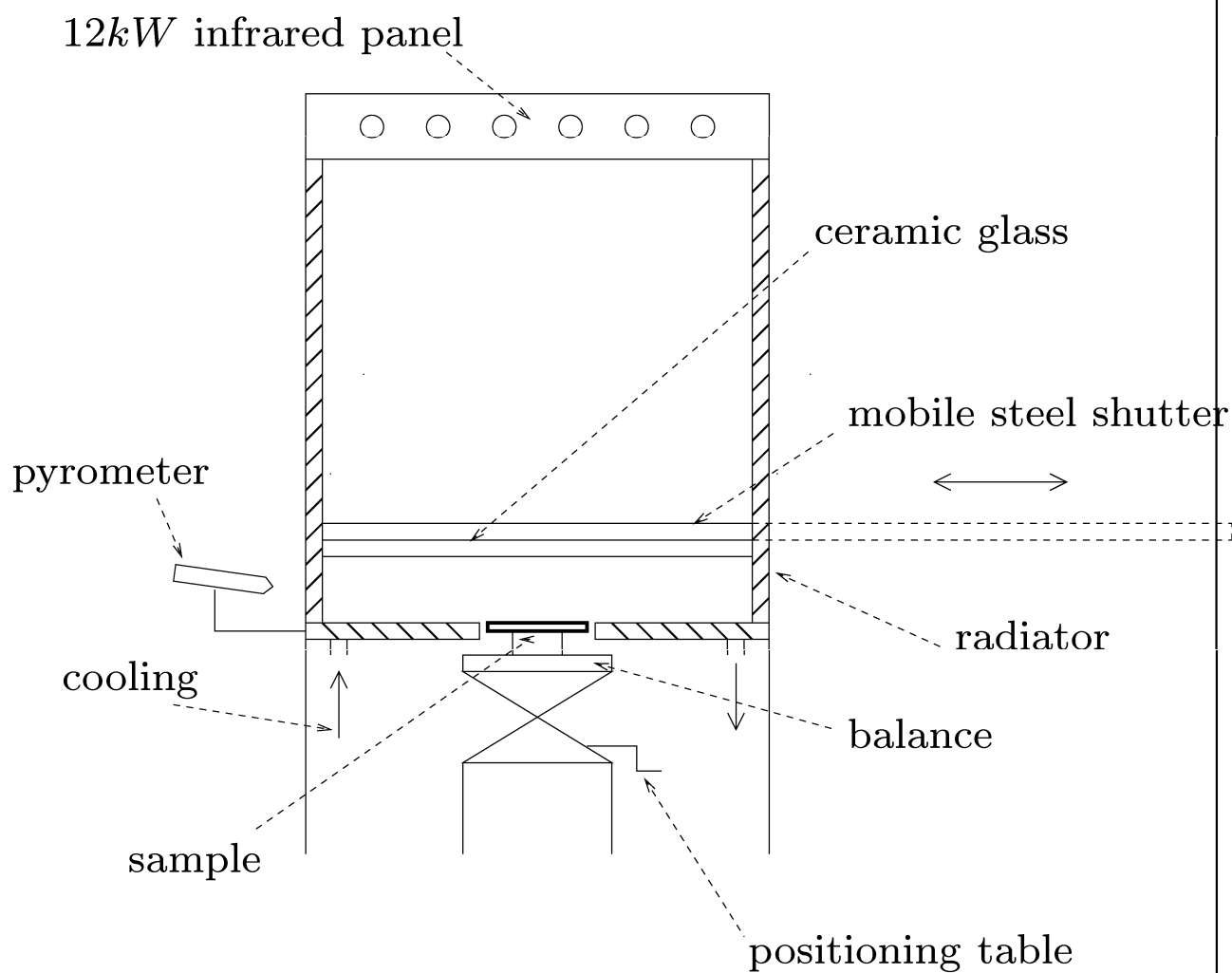
↪ **Necessity:**

- *reduce the time required* by the optimization task for *on-line implementation* of such a controller.

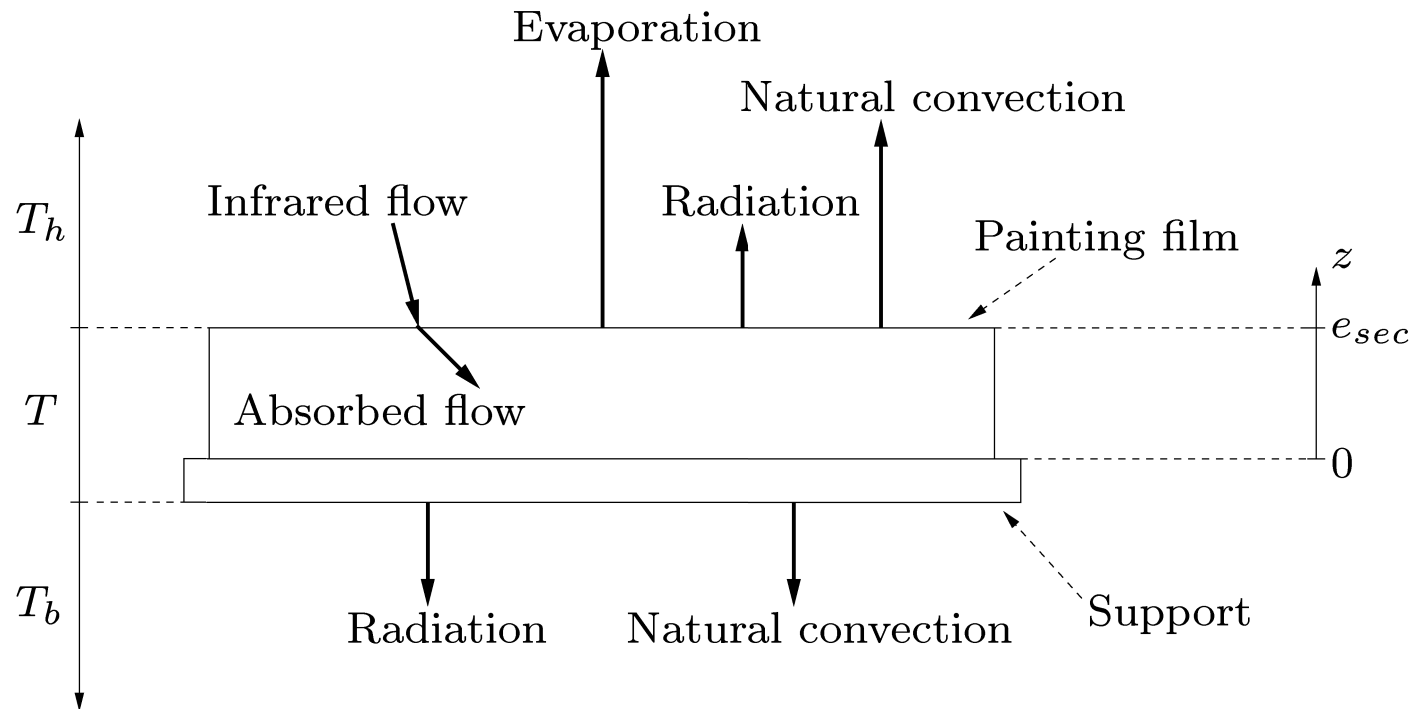
Infrared drying process

of painting film

[Blanc et al., 11th IHTC, pp.181-186]



Thermal flows



Energy balance

$$\begin{cases} (\rho_p C_p(\bar{\chi}, T)e_p + \rho_s C_s e_s) \frac{\partial T}{\partial t} = - \sum_{i=1}^5 P_i + P_{abs}, & t > 0 \\ T(0) = T_0 \end{cases} \quad (1)$$

with the losses P_i :

$$\begin{cases} P_1 = h_c(T - T_b), & P_2 = \sigma_2(T^4 - T_b^4) \\ P_3 = h_c(T - T_h), & P_4 = \sigma_4(T^4 - T_h^4) \\ P_5 = l_v(T)\dot{m}(\bar{\chi}, T) \end{cases} \quad (2)$$

and the absorbed **infrared flow**:

$$P_{abs} = \alpha_{ir}(\bar{\chi})\varphi_{ir}(t) \quad (3)$$

Mass balance

$$\left\{ \begin{array}{l}
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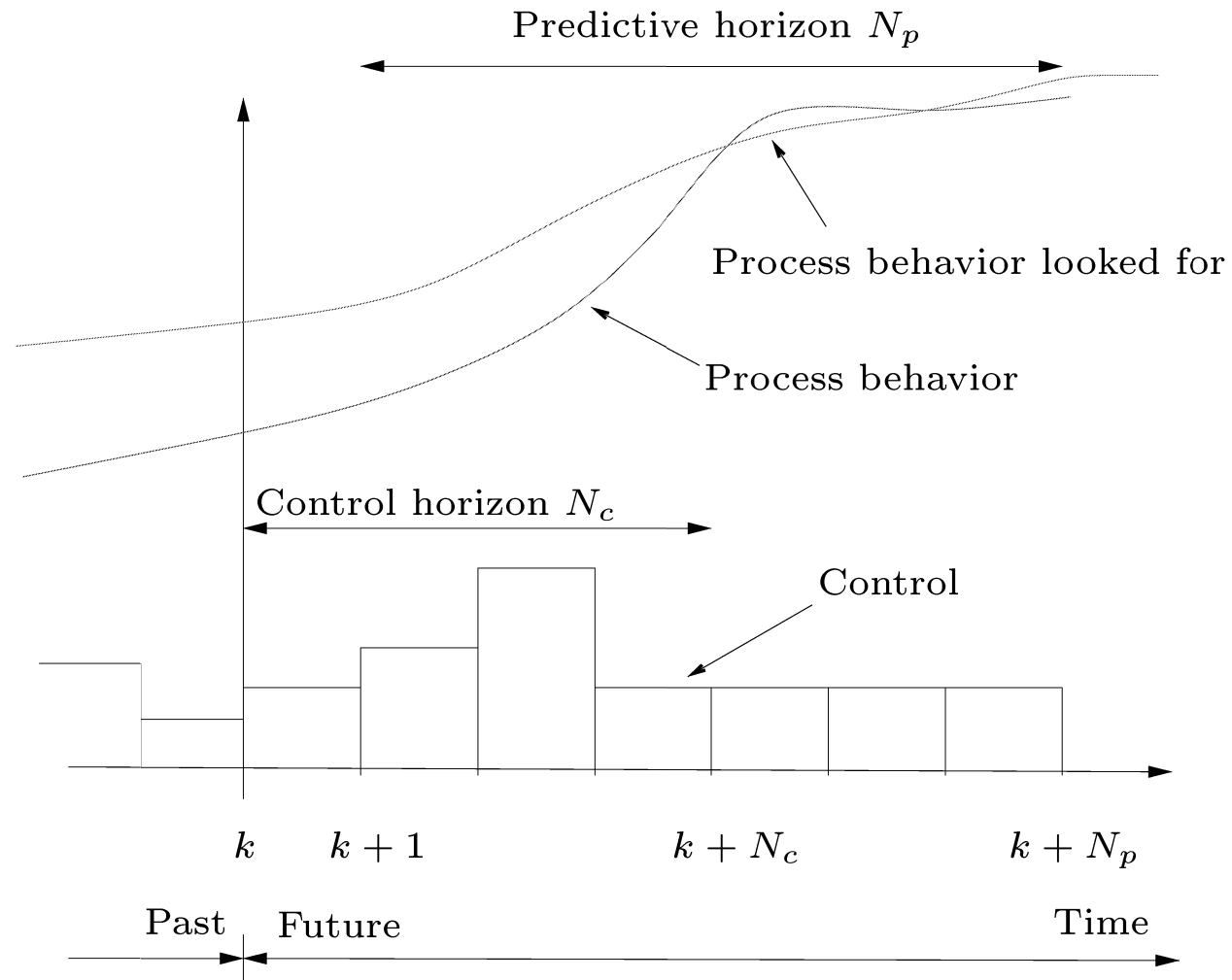
Nonlinear dynamic PDE model (\mathcal{S}_{NL})

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the state vector is $x(z, t) = (T(t), \chi(z, t))^T$, the **controlled variable** (CV) $y(t)$ is the temperature $T(t)$, the **manipulated variable** (MV) $u(t)$ is the infrared flow $\varphi_{ir}(t)$.

MPC framework: the paradigm

[Morari & Lee, *Comp. & Chem. Eng.*, 23(4-5), 1999][Rawlings, *Control Systems*, 20(3), 2000]



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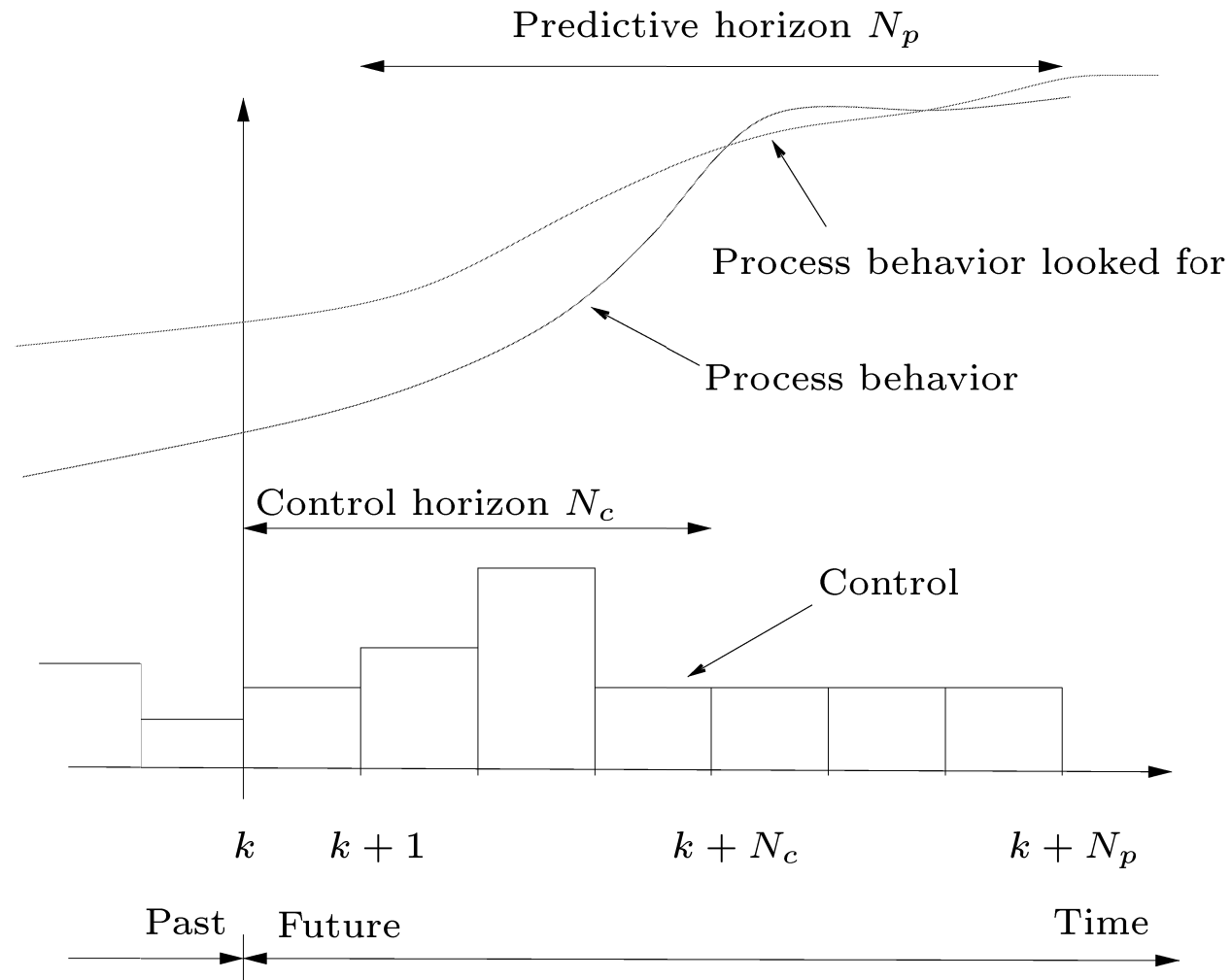
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MPC framework: main features

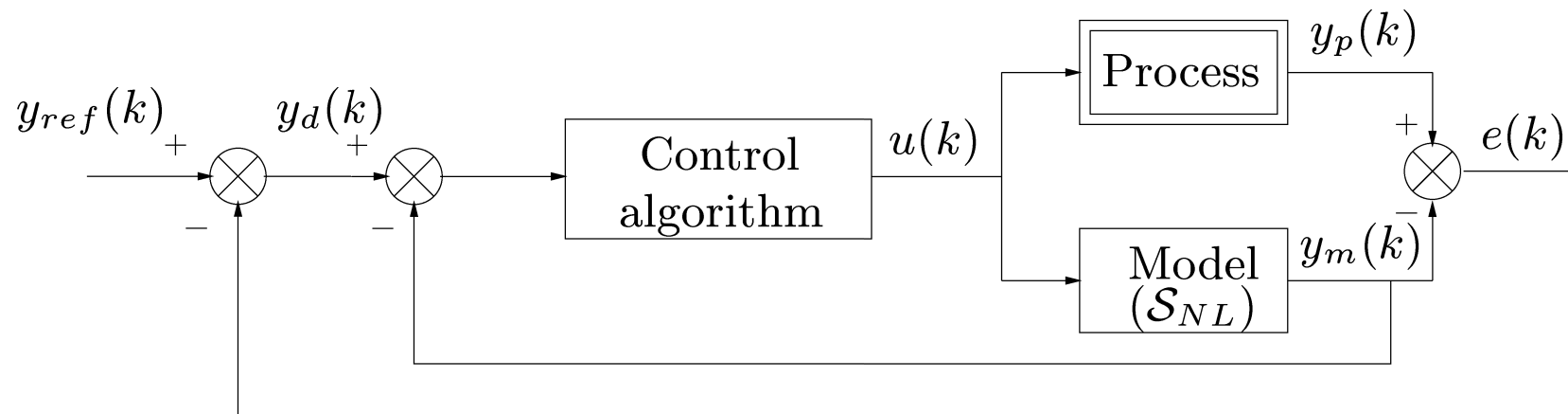
- Key idea is to **forecast future process behavior: model needed.**
 - ★ Historically, models obtained from **experimental step response** were used: simple but local,
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 - ★ Need to **reduce the on-line computational burden**,
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Internal model control structure

[Morari & Zafiriou, Robust Process Control, 1989]



Real time aspect:

variation model

A model **linearization** about (\mathcal{S}_0) characterised by (u_0, x_0, y_0) , with variations checking:

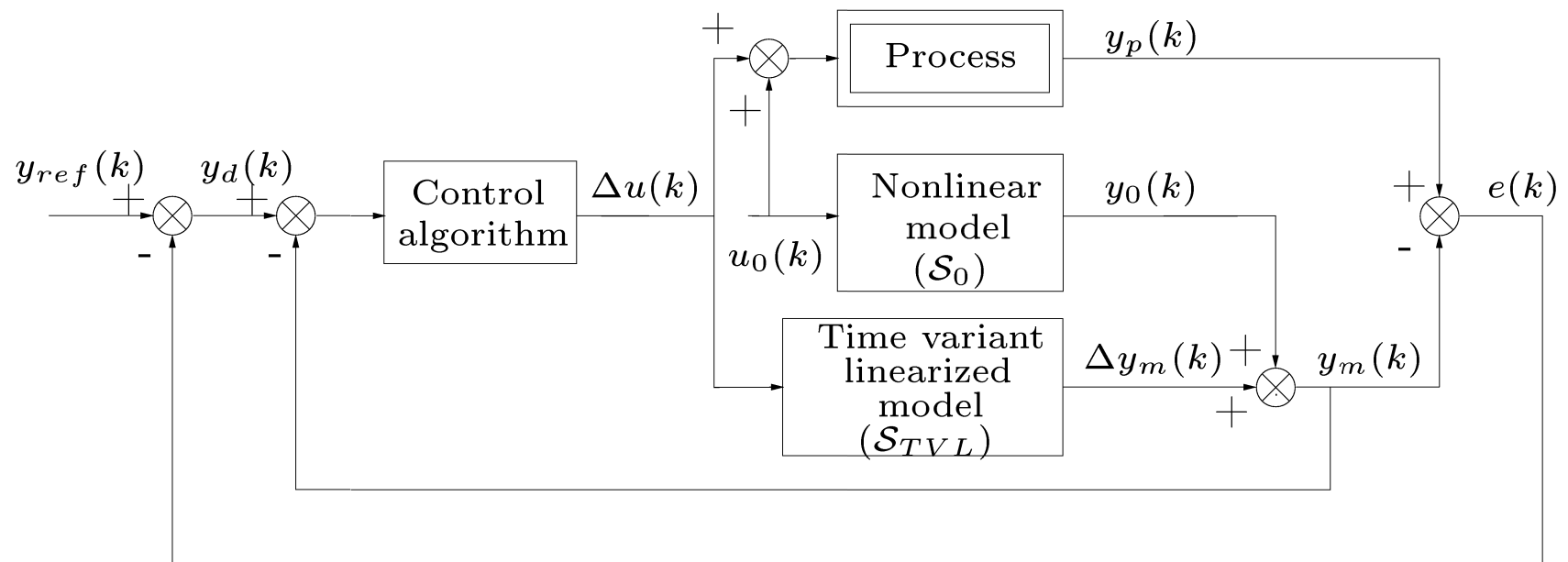
$$\begin{cases} u(k) = u_0(k) + \Delta u(k) \\ x(k) = x_0(k) + \Delta x(k) \\ y_m(k) = y_0(k) + \Delta y_m(k) \end{cases} \quad (6)$$

leads to (\mathcal{S}_{LTV}) :

$$\begin{cases} \frac{\partial \Delta x}{\partial t} = \frac{\partial F_d}{\partial x|(\mathcal{S}_0)} \Delta x \text{ for } 0 < z < e_{sec}, t > 0 \\ \frac{\partial F_b}{\partial x|(\mathcal{S}_0)} \Delta x + \frac{\partial F_b}{\partial u|(\mathcal{S}_0)} \Delta u = 0 \text{ for } z \in \{0, e_{sec}\}, t > 0 \\ \Delta y_m = C \Delta x \text{ for } 0 \leq z \leq e_{sec}, t > 0 \\ \Delta x(z, 0) = 0 \text{ for } 0 \leq z \leq e_{sec} \end{cases} \quad (7)$$

Internal time-varying linearized

model control structure



Real time constrained optimization problem

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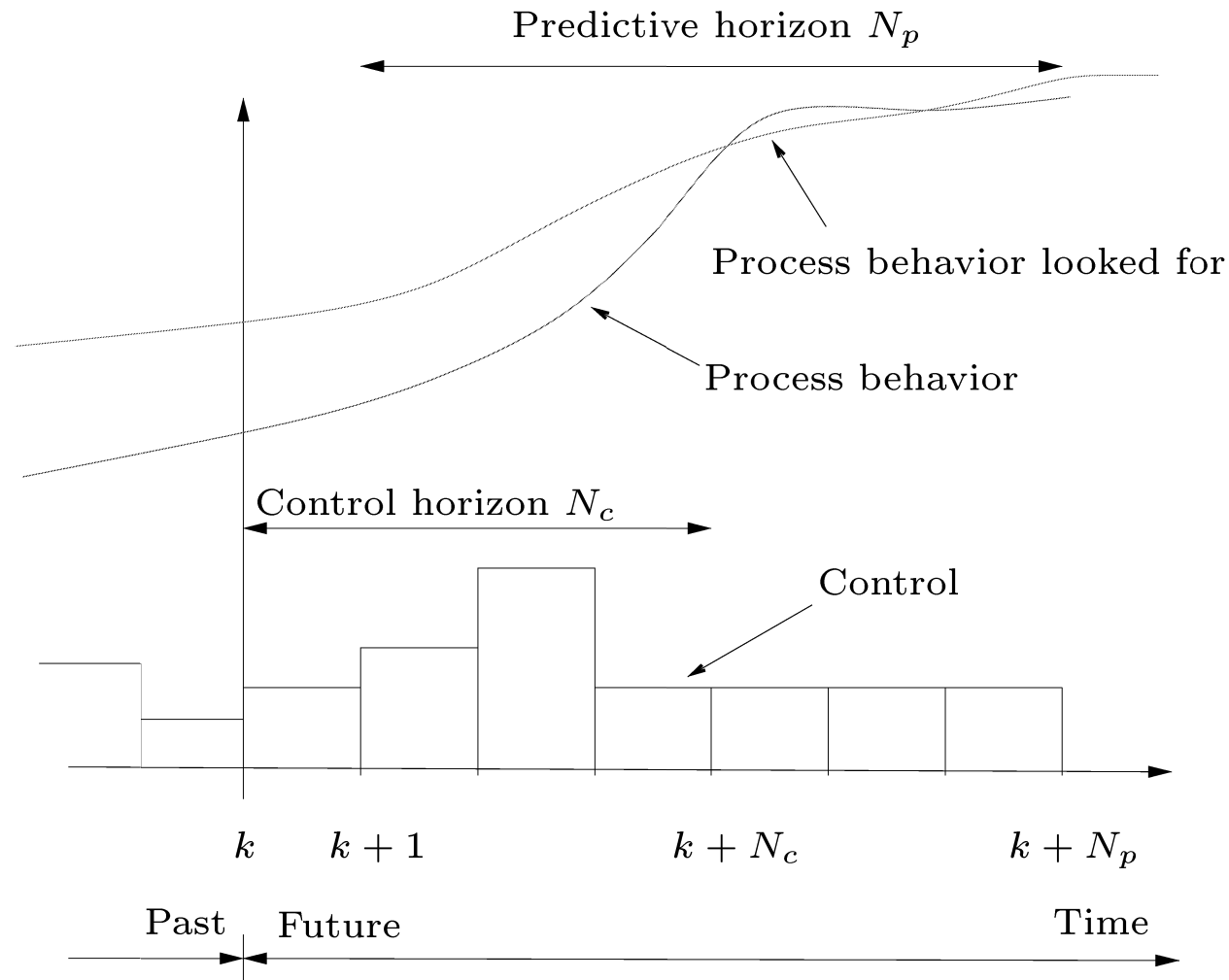
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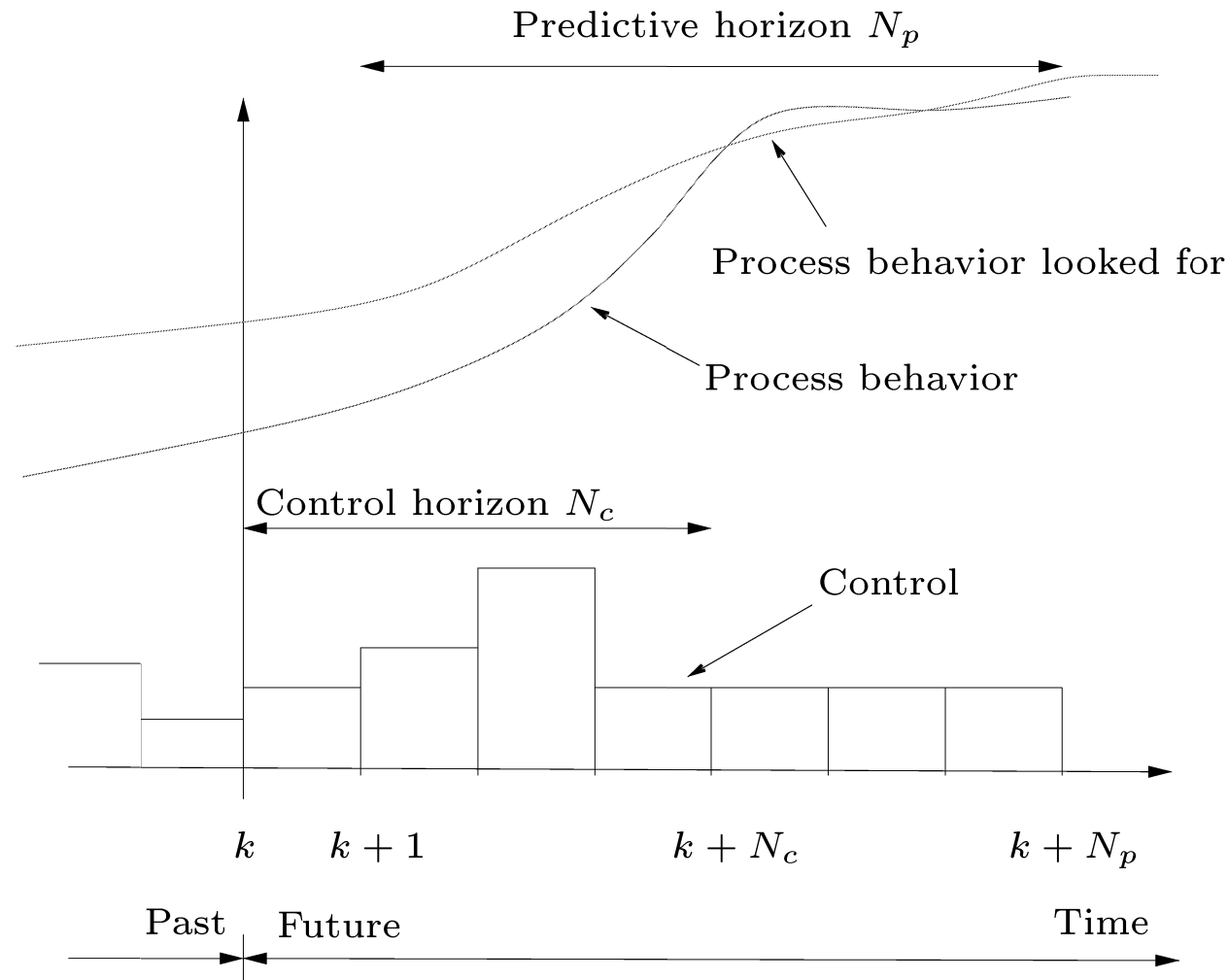
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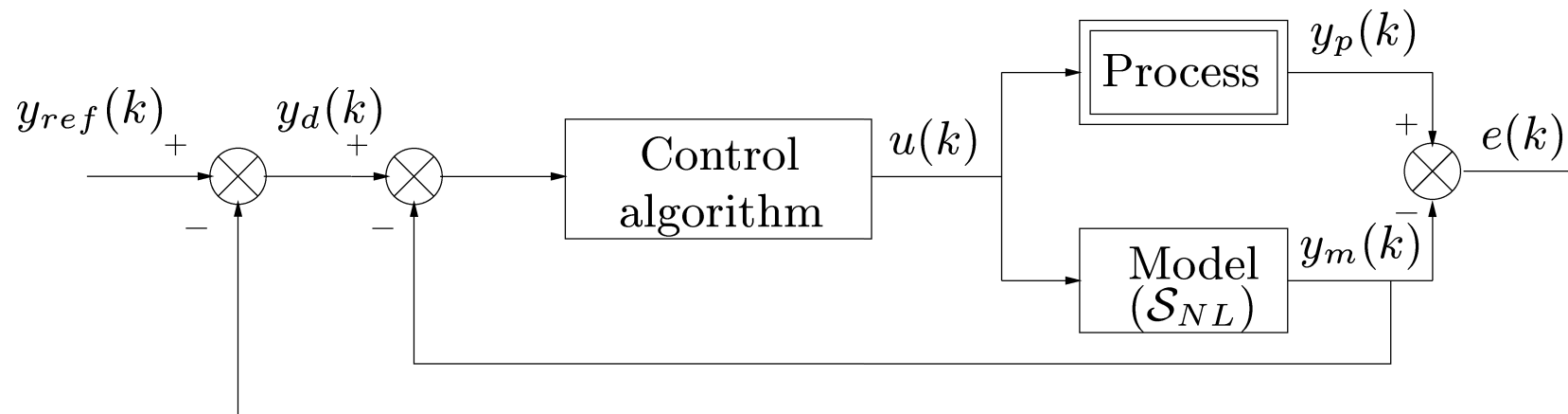
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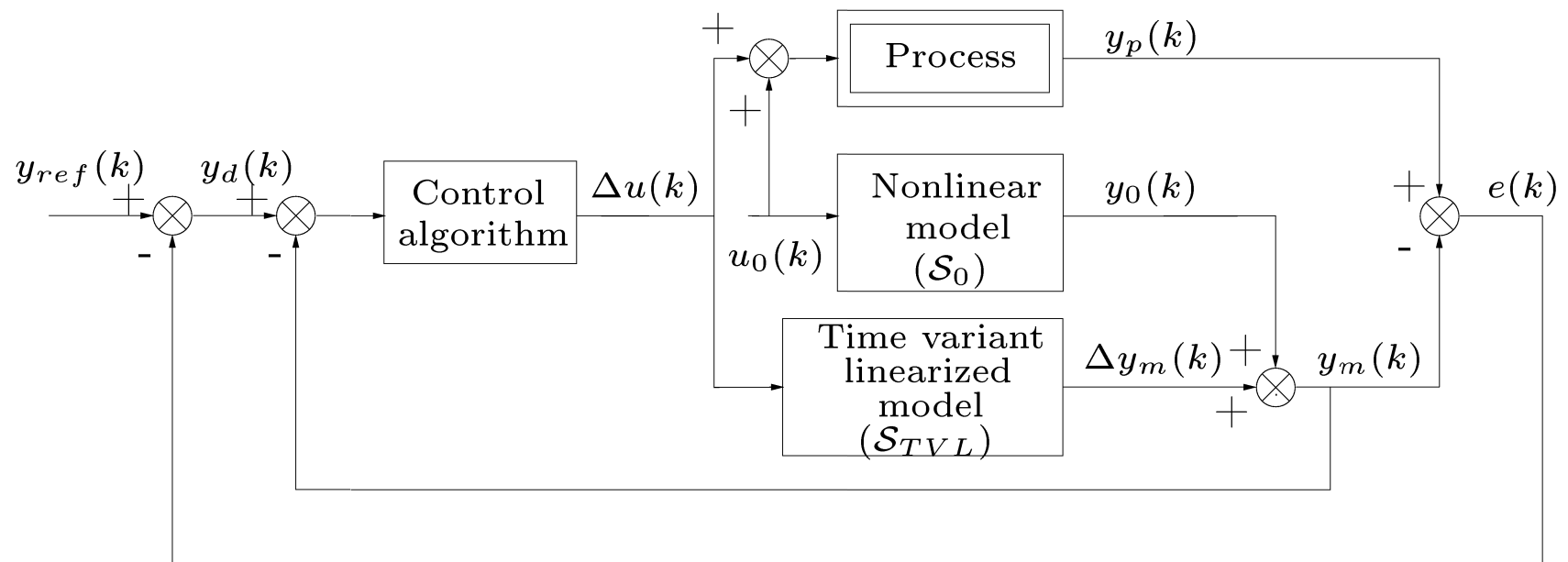
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Constraints on manipulated variable

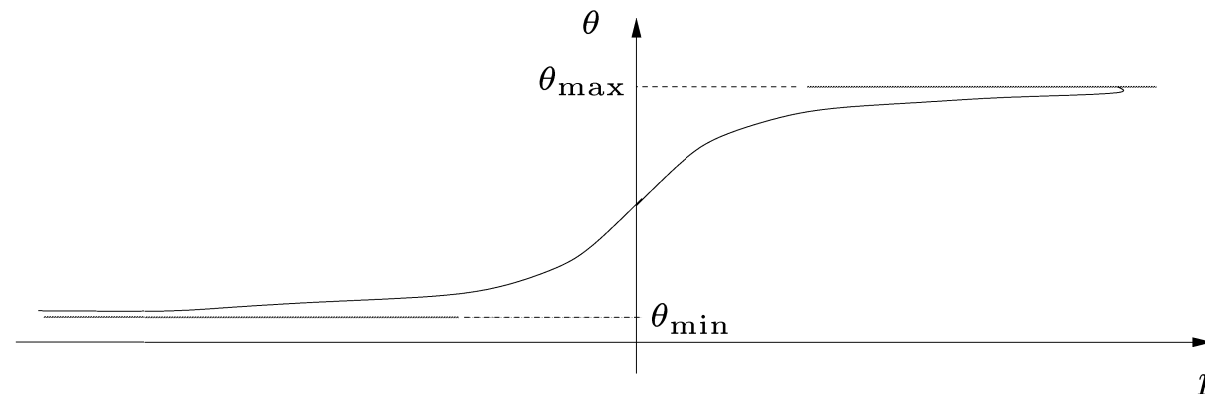
magnitude and velocity: a transformation method

From the **constrained variable** θ :

$$\theta_{\min} \leq \theta \leq \theta_{\max} \quad (9)$$

an **unconstrained variable** p is defined such that:

$$\theta = f(p) = \frac{1}{2}(\theta_{\max} + \theta_{\min}) + \frac{1}{2}(\theta_{\max} - \theta_{\min}) \tanh(p) \quad (10)$$



Final unconstrained optimization problem

$$\min_{\Delta \tilde{p}} J(\Delta \tilde{p}) = \frac{1}{N_p} \sum_{j=k+1}^{j=k+N_p} g(y_{ref}(j), \Delta y_m(j), e(k))$$

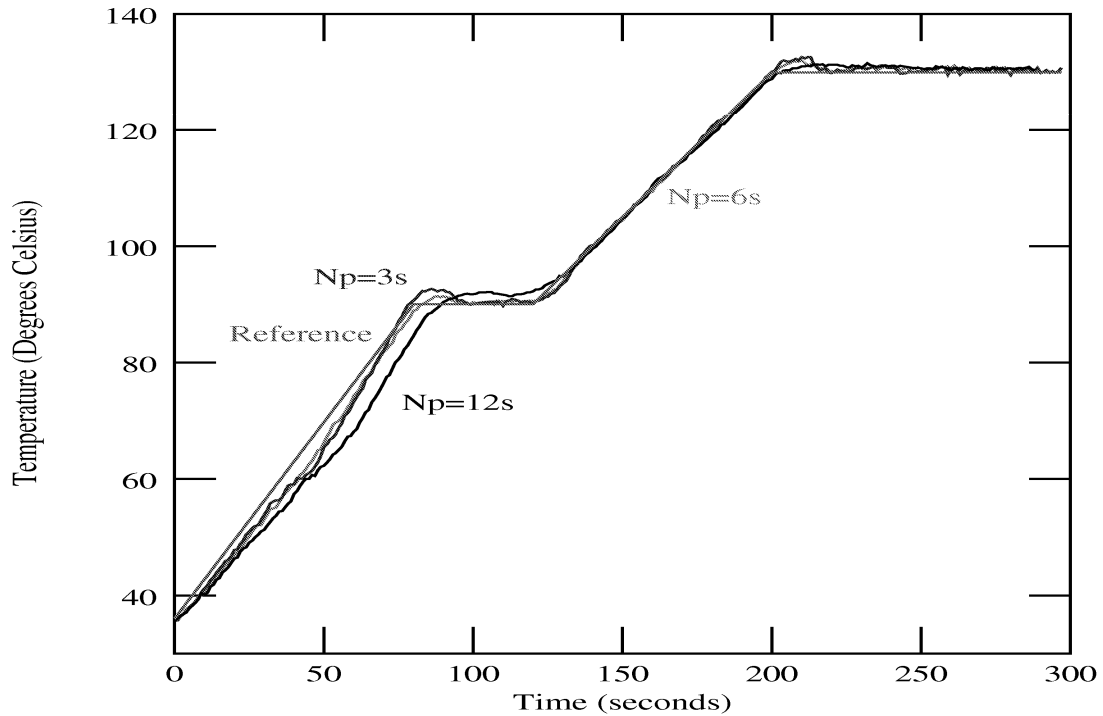
$$\Delta \tilde{p} = [\Delta p(k) \dots \Delta p(k + N_c - 1)]^T$$

based on the transformation method: $u \rightleftharpoons p$, $\Delta u \rightleftharpoons \Delta p$

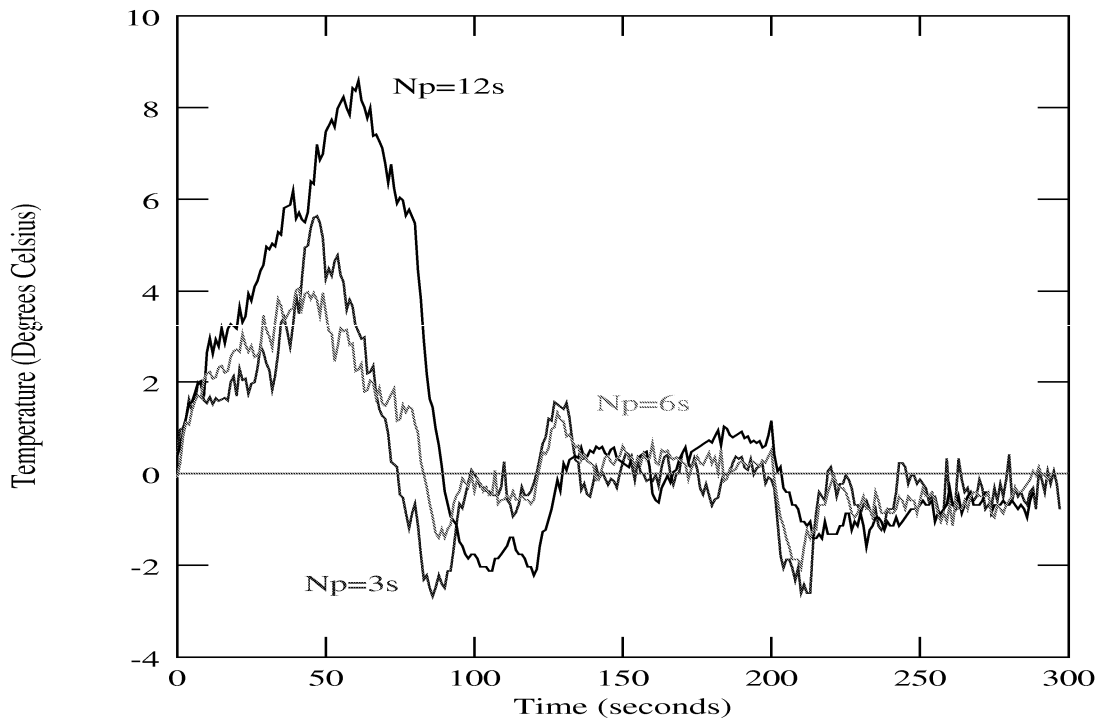
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\implies Optimization problem solved by the Levenberg-Marquardt's algorithm.

Experimental results

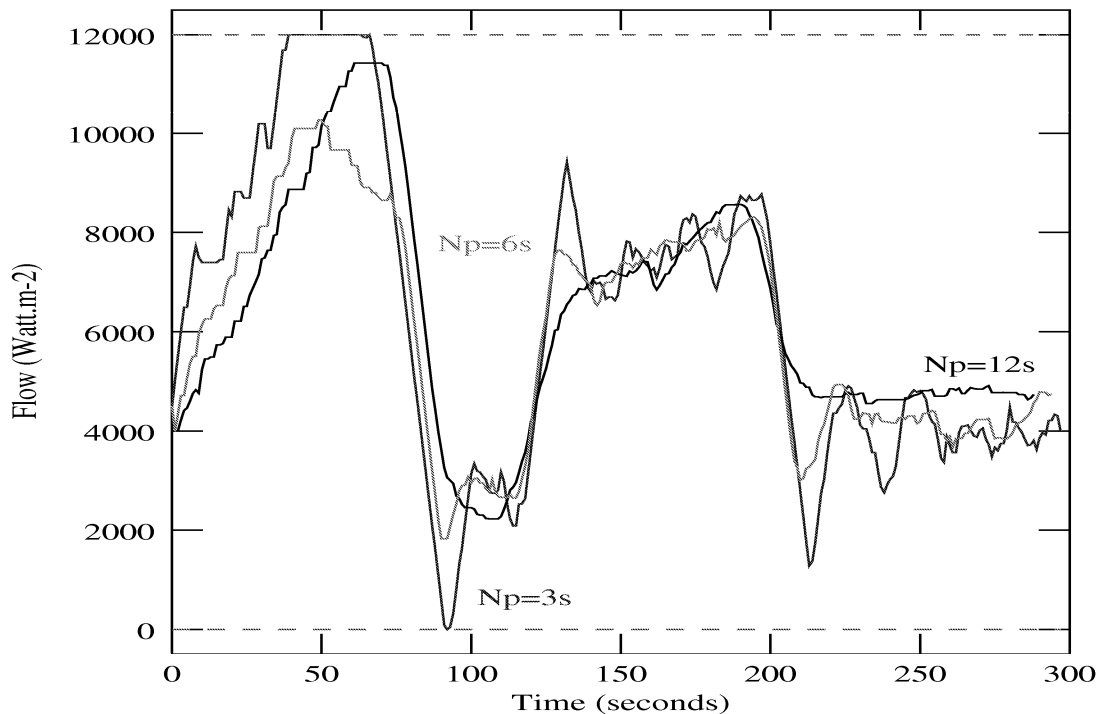


Temperature trajectory tracking

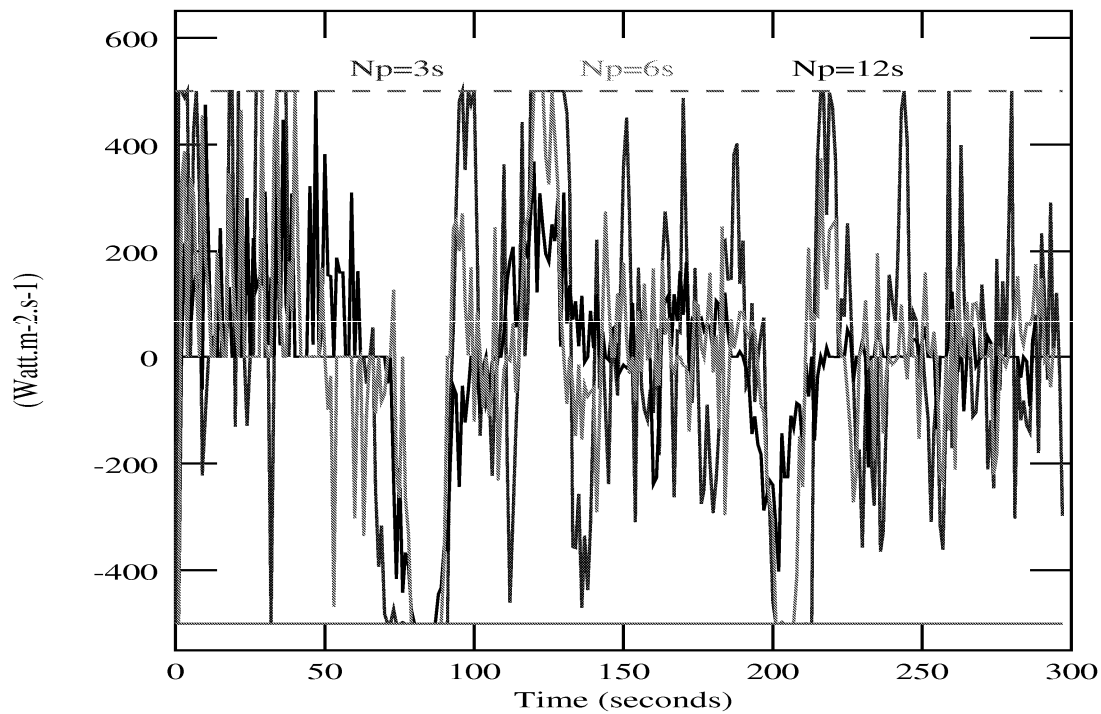


Temperature tracking error

Experimental results



Infrared flow magnitude

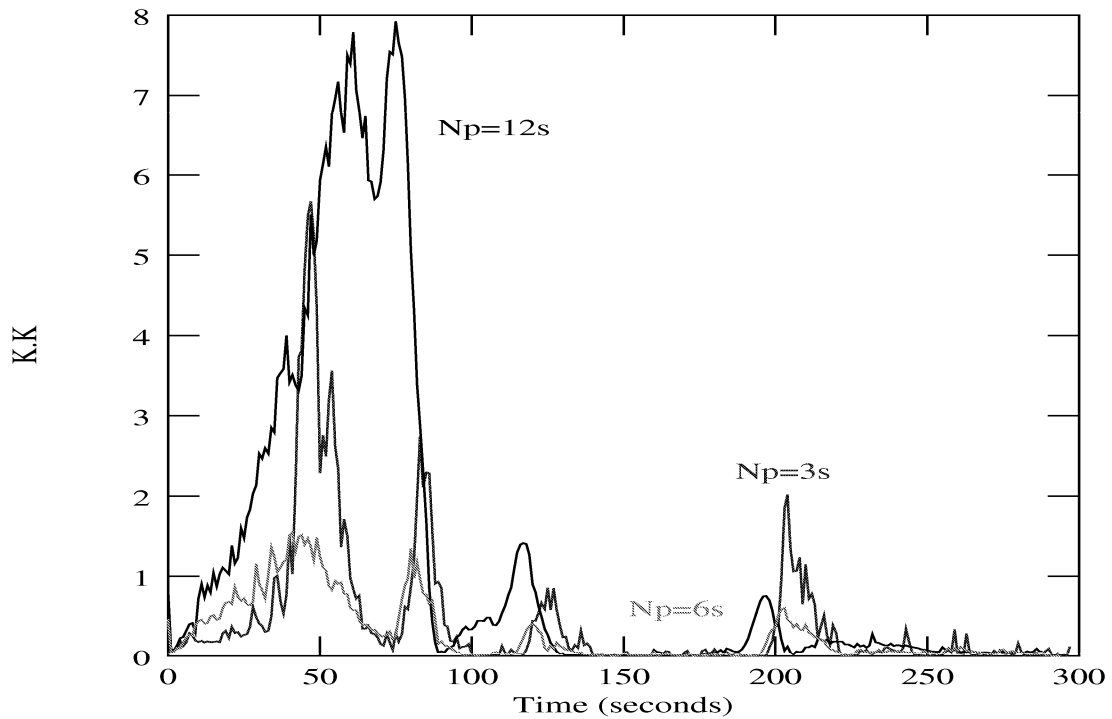


Infrared flow velocity

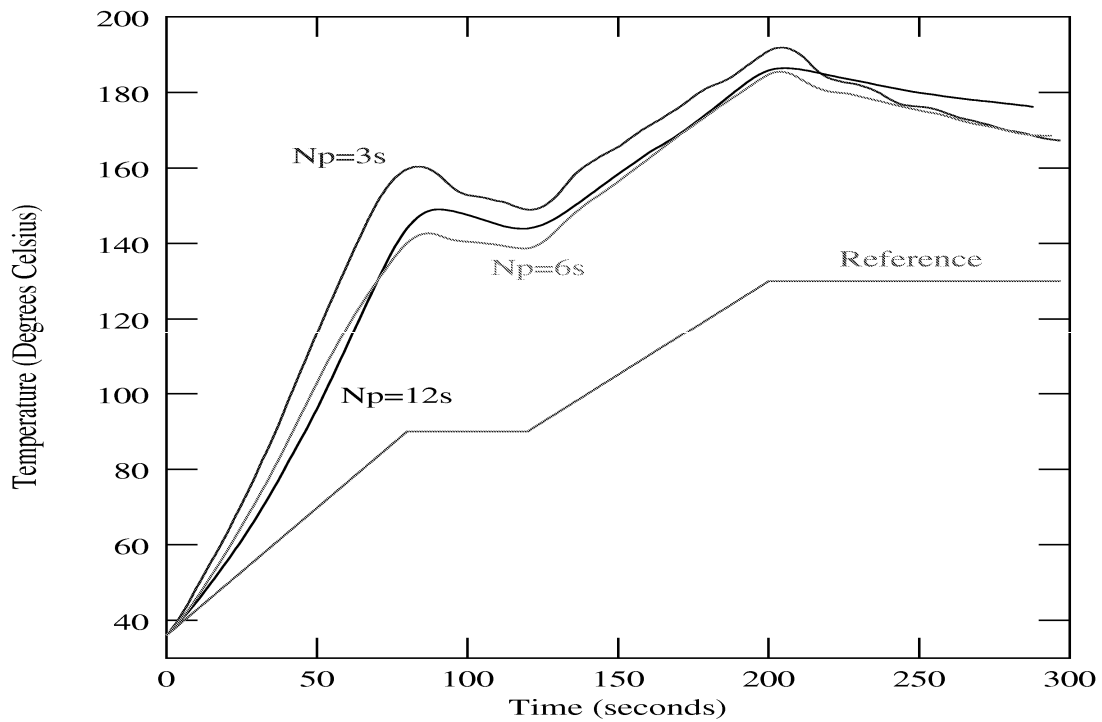
Conclusions

- **Implementable control strategy:**
 - ★ for **trajectory tracking** or for **any optimization task**,
 - ★ handles **constraints dealing with performance and process limitations**.
- **Computational time reduction:**
 - ★ **nominal behavior given off-line**,
 - ★ **on-line correction**,
 - ★ **transformation method for MV constraints**,
 - ★ **constraints on CV can be accounted for in a exterior penalty method**.
- Influences of N_p on experimental results.
- (Noisy) mean water content trajectory tracking: use of a **receding horizon observer**.
- Need for **multi disciplinary** approaches.

Experimental results



Cost function J



Modeled temperature