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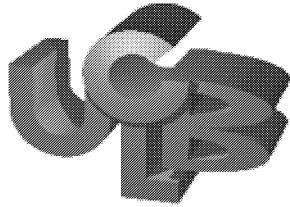
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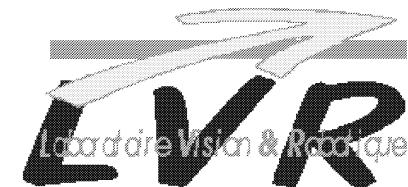
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**Predictive control of a  
nonlinear distributed parameter system :  
real time control of a painting film drying process**

Co-authors : M.C. Larabi, P. Dufour, P. Laurent, Y. Touré

Presented by : P. Dufour



## Context

Process control difficulties are increasing :

→ necessity of more accurate models :

- dynamic aspect
- independant space variables
- nonlinear physical phenomena

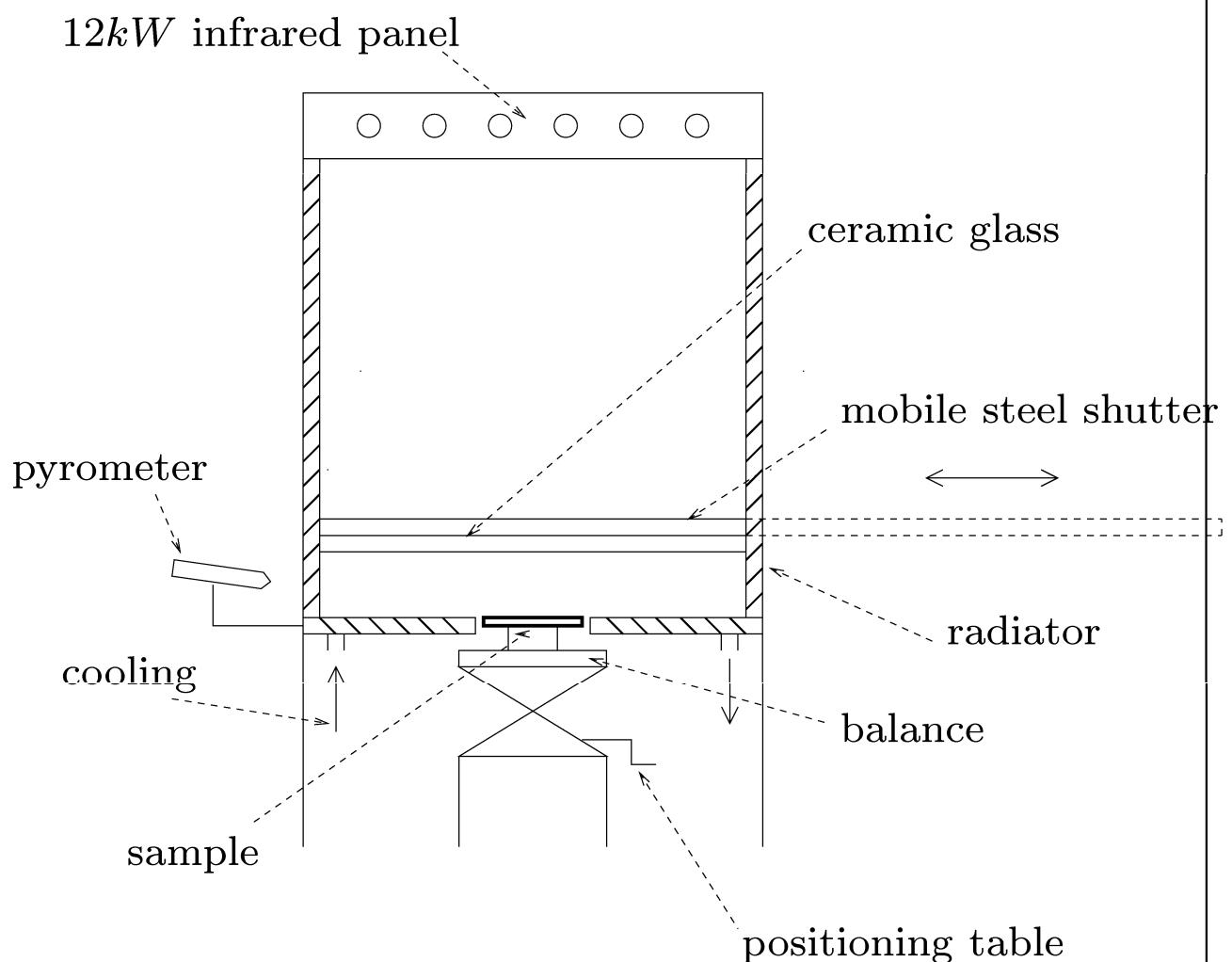
→ nonlinear partial differential equations.

## Objectives

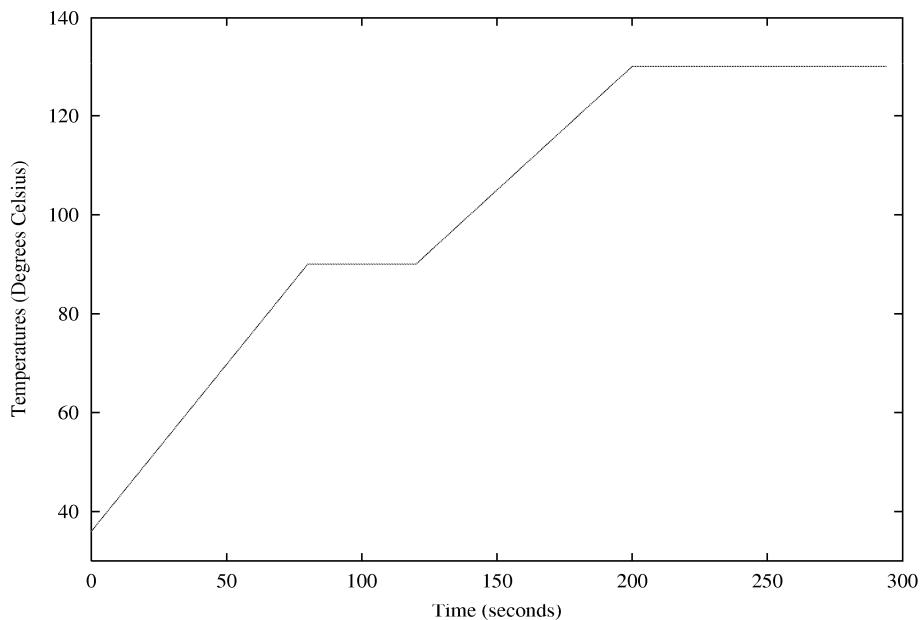
**Extend** the use of the **predictive control strategy** from systems described by **ordinary differential equations** to systems described by **partial differential equations**.

## Infrared drying process

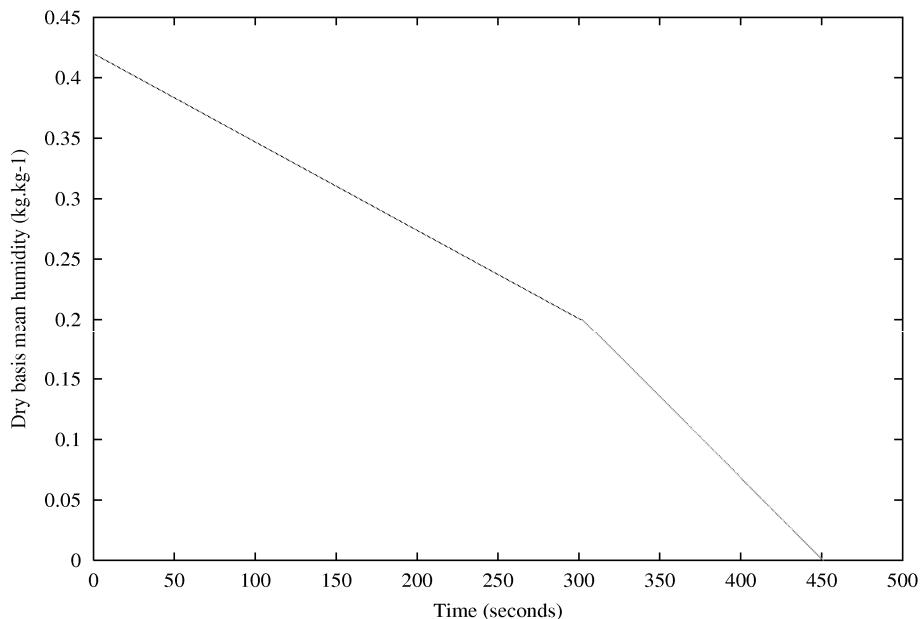
of painting film [Blanc et al., 1998]



## Control problem : trajectory tracking



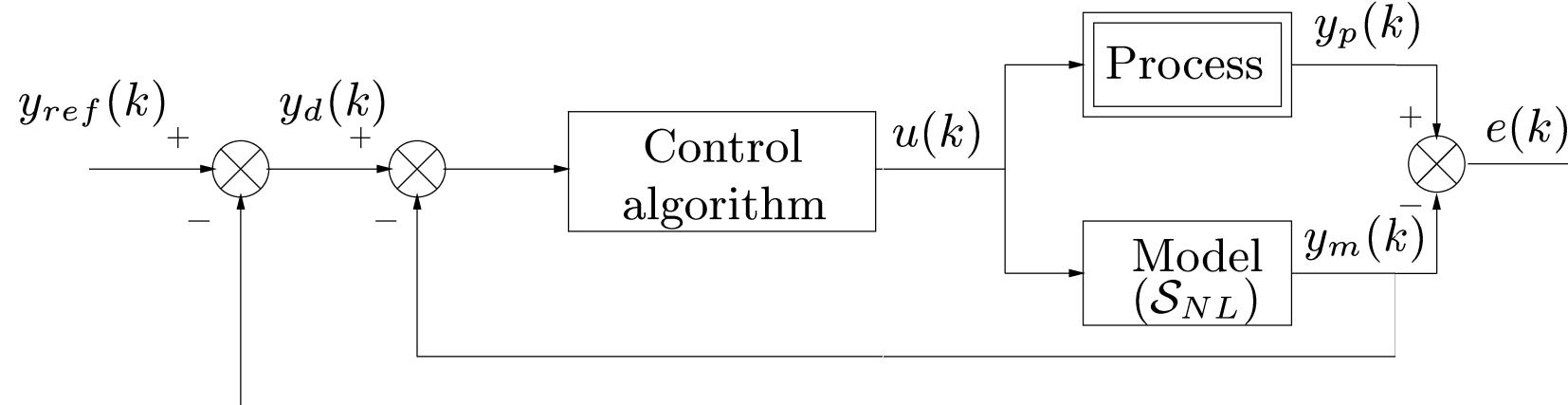
Temperature reference trajectory



Humidity reference trajectory

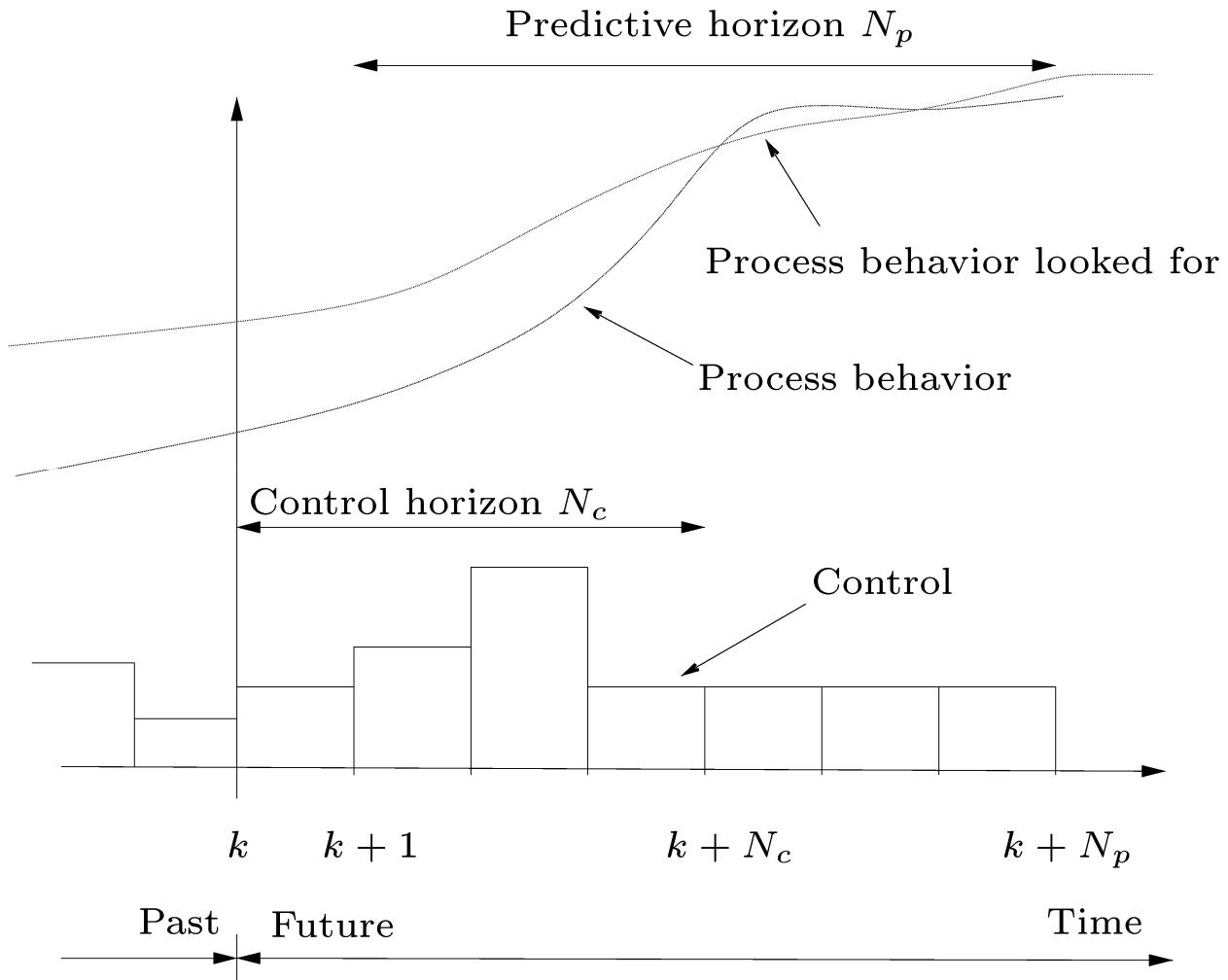
## Internal model control structure

[Morari and Zafiriou, 1983]



## Predictive control strategy

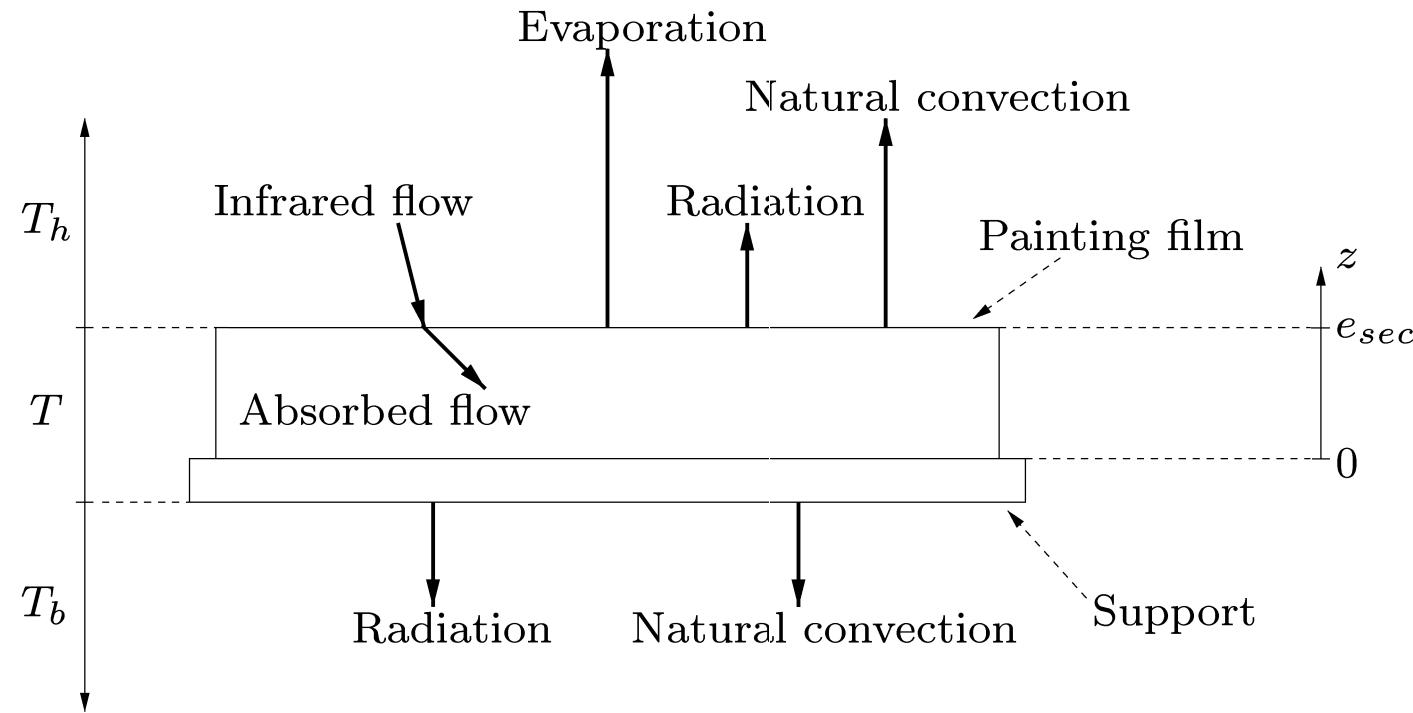
[Richalet et al., 1976]



Problems encountered :

- real time resolution
- horizons choice
- structural aspects (feasibility, stability)

## Thermal flows



## Energy balance

$$\begin{cases} (\rho_p C_p(\bar{\chi}, T) e_p + \rho_s C_s e_s) \frac{\partial T}{\partial t} = - \sum_{i=1}^5 P_i + P_{abs} \\ T(0) = T_0 \end{cases} \quad (1)$$

with the losses  $P_i$  :

$$\begin{cases} P_1 = h_c(T - T_b), \quad P_2 = \sigma_2(T^4 - T_b^4) \\ P_3 = h_c(T - T_h), \quad P_4 = \sigma_4(T^4 - T_h^4) \\ P_5 = l_v(T) \dot{m}(\bar{\chi}, T) \end{cases} \quad (2)$$

and the absorbed infrared flow :

$$P_{abs} = \alpha_{ir}(\bar{\chi}) \varphi_{ir}(t) \quad (3)$$

## Mass balance

$$\frac{\partial \chi}{\partial t} = \frac{\partial}{\partial z} [D_{eff}(\chi, T) \frac{\partial \chi}{\partial z}] \text{ for } 0 < z < e_{sec}, \quad t > 0$$

$$\text{with } D_{eff}(\chi, T) = \frac{D_0 \exp^{(-\frac{a}{\chi})} \exp^{(-\frac{Ea}{RT})}}{(1+\phi\chi)^2}$$

with the boundary conditions :

$$-D_{eff}(\chi, T) \frac{\partial \chi}{\partial z} = \frac{\dot{m}(\bar{\chi}, T)}{\rho} \text{ for } z = e_{sec}, \quad t > 0 \quad (4)$$

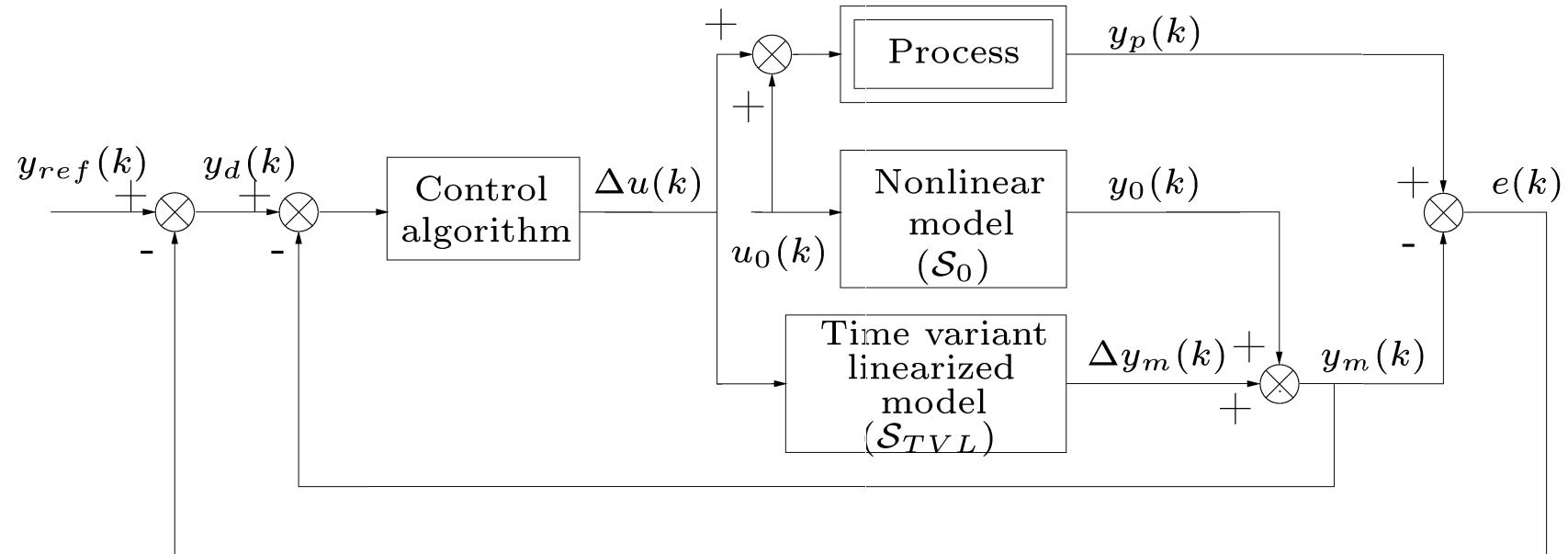
$$\frac{\partial \chi}{\partial z} = 0 \text{ for } z = 0, \quad t > 0$$

with the initial condition :

$$\chi(z, 0) = x_i(z) \text{ for } 0 \leq z \leq e_{sec}$$

## Internal linearized time-variant

### model control structure



## Real time aspect :

### variation model

A model **linearization** about  $(\mathcal{S}_0)$  characterised by  $(u_0, x_0, y_0)$ , with variations checking :

$$\begin{cases} u(k) = u_0(k) + \Delta u(k) \\ x(k) = x_0(k) + \Delta x(k) \\ y_m(k) = y_0(k) + \Delta y_m(k) \end{cases} \quad (5)$$

leads to :

$$(\mathcal{S}_{LTV}) \left\{ \begin{array}{l} \frac{\partial \Delta x}{\partial t} = \frac{\partial F_d}{\partial x|_{(S_0)}} \Delta x \text{ over } \Omega \times ]0, +\infty[ \\ \frac{\partial F_b}{\partial x|_{(S_0)}} \Delta x + \frac{\partial F_b}{\partial u|_{(S_0)}} \Delta u = 0 \text{ over } \partial \Omega \times ]0, +\infty[ \\ \Delta y_m = C \Delta x \text{ over } \bar{\Omega} \times ]0, +\infty[ \\ \Delta x(z, 0) = 0 \text{ with } z \in \bar{\Omega} \end{array} \right. \quad (6)$$

## Real time constrained optimization problem

$$\left\{ \begin{array}{l}
 \min_{\Delta \tilde{u}} J(\Delta \tilde{u}) = \frac{1}{N_p} \sum_{j=k+1}^{j=k+N_p} g(\Delta u(j-1), \Delta y_m(j)) \\
 \Delta \tilde{u} = [\Delta u(k) \dots \Delta u(k + N_c - 1)]^T \\
 c_i(\Delta u(j-1)) \leq 0 \quad (i \in \mathcal{I}, j \in \{k+1, k+N_c\}) \\
 (\mathcal{S}_{LTV}) \left\{ \begin{array}{l}
 \frac{\partial \Delta x}{\partial t} = \frac{\partial F_d}{\partial x}|_{(S_0)} \Delta x \text{ over } \Omega \times ]0, +\infty[ \\
 \frac{\partial F_b}{\partial x}|_{(S_0)} \Delta x + \frac{\partial F_b}{\partial u}|_{(S_0)} \Delta u = 0 \text{ over } \partial \Omega \times ]0, +\infty[ \\
 \Delta y_m = C \Delta x \text{ over } \bar{\Omega} \times ]0, +\infty[ \\
 \Delta x(z, 0) = 0 \text{ with } z \in \bar{\Omega}
 \end{array} \right. \end{array} \right. \quad (7)$$

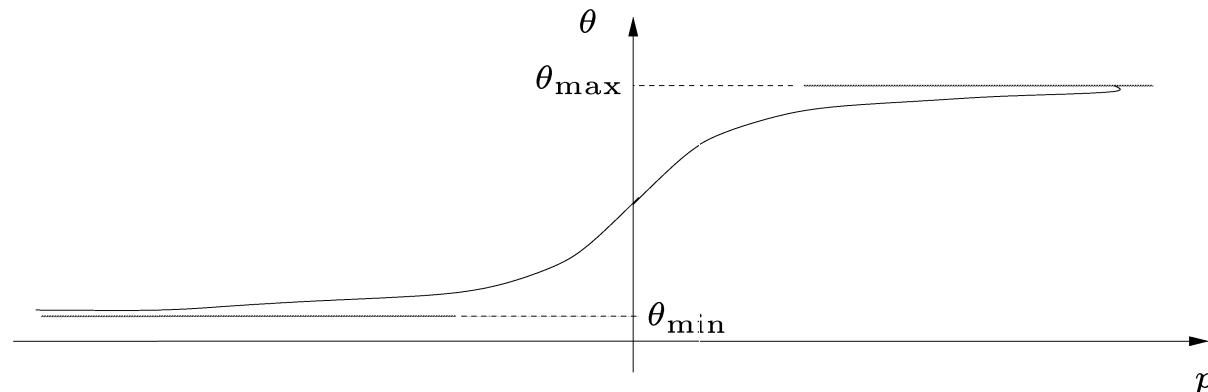
## Hard constraints handling : transformation method

From the **constrained variable**  $\theta$  :

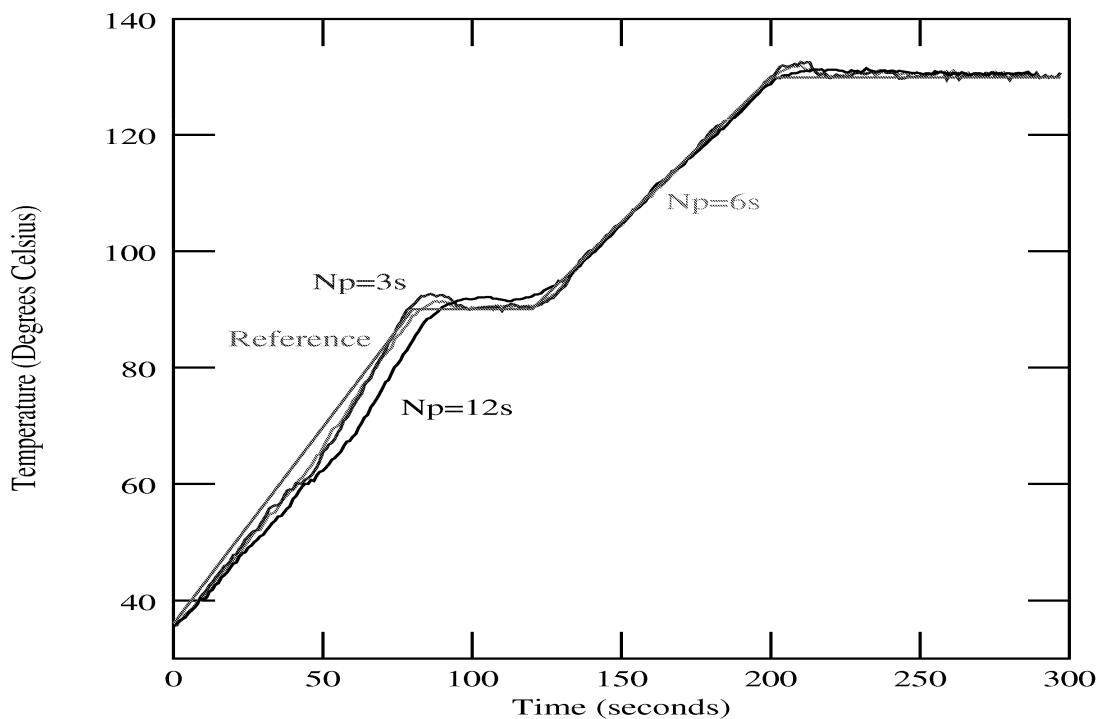
$$\theta_{\min} \leq \theta \leq \theta_{\max} \quad (8)$$

an **unconstrained variable**  $p$  is defined such that:

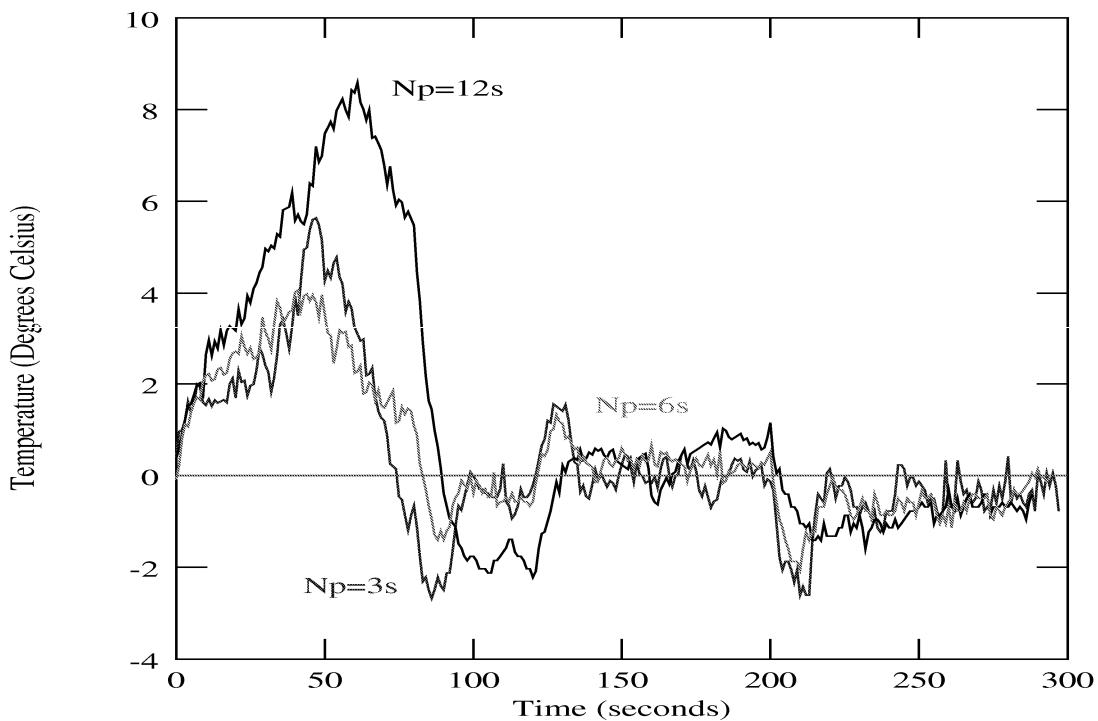
$$\theta = f(p) = \frac{1}{2}(\theta_{\max} + \theta_{\min}) + \frac{1}{2}(\theta_{\max} - \theta_{\min}) \tanh(p) \quad (9)$$



## Experimental results

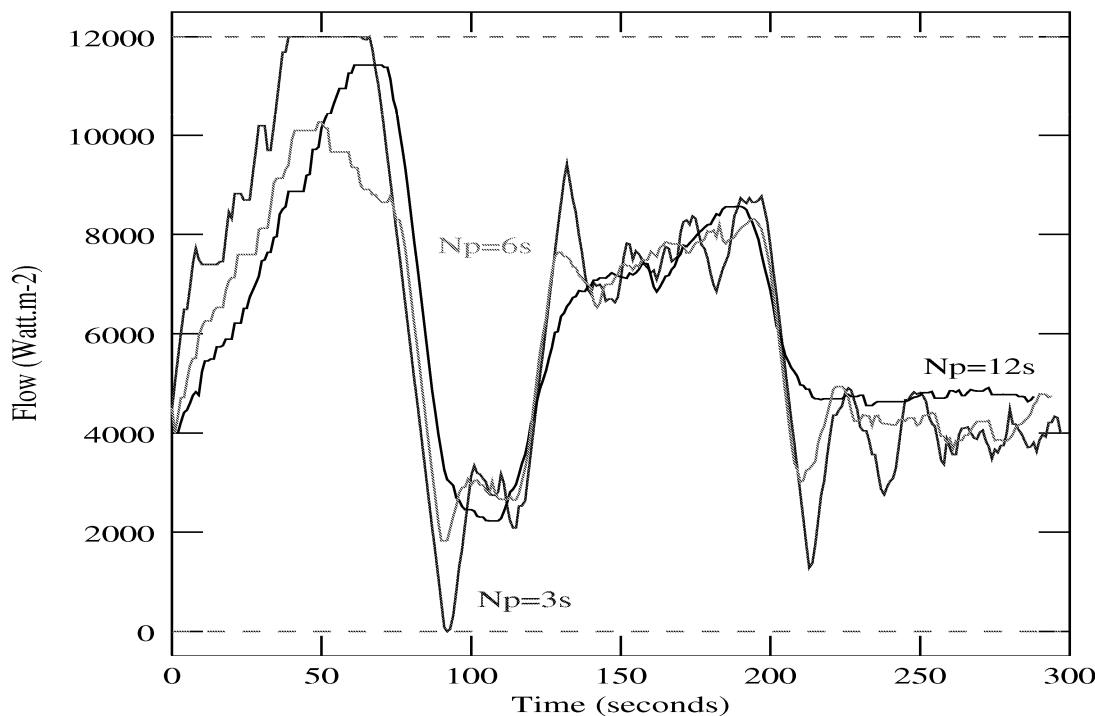


Trajectory tracking

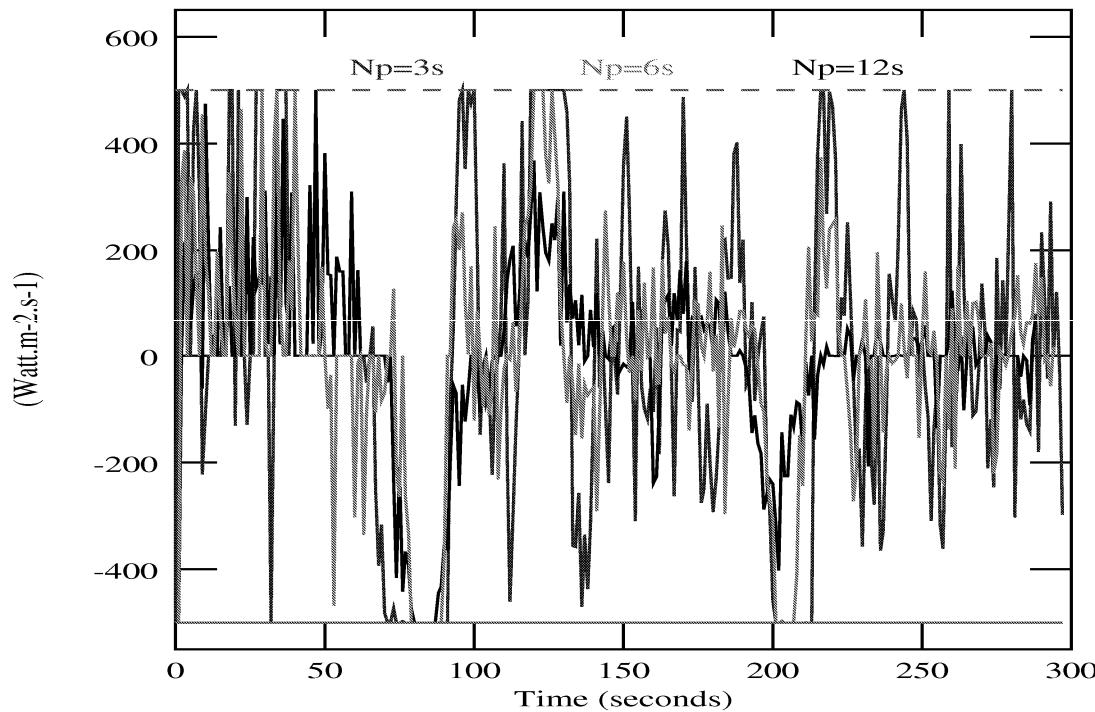


Tracking error

## Experimental results

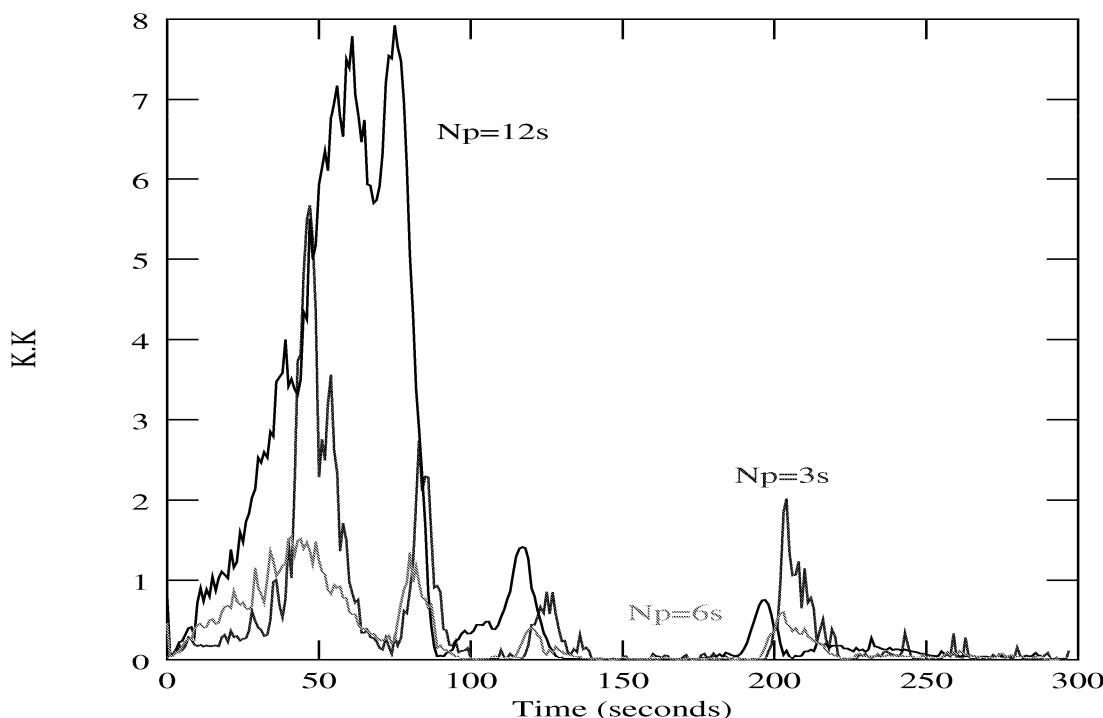


Control magnitude

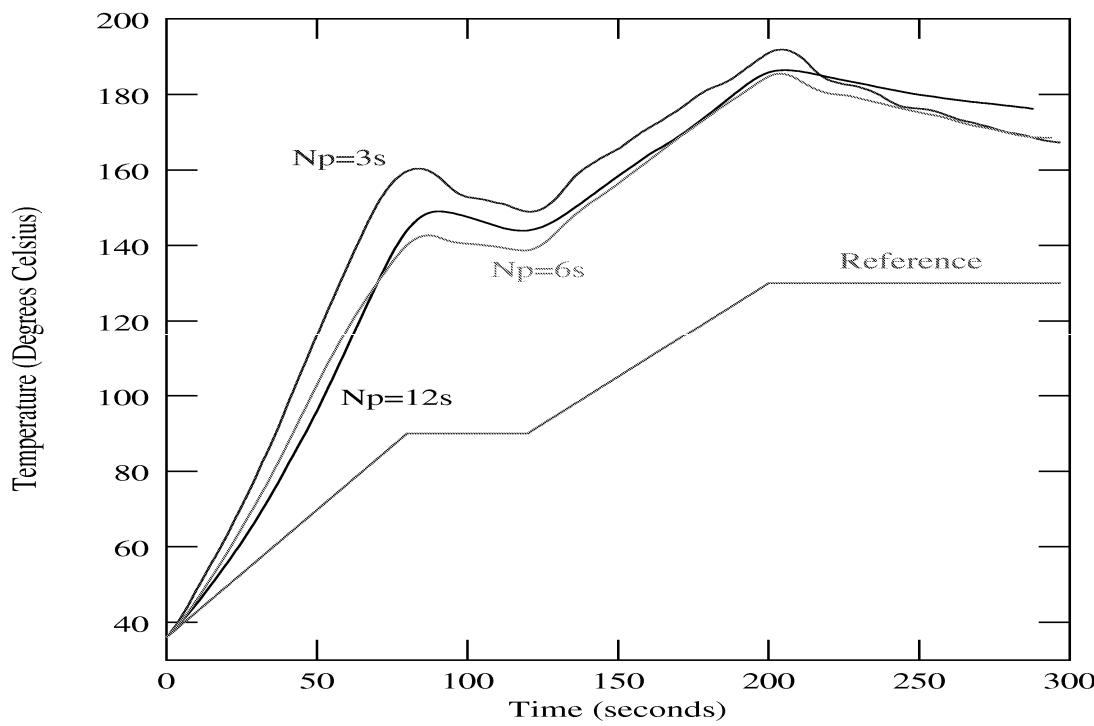


Control velocity

## Experimental results



Cost function



Model output

## Conclusions

- Extend the use of the **model predictive control** strategy to complex systems :
  - described by **partial differential equations**
  - strongly **nonlinear phenomena**
- Real time control aspect :
  - **nominal behavior** given **off line**
  - **on line correction**
- Horizons influence
- Behavior interpretation :
  - **multi disciplinary** study.

## Prospects

- For a **strongly noisy output** : use of a **receding horizon observer**.
- Stability of the closed loop system.