

*Optimal input design for online identification :  
a coupled observer-MPC approach*

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# Optimal input design

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- ✓ Problem of **parameter estimation** needs to be addressed
  - ✓ A natural question: how can the **input** sequence be chosen in such a way that the **parameters are optimally estimated**?
  - ✓ **1974 & 1985**: studies on optimal input in **linear systems**
  - ✓ **2002**: Kessman and Stigter include **analytical solution** of optimal input design
  - ✓ **2004**: Stigter and Kessman have found **recursive algorithm solutions**
  - ✓ But few studies exist for complex model based systems: nonlinear **PDE** system
- Control problem** tackled here: optimally control online a process based on a nonlinear PDE model to **estimate online** a set of the model parameters, under **input/output constraints**



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# ➤ Parametric sensitivity model

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Let the model structure:

$$\begin{cases} \dot{x}(t) = f(x(t), \theta) + g(u(t), \theta) \\ y(t) = h(x(t)) \end{cases} \quad (1)$$

$x(t)$  : n-dimensional state vector

$u(t)$  : r-dimensional input vector

$y(t)$  : m-dimensional output vector

$\theta$  : p-dimensional vector of time-invariant parameters

The associated **parametric sensitivity model** is derived from the model (1) :

$$\begin{cases} \dot{x}_\theta(t) \approx \frac{\partial f}{\partial x}(x(t), \theta) \cdot x_\theta(t) + \frac{\partial f}{\partial \theta}(x(t), \theta) + \frac{\partial g}{\partial \theta}(u(t), \theta) \\ y_\theta(t) \approx \frac{\partial h}{\partial x}(x(t)) \cdot x_\theta(t) \end{cases} \quad (2)$$

$$x_\theta \equiv \frac{\partial x}{\partial \theta} \quad \text{and} \quad y_\theta \equiv \frac{\partial y}{\partial \theta}$$



# ➤ Observer for state-affine systems

[Hammouri and De Leon, 1990]

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**Observer** : dynamic system obtained from the nominal model by adding a correction term.

**Advantage**: estimate the states not measured and unknown parameters.

The state affine system is in the following form :

$$\begin{cases} \dot{x} = A(u)x + B(u) \\ y = C(t)x \end{cases} \quad (3)$$

$$A(u) \in \mathcal{R}^n \times \mathcal{R}^n, B(u) \in \mathcal{R}^r \times \mathcal{R}^r \text{ et } C(t) \in \mathcal{R}^m \times \mathcal{R}^n$$

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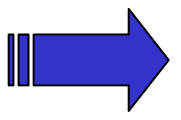
# ➤ Observer for state-affine systems

## [Hammouri and De Leon, 1990]

The observer for system (3) :

$$\begin{cases} \dot{\hat{x}} = A(u)\hat{x} + b(u) - S^{-1}C^T (C\hat{x} - y) \\ \dot{S} = -\rho S - A^T(u)S - SA(u) - C^T C \end{cases} \quad (4)$$

$\rho > 0; \hat{x}(0) \in \mathfrak{R}^n$  is the initial condition and  $S(t) \in PS(n)$ ,



**Main idea** : find a control law that maximizes  $y_{\theta}$ , based on the model (1) coupled with the sensitivity function (2) and the observer (4).



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# ➤ Model Predictive Control (MPC)

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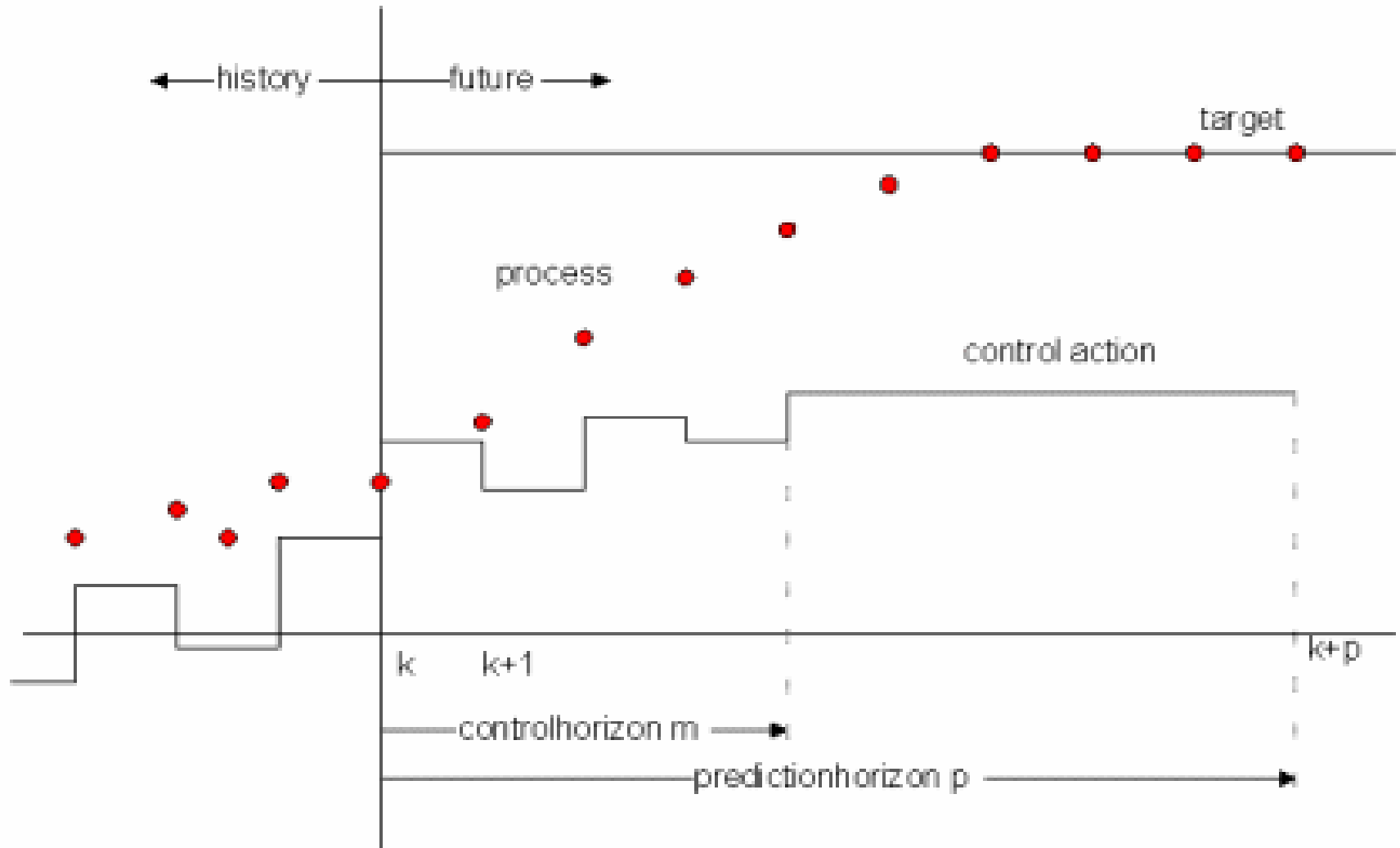
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Advantages:

- ✓ **constraints** (such as manipulated variables physical limitations, constraints due to operating procedures safety reasons...) may be specified
- ✓ a model aims **to predict** the future behavior of the process and the best one is chosen by **a correct optimal control** of the manipulated variables

Drawbacks:

- ✓ **computational time** needed may limit online time
- ✓ **suboptimal solutions**
- ✓ how to handle **unfeasibilities**

# ➤ Model Predictive Control (MPC)

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$$\left\{ \begin{array}{l} \min_u J(u) = \sum_{j=k+1}^{j=k+p} [a(y_{ref}(j), y_p(j))] \\ \text{subject to constraints on the manipulated variable :} \\ u_{\min} \leq u(j) \leq u_{\max} \forall j \in \{k+1, k+p\} \\ \Delta u_{\min} \leq u(j) - u(j-1) \leq \Delta u_{\max} \forall j \in \{k+1, k+p\} \\ \text{subject to constraints on the controlled variable :} \\ c(y_p(j), y_{ref}(j)) \leq 0 \forall j \in \{k+1, k+p\} \end{array} \right.$$

The function **a** means: trajectory tracking, processing time minimization, energy consumption minimization, **sensitivity**

**maximization**, ...



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# ➤ Model Predictive Control (MPC)

*[Dufour et al, IEEE TCST 11(5) 2003]*

- ✓ Originally developed for **nonlinear PDE model control**
- ✓ Main idea: **decrease the online time needed to compute** the PDE model based **control**

Approach:

- ✓ Input constraints: **hyperbolic transformation**
- ✓ Output constraints: **exterior penalty method**
- ✓ **Linearization** + sensitivities computed **off line**
- ✓ **One line** use of a time varying **linear model**
- ✓ One line resolution of the **penalized** (and so unconstrained) **optimization control problem**: as modified Levenberg Marquart Algorithm
- ✓ The turnkey **MPC@CB software** is used

# ➤ Final control structure

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To **maximize** the **parametric sensitivity** of the process output  $\frac{\partial y_p}{\partial \theta}$

⑤

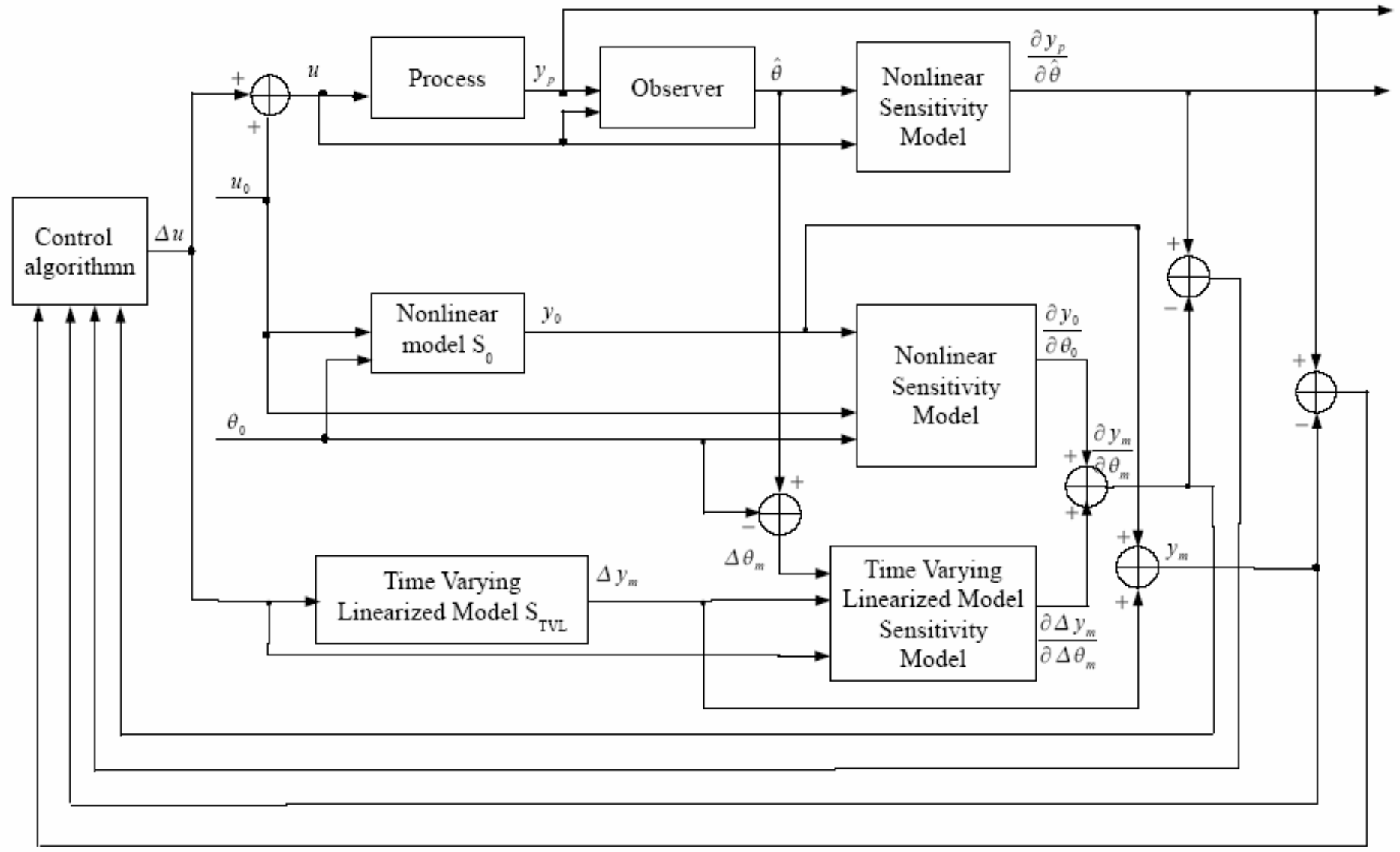
Cost function  $J$  to minimize:

$$J(u) = \sum_{j=k+1}^{j=k+N_p} \frac{1}{\left\| \varepsilon + \frac{\partial y_p}{\partial \hat{\theta}}(k) \right\|^2}$$

# ➤ Final control structure

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## Linearized IMC-MPC observer based structure for coupled input design and parameter estimation



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# ➤ Model for powder coating curing process

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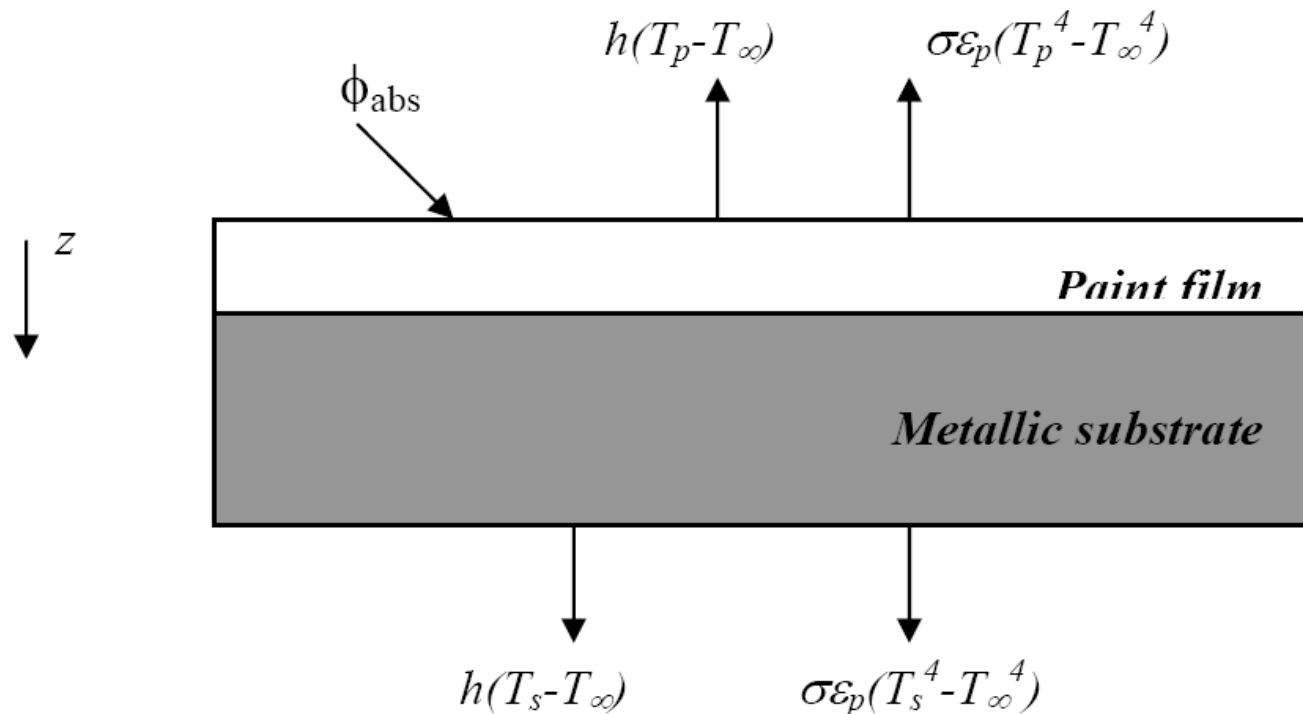
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**Powder coatings** = fine particles of: resin + cross-linker in thermosetting or thermoplastic powder coatings + pigments + extenders + flow additives and fillers to achieve specific properties (color, ...)



**Schematic drawing of the “substrat + powder” sample**



# ➤ Model for powder coating curing process

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- ✓ Thermal model based on the Fourier law of heat conduction uses:
  1. the **temperature** variable varying in the thickness of the powder coating metal sample
  2. the **degree of cure conversion** variable (ranging from 0+ to 1 and the end)
- ✓ A **non linear PDE Boundary control problem** has to be tackled

# ➤ Model for powder coating curing process

[Bombard et al, 2006]

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$$\frac{\partial T_p(z, t)}{\partial t} = \frac{\lambda_{c,p}}{\rho_p C_{pp}} \frac{\partial^2 T_p(z, t)}{\partial z^2} - \frac{e_p \Delta H_0}{C_{pp}} k_0 e^{\left(\frac{-E_a}{RT_p(z, t)}\right)} x^m (1-x)^n \quad \forall z \in ]0, e_p[ \quad \forall t > 0$$

$T_p(z, t)$  = **temperature** across the powder film thickness

$e_p$  = film thickness (~0.1 mm)

$x(z, t)$  = degree of cure

$$\frac{\partial T_s(z, t)}{\partial t} = \frac{\lambda_{c,s}}{\rho_s C_{ps}} \frac{\partial^2 T_s(z, t)}{\partial z^2} \quad \forall z \in ]e_p, e_p + e_s[ \quad \forall t > 0$$

$T_s(z, t)$  = **temperature** across substrate

$e_s$  = film thickness (~ 1 mm)

# ➤ Model for powder coating curing process

[Bombard et al, 2006]

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3 boundary conditions for the temperature:

$$-\lambda_p \frac{\partial T_p(z, t)}{\partial z} = \alpha_p \varphi_{ir}(t) - \sigma \varepsilon_p (T_p^4(z, t) - T_{ext}^4) - h_p (T_p(z, t) - T_{ext}) \quad \text{at } z = 0, \forall t > 0$$

**Estimated parameter**
**Manipulated variable**

$$-\lambda_{c,p} \frac{\partial T_p(z, t)}{\partial z} = -\lambda_{c,s} \frac{\partial T_p(z, t)}{\partial z} \quad \text{at } z = e_p, \forall t > 0$$

$$-\lambda_s \frac{\partial T_s(z, t)}{\partial z} = -\sigma \varepsilon_s (T_s^4(z, t) - T_{ext}^4) - h_s (T_s(z, t) - T_{ext}) \quad \text{at } z = e_p + e_s, \forall t > 0$$

# ➤ Model for powder coating curing process

*[Bombard et al, 2006]*

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The degree of cure  $x(z,t)$  of the powder:

$$\frac{\partial x(z,t)}{\partial t} = k_0 e^{\left(\frac{-E_a}{RT_p(z,t)}\right)} x^m (1-x)^n \quad \forall z \in [0, e_p] \forall t > 0$$

Initial conditions:

$$T_p(z,t) = T_s(z,t) = T_{ext} \quad \forall z \in [0, e_p + e_s], t = 0$$

$$x(z,t) = 0^+ \quad \forall z \in [0, e_p], t = 0$$

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- ✓ Developed under Matlab, **MPC@CB©** solvers **any users defined:**
  - trajectory tracking problem
  - maximization of the parameter sensitivity
  - operating time minimization problem
  - any cost function
  - input/output constraint handled
- ✓ **Any user defined** continuous **model** (SISO, MISO, SIMO, MIMO model), including large scale **PDE model**
- ✓ **Easy** to introduce a user defined observer
- ✓ **Easy** to apply software for simulation or real time application

**MPC@CB©: flexibility/ ease for a quick use in control!**

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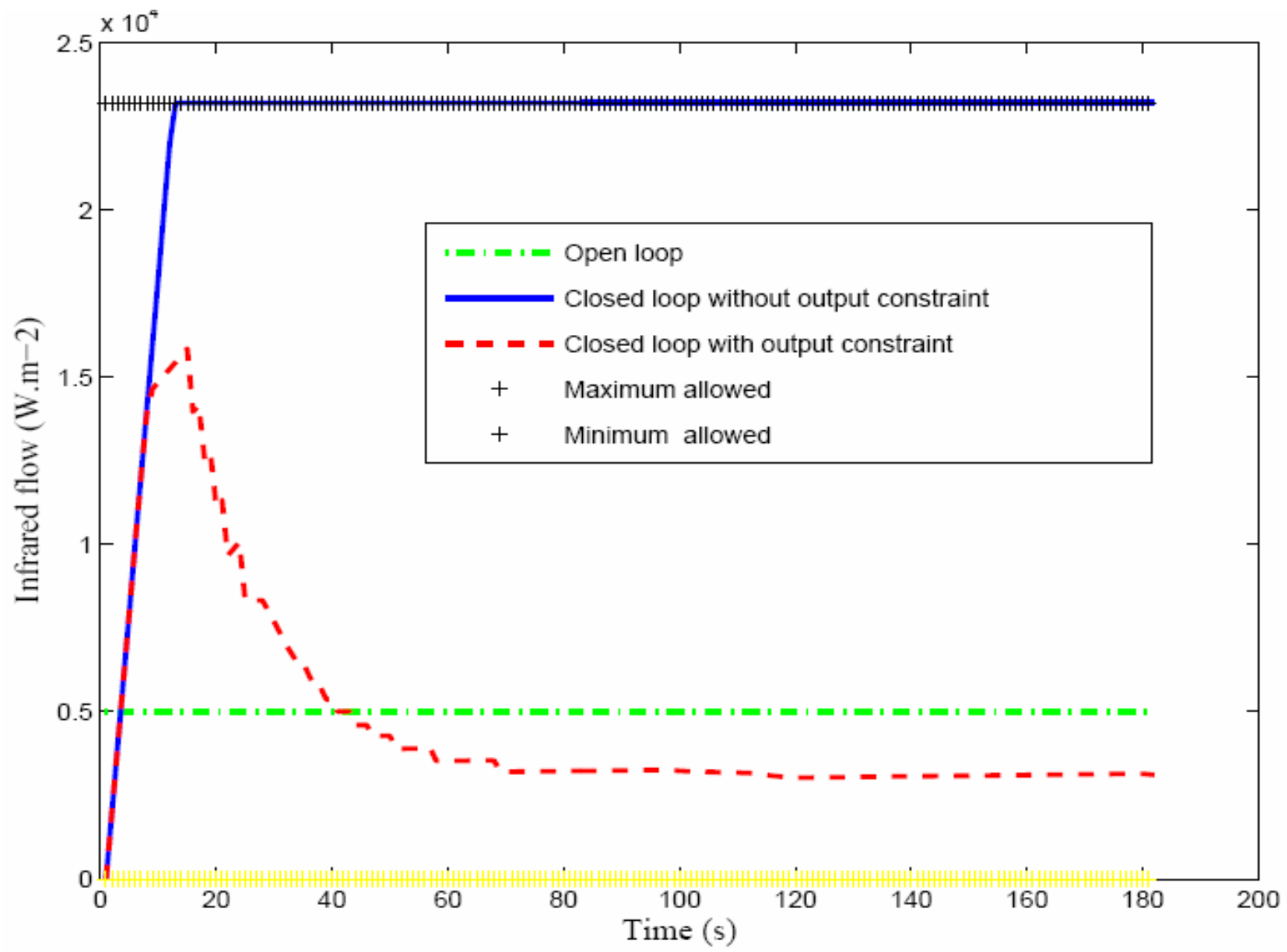
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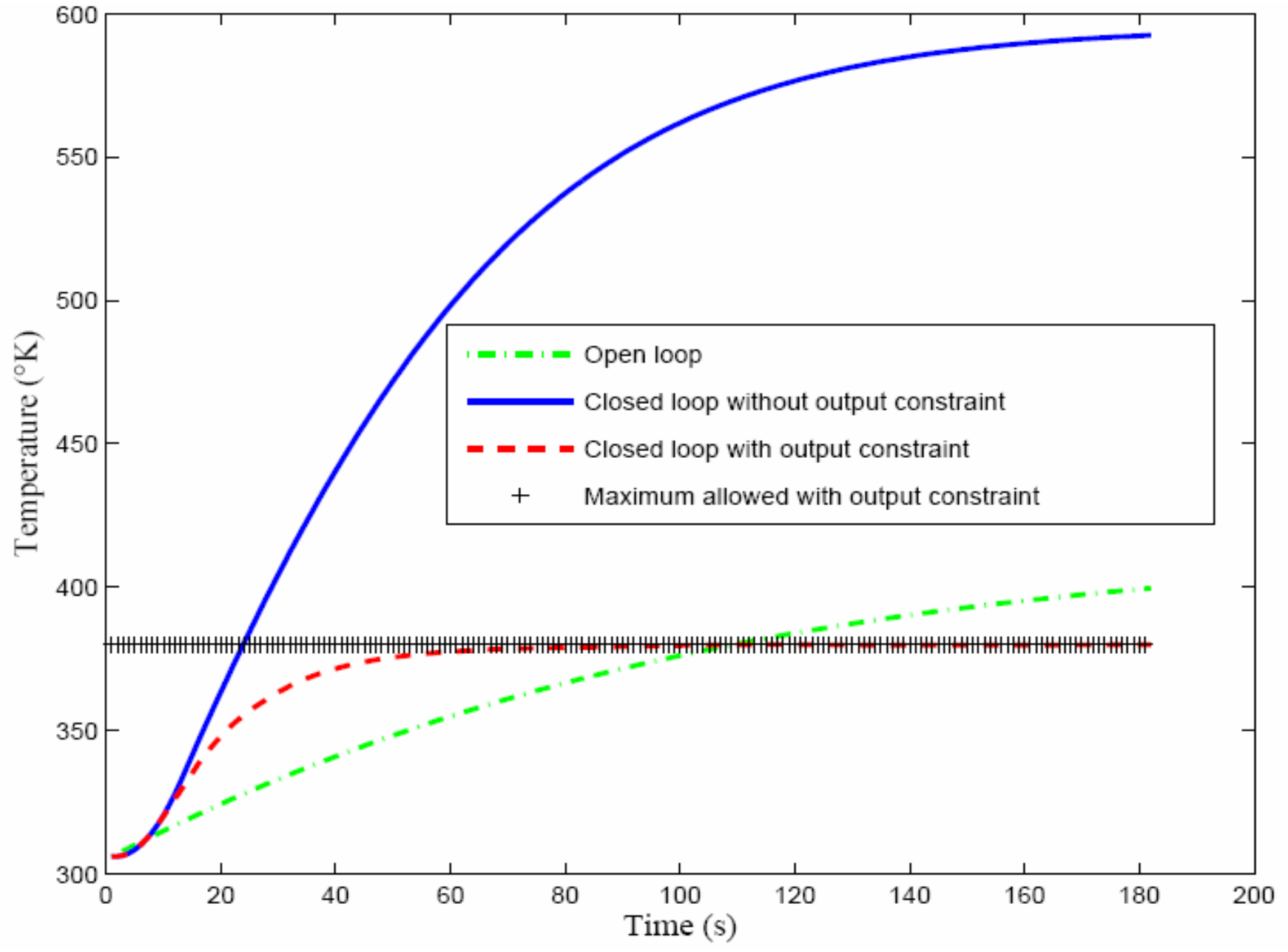
# Optimal infrared flow magnitude (input), with output constraint (sample time= 1s)





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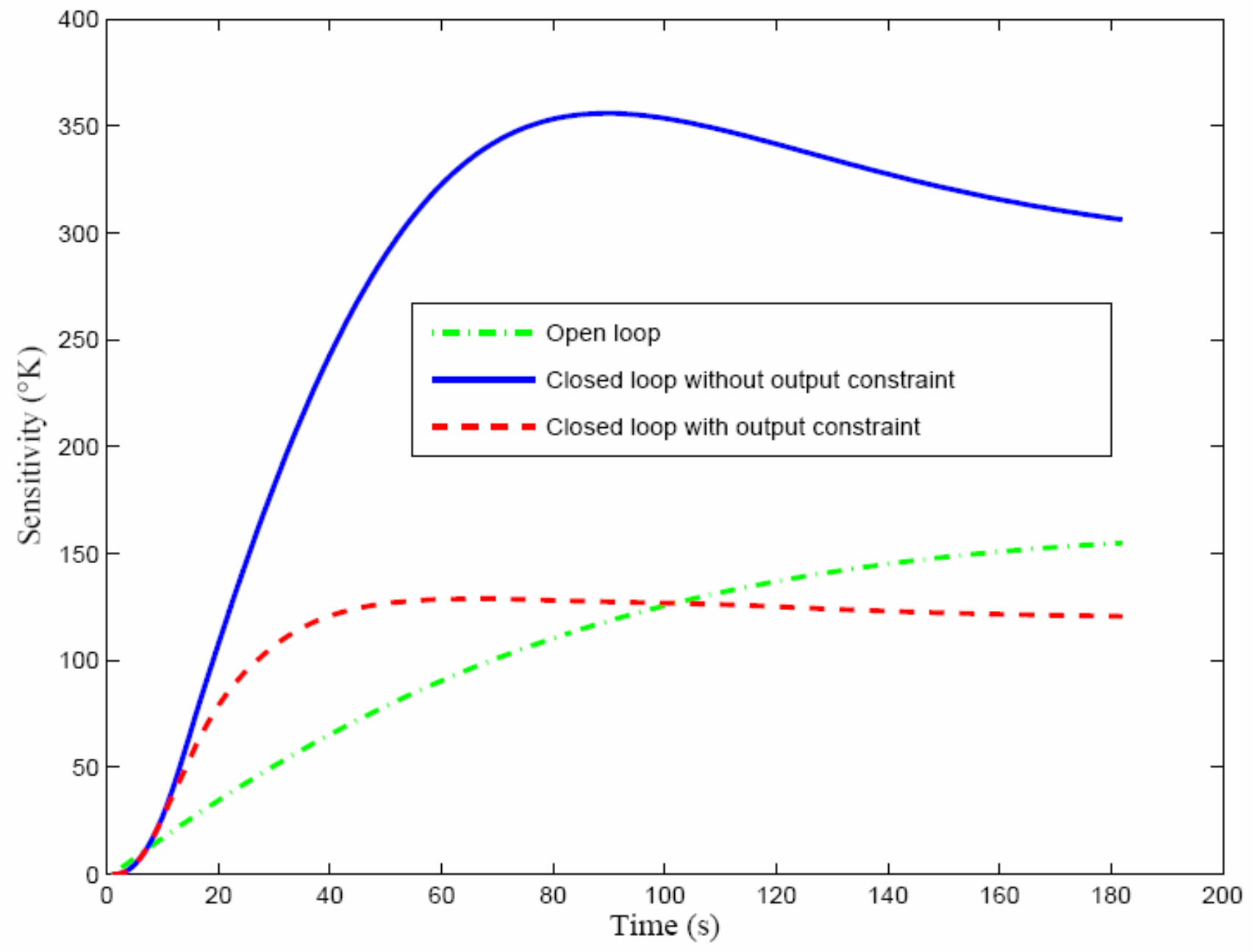
# Temperature in process output, with magnitude+ velocity input constraints+ output constraint (sample time= 1s)



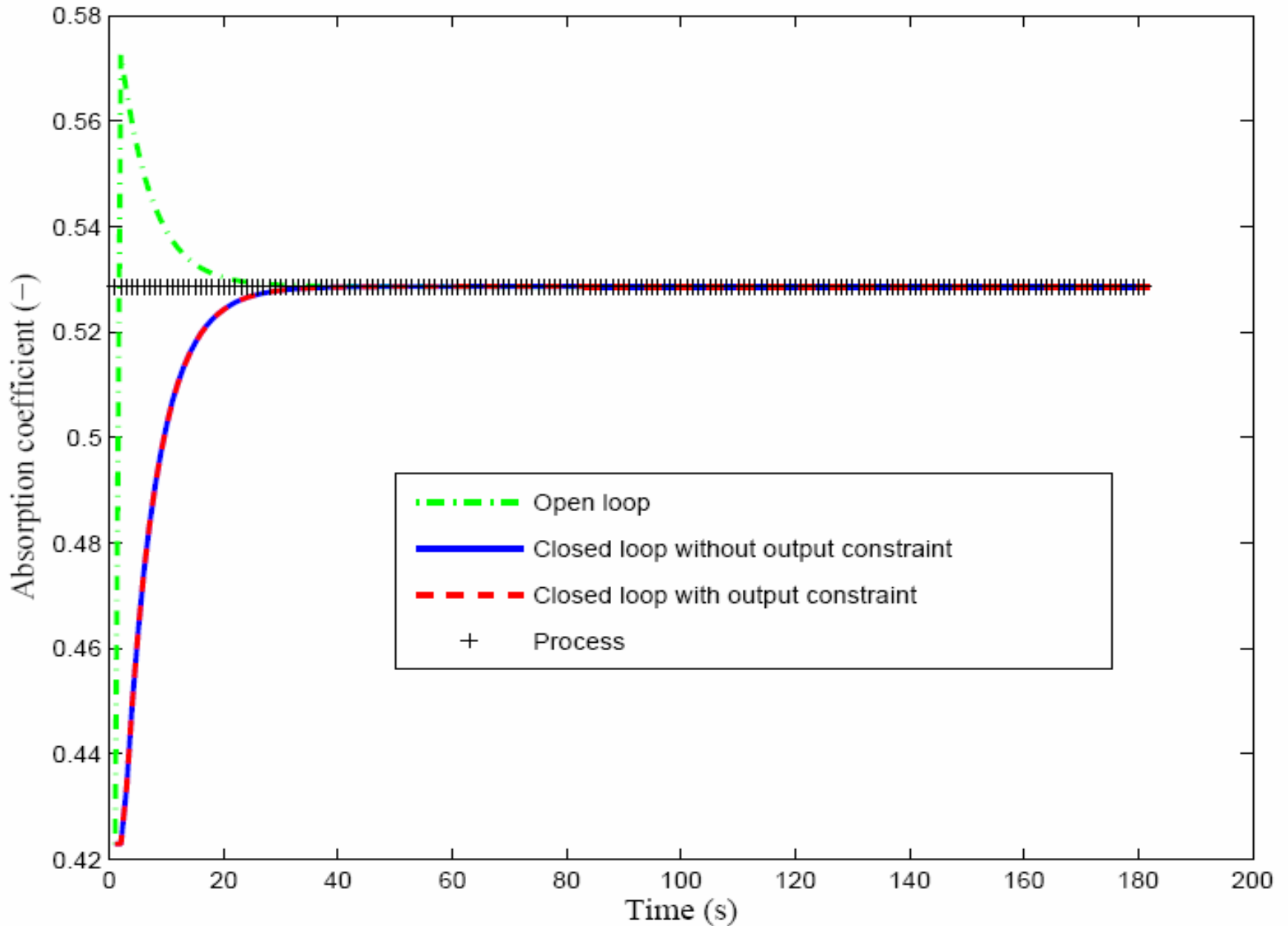


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# Sensitivity of the process output, with magnitude+ velocity input constraints+ output constraint (sample time= 1s)



**Parameter estimation**, with magnitude+ velocity input constraints+ output constraint (sample time= 1s)



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# Conclusions

- ✓ Approach coupling a **process model**, a **parametric sensitivity model**, an **observer** and a **predictive controller** was used on line for **optimal constrained optimization**
- ✓ **Optimal identification** of PDE system by a **general MPC@CB© software** has been shown

# Perspectives

- ✓ Extended for other process models and for **parameter vector case**
- ✓ To use **MPC@CB©** : [dufour@lagep.univ-lyon1.fr](mailto:dufour@lagep.univ-lyon1.fr)

*Thank you*

*Any questions ?*