



**HAL**  
open science

# Trajectory tracking strategy for a non-linear distributed parameter process

Pascal Dufour, Estelle Courtial, Youssoufi Touré, Pierre Laurent

► **To cite this version:**

Pascal Dufour, Estelle Courtial, Youssoufi Touré, Pierre Laurent. Trajectory tracking strategy for a non-linear distributed parameter process. IFAC-IEEE European Control Conference (ECC), Sep 2001, Porto, Portugal. Paper 4993. hal-00353049

**HAL Id: hal-00353049**

**<https://hal.science/hal-00353049v1>**

Submitted on 22 Jan 2009

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

**This document must be cited according to its final version  
which is published in a conference proceeding as:  
P. Dufour<sup>1</sup>, E. Courtial<sup>2</sup>, Y. Touré<sup>3</sup>, P. Laurent<sup>1</sup>,  
"Trajectory tracking strategy  
for a non-linear distributed parameter process",  
Proceedings of the 6<sup>th</sup> European Control Conference (ECC) 2001,  
Paper 4993,  
Porto, Portugal, september 4-7, 2001.**

**All open archive documents of Pascal Dufour are available at:  
<http://hal.archives-ouvertes.fr/DUFOUR-PASCAL-C-3926-2008>**

**The professional web page (Fr/En) of Pascal Dufour is:  
<http://www.lagep.univ-lyon1.fr/signatures/dufour.pascal>**

1

Université de Lyon, Lyon, F-69003, France; Université Lyon 1;  
CNRS UMR 5007 LAGEP (Laboratoire d'Automatique et de GÉnie des Procédés),  
43 bd du 11 novembre, 69100 Villeurbanne, France  
Tel +33 (0) 4 72 43 18 45 - Fax +33 (0) 4 72 43 16 99  
<http://www.lagep.univ-lyon1.fr/> <http://www.univ-lyon1.fr> <http://www.cnrs.fr>

2

Université de Picardie Jules Verne,  
EA 3699 CREA (Centre de Robotique, d'Electrotechnique et d'Automatique d'Amiens),  
7 rue du moulin neuf, 80029 Amiens Cedex 1, France  
<http://www.crea.u-picardie.fr/>

3

Université d'Orléans,  
UPRES EA 2078 LVR (Laboratoire de Vision et de Robotique ),  
63 Av de Lattre de Tassigny, 18020 Bourges Cedex, France  
<http://www.bourges.univ-orleans.fr/rech/lvr/>

# TRAJECTORY TRACKING STRATEGY FOR A NONLINEAR DISTRIBUTED PARAMETER PROCESS

P. Dufour<sup>†</sup>, E. Courtial<sup>\*</sup>, Y. Touré<sup>†</sup>, P. Laurent<sup>+</sup>

<sup>†</sup> LVR UPRES EA 2078 - University of Orleans, 63 av de Lattre de Tassigny, 18020 Bourges Cedex, France  
e-mail: Pascal.Dufour, Youssoufi.Toure@bourges.univ-orleans.fr

<sup>\*</sup> CREA EA 3299 - University of Picardie Jules Verne, 7 rue du moulin neuf, 80000 Amiens, France  
e-mail: estelle.courtial@sc.u-picardie.fr

<sup>+</sup> LAGEP UMR CNRS 5007 - UCBL, 43 Bd du 11 novembre 1918, 69622 Villeurbanne Cedex, France  
e-mail: laurent@lagep.univ-lyon1.fr

**Keywords:** Trajectory Tracking, Process Control, Predictive Control, Infinite Dimensional Systems.

## Abstract

A trajectory tracking strategy is presented for processes described by a parabolic nonlinear distributed parameter model. A model-based predictive control approach, combined with an internal model control structure and a state estimation method is extended to this kind of process. On-line requirements such as computational time and constraint satisfaction are outlined and discussed. This method is then implemented for the control of a drying process and experimental results are discussed.

## 1 Introduction

Most industrial control needs involve the design of a robust controller capable of achieving a trajectory tracking in spite of inevitable model uncertainties and physical constraints. In the meantime, the developed process models have become more and more complex and nonlinear in order to describe changes in operating conditions, parameter variations, complex dynamic behavior. Used in food processing as well as in drying processing, microscopic description of phenomena leads to a model described by a set of partial differential equations. This model is used for the control of the temperature historic during the drying procedure.

The purpose of this paper is to present a trajectory tracking strategy for a class of nonlinear distributed parameter systems. Only few practical works directly deal with the control of such systems. Even if the existing ones are based on interesting structures, they deal neither with complex nonlinearities nor with a set of partial differential equations [6, 8]. Furthermore, a great challenge is to be able to control this kind of process in real-time. The Model Predictive Control (MPC) approach has been successfully applied in industry to processes described by a set of linear or nonlinear ordinary differential equations [1]. In this paper, the MPC formulation is extended for the control of process described by a nonlinear distributed parameter system subject to constraints on the manipulated variable. Modifications are necessary due to the on-line control requirements.

This paper is organised as follows: in section 2, the painting film drying plant - modeled by a nonlinear distributed parameter system- is presented. In section 3, the MPC methodology is briefly mentioned and the considered control problem is stated. Then, on-line requirements such as computational time, constraint satisfaction, disturbed measurements are addressed. A solution is presented to reduce the computational time needed for the model resolution and to take into account the constraints. Finally, experimental results on a real drying process illustrate the capabilities of this approach in section 4.

## 2 Drying process

A laboratory plant has been developed in order to dry a painting film sample coated on a car iron support by infrared radiation [2]. The plant is described Figure 1 with the infrared part and with the instrumentation part.

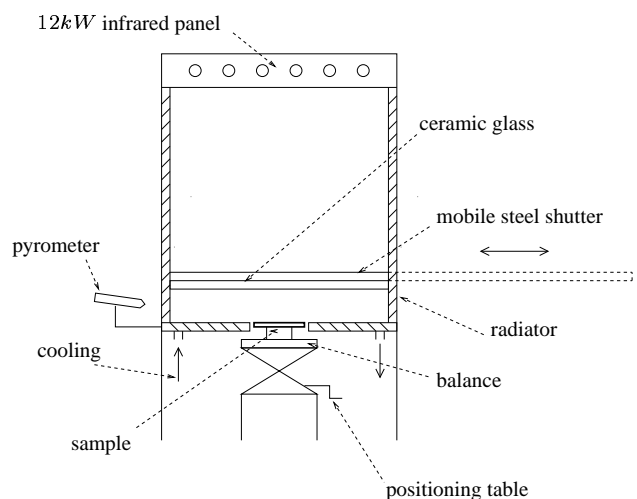


Figure 1: Drying process.

The thin painting film sample is characterised by its tempera-

ture assumed to be uniform  $T(t)$  and by its dry basis humidity  $\chi(z, t)$  assumed to vary according to the thickness  $z$  of the sample. These assumptions have been checked experimentally [2]. The drying procedure leads to water losses that produces a variation in the sample geometry. Considering the surface size and the low thickness of the sample, the water extraction leads only to the linear reduction of the sample thickness  $e_p$  with respect to the mean humidity  $\bar{\chi}$ :

$$e_p = e_{sec}(1 + \phi \bar{\chi}(t)) \quad (1)$$

where  $e_{sec}$  is the final constant dried thickness of the sample and:

$$\bar{\chi}(t) = \frac{1}{e_{sec}} \int_0^{e_{sec}} \chi(z, t) dz \quad (2)$$

## 2.1 Energetical balance

The temperature  $T(t)$  is assumed to be uniform on the sample and support set. Taking into account of the different losses  $P_i$  as well as the absorbed infrared flow represented Figure 2, the energetical balance leads to:

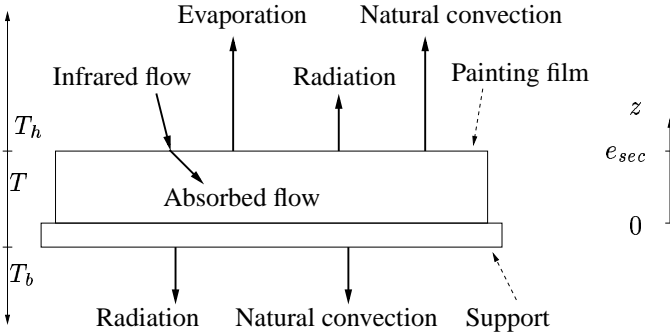


Figure 2: Thermal flows.

$$(\rho_p C_p(\bar{\chi}, T) e_p + \rho_s C_s e_s) \frac{dT}{dt} = - \sum_{i=1}^5 P_i + P_{abs} \quad (3)$$

$\rho_p C_p(\bar{\chi}, T) e_p$  and  $\rho_s C_s e_s$  are respectively the surface thermal capacity of the painting film sample and the surface thermal capacity of the support.

Different losses due to the natural convection and radiation phenomena on both surfaces are expressed as:

$$P_1 = h_c(T - T_h) \quad (4)$$

$$P_2 = \sigma_h(T^4 - T_h^4) \quad (5)$$

$$P_3 = h_c(T - T_b) \quad (6)$$

$$P_4 = \alpha_s \sigma(T^4 - T_b^4) \quad (7)$$

The water loss  $P_5$  depends on the drying velocity  $\dot{m}(\bar{\chi}, T)$ :

$$P_5 = l_v(T) \dot{m}(\bar{\chi}, T) \quad (8)$$

The absorbed flow  $P_{abs}$  depends on the manipulated variable, i.e. the infrared flow  $\varphi_{ir}(t)$ :

$$P_{abs} = \alpha_{ir}(\bar{\chi}) \varphi_{ir} \quad (9)$$

## 2.2 Mass balance

Since there is no macroporous structure, the water migrates into the upper radiated surface only by diffusion phenomenon. Using the Fick law, the mass balance can be written as:

- for  $z \in \Omega = ]0, e_{sec}[$ :

$$\frac{\partial \chi}{\partial t} = \frac{\partial}{\partial z} [D_{eff}(\chi, T) \frac{\partial \chi}{\partial z}] \quad (10)$$

with the effective diffusion coefficient  $D_{eff}$  depending on the humidity and the temperature:

$$D_{eff}(\chi, T) = \frac{D_0 \exp(\frac{-a}{\chi}) \exp(\frac{-E_a}{RT})}{(1 + \phi \chi)^2} \quad (11)$$

- at  $z = 0$ , i.e. at the painting film sample lower surface, there is no water transfer:

$$\frac{\partial \chi}{\partial z} = 0 \quad (12)$$

- at  $z = e_{sec}$ , the outgoing flow is linked to the drying velocity through:

$$-D_{eff}(\chi, T) \frac{\partial \chi}{\partial z} = \frac{\dot{m}(\bar{\chi}, T)}{\rho} \quad (13)$$

## 2.3 Distributed parameter model

From the previous heat and mass balances, the drying process can be represented by the parabolic nonlinear distributed parameter system ( $\mathcal{S}_{NL}$ ):

$$(\mathcal{S}_{NL}) \left\{ \begin{array}{l} \frac{\partial \chi}{\partial t} = \frac{\partial}{\partial z} [D_{eff}(\chi, T) \frac{\partial \chi}{\partial z}] \text{ for } z \in ]0, e_{sec}[ , t > 0 \\ \frac{dT}{dt} = \frac{- \sum_{i=1}^5 P_i + P_{abs}}{\rho C_p(\bar{\chi}, T) e_p + \rho_s C_s e_s} \forall z \in [0, e_{sec}] , t > 0 \\ \text{with the scalar input:} \\ u(t) = \varphi_{ir}(t) \text{ for } t > 0 \\ \text{with the output:} \\ y(t) = \bar{\chi}(t) = \frac{1}{e_{sec}} \int_0^{e_{sec}} \chi(z, t) dz \\ \text{with the boundary conditions:} \\ \frac{\partial \chi}{\partial z} = 0 \text{ for } z = 0, t > 0 \\ -D_{eff}(\chi, T) \frac{\partial \chi}{\partial z} = \frac{\dot{m}(\bar{\chi}, T)}{\rho} \text{ for } z = e_{sec}, t > 0 \\ \text{with the initial conditions :} \\ \chi(z, 0) = \chi_i, \forall z \in [0, e_{sec}], \\ T(0) = T_i \end{array} \right. \quad (14)$$

All parameter values and remaining expressions can be found in [7].

**Remark 2.1** According to the spatial uniform property assumption on the temperature, the control problem is a distributed control problem: indeed, the manipulated variable, i.e. the infrared flow  $\varphi_{ir}(t)$ , acts instantaneously at the boundary ( $z = e_{sec}$ ) as well as over the painting film sample.

### 3 Model Predictive Control strategy

#### 3.1 MPC methodology

For a SISO (single-input, single-output) system, the MPC objective is to determine a sequence of  $N_c$  future controls  $u(k+i)$  (with  $i = 0, \dots, N_c - 1$ ) by the minimization of the cost function  $J$  over a finite horizon  $N_p$  at each sampled time  $k$ :

$$J = \sum_{j=k+1}^{j=k+N_p} [y_{ref}(j) - y_p(j)]^2 \quad (15)$$

$N_c$  and  $N_p$  respectively denote the control horizon and the prediction horizon.  $y_{ref}$  is the reference to be tracked by the process output  $y_p$ . At each sampled time, only the first component of the optimal control sequence is applied to the plant. At the next sampling instant, measurements are updated and the optimization problem is solved again. This methodology is also referred to as receding or moving horizon control. One of the reasons of its successful application in industry since 25 years is its ability to explicitly handle constraints in the problem formulation [9].

#### 3.2 Control problem statement

For real applications, the dried painting film sample should satisfy quality requirements: neither bubbles nor fissures are allowed. To ensure the final product quality, paint producers commonly provide a temperature reference profile during the drying cycle. However, a humidity reference trajectory could also be a relevant profile for the drying procedure since it is the most representative data.

In this study, the objective is to track a humidity reference trajectory in presence of constraints on the manipulated variable  $u(t)$ . A MPC strategy involving a distributed parameter system is well-adapted to satisfy these control requirements [4, 5]. The reference trajectory,  $y_{ref}$ , is the humidity reference. The process outputs,  $y_p(j)$ , over the prediction horizon are unknown but they are predicted from the process model.

##### 3.2.1 Computational time requirement

The process model plays a crucial role in terms of control performance. A great part of the computational time is devoted to the model resolution. The resolution of this nonlinear distributed parameter model can be prohibitive in terms of time especially with small sampled time like here (1s). As a consequence, the initial model ( $\mathcal{S}_{NL}$ ) is divided into a nominal

nonlinear model ( $\mathcal{S}_0$ ) computed off-line and a time-variant linearized model ( $\mathcal{S}_{TVL}$ ) that has to be solved on-line. ( $\mathcal{S}_{TVL}$ ) represents the variation model around the nominal behavior and is obtained by a linearization of ( $\mathcal{S}_{NL}$ ) over the given reference trajectory like in [4]. All the models ( $\mathcal{S}_{NL}$ ), ( $\mathcal{S}_0$ ) and ( $\mathcal{S}_{TVL}$ ) are distributed parameter models. For batch reactor like here, ( $\mathcal{S}_0$ ) can be solved off-line and  $u_0$  can be tuned arbitrary. The on-line resolution of the nonlinear system ( $\mathcal{S}_{NL}$ ) is transformed into the on-line resolution of ( $\mathcal{S}_{TVL}$ ) consequently reducing the computational time. Taking into account these modifications, the internal model control structure becomes a time-variant linearized internal model control (TVLIMC) structure depicted Figure 3.

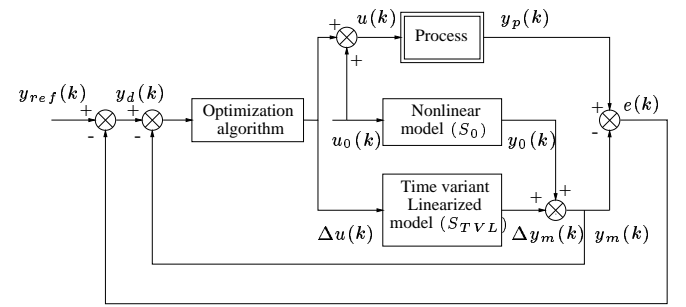


Figure 3: Time Variant Linearized Internal Model Control (TVLIMC) structure.

$y_0(k)$  is the ( $\mathcal{S}_0$ ) nominal output computed off-line,  $\Delta y_m(k)$  is the ( $\mathcal{S}_{TVL}$ ) output variation computed on-line during the optimization resolution and  $y_m(k)$  is the model output:  $y_m(k) = y_0(k) + \Delta y_m(k)$ .

The objective is now to find the control variation  $\Delta u(k)$  of the manipulated variable  $u(k)$  around a chosen trajectory  $u_0(k)$  leading to the best possible optimization result.

**Remark 3.1** Due to the TVLIMC structure and assuming that the error  $e(j) = y_p(j) - y_m(j)$  remains constant over the prediction horizon, the tracking problem can be reformulated as:

$$y_{ref}(j) - (y_p(j) - y_m(j)) = y_d(j) \quad (16)$$

$$y_{ref}(j) - y_p(j) = y_d(j) - y_m(j) \quad (17)$$

According to the TVLIMC structure, the trajectory tracking can be expressed as the following constrained optimization

problem:

$$\begin{cases}
 \min_{\Delta \tilde{u}} J(\Delta \tilde{u}) = \sum_{j=k+1}^{j=k+N_p} [y_d(j) - (y_0(j) + \Delta y_m(j))]^2 \\
 \Delta \tilde{u} = [\Delta u(k) \dots \Delta u(k + N_c - 1)]^T \\
 \Delta u(j) = \Delta u(k + N_c - 1) \\
 \forall j \in \{k + N_c, \dots, k + N_p - 1\} \\
 \text{subject to constraints on the manipulated variable :} \\
 u(j) = u_0(j) + \Delta u(j), T_e = \text{the sampling period} \\
 u_{\min} \leq u(j) \leq u_{\max} \\
 \Delta u_{\min} \leq \frac{u(j) - u(j-1)}{T_e} \leq \Delta u_{\max} \\
 \text{and subject to the } (\mathcal{S}_{TVL}) \text{ model resolution.}
 \end{cases} \quad (18)$$

### 3.2.2 Constraints handling

The constrained nonlinear optimization problem (18) is transformed into an unconstrained nonlinear optimization problem by using a hyperbolic transformation law for the constraints. This law transforms a variable with upper and lower bound constraints (i.e magnitude, velocity and acceleration rate) into a new unconstrained variable. Then, any unconstrained optimization algorithm can be applied for the resolution. The Levenberg-Marquardt algorithm has been chosen for its efficiency to solve this kind of problem.

### 3.2.3 State-estimator

Practical problems like disturbed measurements or unavailable measurements are investigated. Noisy and disturbed measurements could be prejudicial to an efficient tracking. In order to overcome this problem, a state estimator is then introduced in the control strategy structure (Figure 4). A feasibility condition is that the variable to be estimated, noted  $y_{est}$ , is sensible to the changes of the measured variable  $y_p$ . In all cases, the estimation depends on the quality of the model.

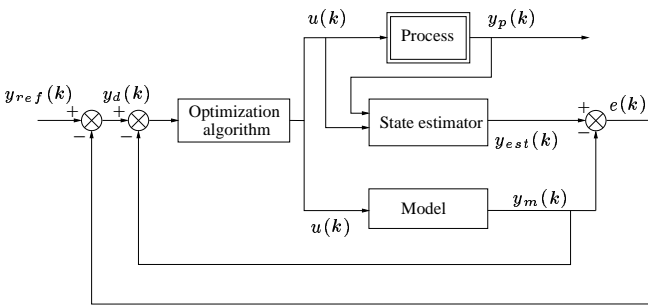


Figure 4: Modified Internal Model Control structure with a state estimator

## 4 Experimental results

### 4.1 Operating conditions

The objective is to track a humidity reference profile for the painting film drying process described Figure 1. The control variable is the infrared radiation  $\varphi_{ir}(t)$ . The nominal input is  $u_0 = 5000 W.m^{-2}.s^{-1}$  and the sampling period is 1 second. The constraints (magnitude and velocity) on the manipulated variable are:  $u_{\max} = 12,000 W.m^{-2}$ ,  $u_{\min} = 0 W.m^{-2}$ ,  $\Delta u_{\max} = +500 W.m^{-2}.s^{-1}$ ,  $\Delta u_{\min} = -500 W.m^{-2}.s^{-1}$ . The initial conditions are  $T_i = 36 ^\circ C$  and  $\chi_i = 0.4 kg.kg^{-1}$ . The models ( $\mathcal{S}_0$ ) and ( $\mathcal{S}_{TVL}$ ) are solved by the finite volume method (10 volumes).

### 4.2 Experiments

The prediction horizon influence is pointed out through several experiments. The control horizon  $N_c$  is in these first attempts equal to 1 in order to reduce the computational time.

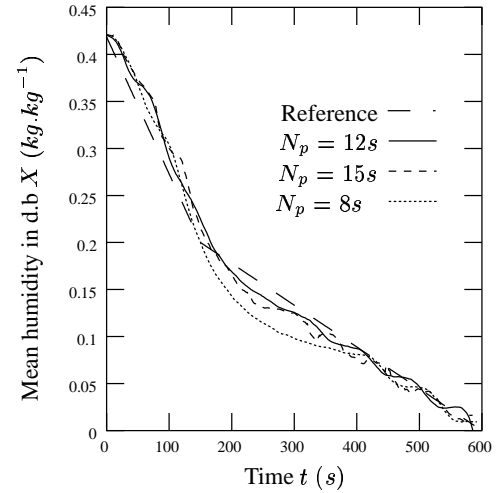


Figure 5: Trajectory tracking for  $N_p = 8s, 12s, 15s$ .

The tracking of the humidity reference trajectory is correctly achieved whatever the prediction horizon value (Figure 5). In all cases, the specified limitations on the actuators are satisfied (Figure 6).

The run with prediction horizon  $N_p = 12s$  seems to satisfy the best compromise between the computational effort and the tracking efficiency. Indeed, a long prediction horizon ( $N_p = 15s$ ) leads to poor tracking performances because of the computational time required and the difficulty to find an optimal solution on time. With a short prediction horizon ( $N_p = 8s$ ), information about the future process behavior is insufficient to anticipate the changes of the reference trajectory behavior and to prevent constraints violation. Much effort is spend on simulation and tests for tuning the prediction horizon

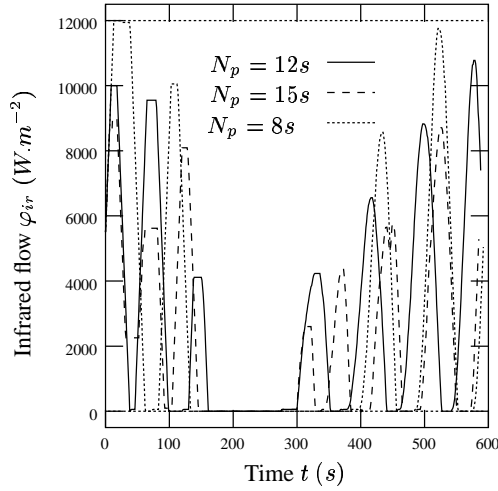


Figure 6: Magnitude of the control variable for  $N_p = 8s, 12s, 15s$ .

parameter. The prediction horizon value depends also on the “quality” of the model. Moreover, the robustness of the IMC structure is confirmed by these experiments since the tracking is achieved in spite of modeling errors and disturbances (Figure 7). The model output  $y_m$  does not track the reference trajectory  $y_p$  but the modified reference  $y_d$ .

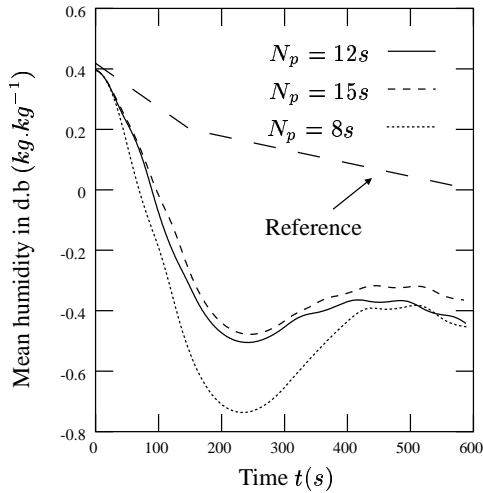


Figure 7: Model output  $y_m$  for  $N_p = 8s, 12s, 15s$ .

The last point is concerned with a practical point. Firstly, the humidity is not directly measured but is calculated from the measurement of the water mass losses by means of a very sensitive balance. This measurement is very noisy and disturbed. Secondly, the humidity is difficult to measure in industrial implementations. So the temperature, easy measurable, is used

to estimate the humidity (Figure 8). Temperatures changes involve humidity changes and then the feasibility condition of estimation in closed-loop is checked. At the end of the experiment, a disturbance appears due to the measurement uncertainty and affects the control performance. With the estimated humidity, the problem is overcome. The difference between the measurements and the estimates is due to the model but the IMC structure is able to cope with model uncertainties (Figure 7).

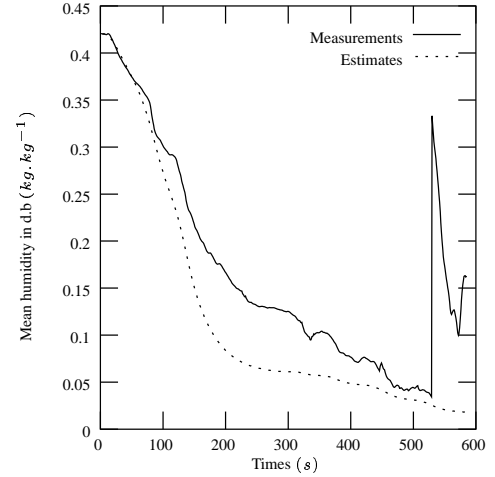


Figure 8: Measured and estimated humidity.

## 5 Conclusion

A trajectory tracking approach for a parabolic nonlinear distributed parameter system has been presented. A model-based predictive control strategy was extended to this kind of model. Experimental results have shown the efficiency and the robustness of this approach. The tuning of the prediction horizon has been discussed. A state estimator has also been used to cope with disturbances on the noisy humidity measurement.

## Acknowledgements

This project is supported by the GdR Automatique (Groupe de Recherche) of the National Center of Scientific Research (CNRS) - France.

## References

- [1] Allgöwer F., Badgwell T.A., Qin J.S., Rawlings J.B., Wright S.J., “Nonlinear predictive control and moving horizon estimation”, *Advances in control. Highlights of ECC’99*, Springer, pp. 391-449, (1999).
- [2] Blanc D., Laurent P., Andrieu J., Gérard J.F., “Modeling of the reactive infrared drying of a model water-based epoxy-amine painting coated on iron support with exper-

imental validation”, *Proc. 11th IHTC*, Kyongju, South Korea, **vol. 5**, pp. 181–186, (1998).

- [3] Courtial E., Touré Y., “ Model predictive controller and state-estimator for nonlinear constrained processes”, *Proc. CESA'96 IEEE IMACS Multiconference*, Lille, France, **vol. 2**, pp. 1168-1172, (1996).
- [4] Dufour P., Touré Y., Laurent P., “ A nonlinear distributed parameter process control : an internal linearized model control approach”, *Proc. CESA'98 IEEE IMACS Multiconference*, Hammamet, Tunisia, **vol. 1**, pp. 134–138, (1998).
- [5] Dufour P., Touré Y., Michaud D.J., Dhurjati P.S., “Optimal trajectory determination and tracking of an autoclave curing process : a model based approach”, *Proc. ECC'99*, Karlsruhe, Germany, Paper No. F1033-6, (1999).
- [6] Ibragimov N.Kh., Shabat A.B., “ Evolutionary equations with nontrivial Lie-Backlund group”, *Functional Analysis and the Application*, **vol. 14**, pp. 19–28, (1980).
- [7] Larabi M.C., Dufour P., Touré Y., Laurent P., “ Predictive control of a nonlinear distributed parameter system: real-time control of a painting film drying process”, *Proc. Mathematical Theory of Networks and Systems*, Perpignan, France, (2000).
- [8] Magri F., “ Equivalence transformations for nonlinear evolution equations”, *Journal of Math. Physics*, **vol. 18**, No. 7, pp. 1405–1411, (1977).
- [9] Richalet J., Rault A., Testud J.L., Papon J., “Model predictive heuristic control: application to industrial processes”, *Automatica*, **vol. 14**, pp. 413–428, (1978).

## 6 Annexe

### 6.1 Drying velocity

The pressure difference between the sample and the ambient air leads to an inside out water migration. This is characterised by the drying velocity  $\dot{m}(\bar{\chi}, T)$ :

$$\dot{m}(\bar{\chi}, T) = \frac{k_m m_v}{R} P_t \frac{2}{T + T_h} \log_{10} \left[ \frac{P_t - \chi_{air} P_{v_{sat}}(T_h)}{P_t - a_w(\bar{\chi}) P_{v_{sat}}(T)} \right] \quad (19)$$

where the saturated vapor saturation  $P_{v_{sat}}(T)$  is given in millibar by the expression :

$$\log_{10} P_{v_{sat}}(T) = C_0 \left(1 - \frac{T_1}{T}\right) - C_1 \log_{10} \frac{T}{T_1} + \frac{C_2 (1 - 10^{-C_3 (T/T_1 - 1)})}{C_4 (10^{C_5 (1 - T_1/T)} - 1) + C_6} \quad (20)$$

The activity  $a_w(\bar{\chi})$  is the solution of:

$$\frac{a_w(\bar{\chi})}{\bar{\chi}} = A_1 a_w^2(\bar{\chi}) + A_2 a_w(\bar{\chi}) + A_3 \quad (21)$$

with:

$$A_1 = K_k \frac{\frac{1}{c} - 1}{\chi_m} \quad (22)$$

$$A_2 = \frac{1 - \frac{2}{c}}{\chi_m} \quad (23)$$

$$A_3 = \frac{1}{\chi_m c K_k} \quad (24)$$

### 6.2 Energetical balance

The absorption coefficient  $\alpha_{ir}(\bar{\chi})$  is given by:

$$\alpha_{ir}(\bar{\chi}) = \alpha_p(\bar{\chi})(1 - \rho_p) + \alpha_s(1 - \rho_p)(1 - \alpha_p(\bar{\chi})) + \alpha_p(\bar{\chi})(1 - \alpha_s)(1 - \rho_p)(1 - \alpha_p(\bar{\chi})) \quad (25)$$

with:

$$\alpha_p(\bar{\chi}) = 1 - [aa_3 \left(\frac{\bar{\chi}}{\bar{\chi}_0}\right)^3 + aa_2 \left(\frac{\bar{\chi}}{\bar{\chi}_0}\right)^2 + aa_1 \frac{\bar{\chi}}{\bar{\chi}_0} + aa_0] \frac{1}{1 - \rho_p} \quad (26)$$

The latent heat coefficient  $l_v(T)$  and the calorific thermal capacity  $C_p(\bar{\chi}, T)$  are expressed with the temperature in Celsius degrees:

$$l_v(T) = [a_5 T^5 + a_4 T^4 + a_3 T^3 + a_2 T^2 + a_1 T + a_0] * 10^3 \quad (27)$$

$$C_p(\bar{\chi}, T) = [oT + b + \bar{\chi} \{c_{p_3} T^3 + c_{p_2} T^2 + c_{p_1} T + c_{p_0}\}] * 10^3 \quad (28)$$