Grand-mother clocks and quiet lasers

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Abstract

Galileo noted in the 16$^{\text{th}}$ century that the period of oscillation of a pendulum is almost independent of the amplitude. However, such a pendulum is damped by air friction. The latter may be viewed as resulting from air molecules getting in contact with the pendulum. It follows that air friction, not only damps the oscillation, but also introduces randomness. In the so-called "grand-mother" clock, discovered by Huygens in the 18$^{\text{th}}$ century, damping is compensated for, on the average, by an escapement mechanism driven by a falling weight. The purpose of this paper is to show that such a clock is, in its idealized form, a quiet oscillator. By "quiet" we mean that in spite of the randomness introduced by damping, the dissipated power (viewed as the oscillator output) does not fluctuate slowly. Comparison is made with quiet laser oscillators discovered theoretically in 1984. Because the input power does not fluctuate in both the mechanical oscillator and the quiet laser oscillator, the output power does not fluctuate at small Fourier frequencies, irrespectively of the detailed mechanisms involved.

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1 Introduction

Oscillators are devices whose mass position or electrical potential vary sinusoidally as a function of time. The time period T is a constant in the present paper and we are not concerned with frequency fluctuations. Only small amplitude fluctuations about some large oscillation amplitudes are considered, first for the "grand-mother clock", and later for laser oscillators.

In any practical oscillator there is damping. Accordingly, in order to sustain the oscillation some energy must be continuously fed in. In the case of the grand-mother pendulum, the energy originates from a slowly dropping weight. In the case of laser diodes, the energy originates from an electrical current, or some other source of energy such as thermal radiation. These power supplies are then called "pumps". The amplitude fluctuations we referred to above are primarily caused by damping, which should be viewed as a random process. Damping may be caused for example by air molecules contacting the pendulum at random times. What we consider as a measurable quantity is not the oscillation amplitude itself, but the dissipated power $P(t)$. Note that the oscillators treated here involve a single input channel (through which the power is fed in), and a single output channel corresponding to the dissipated power $P(t)$. All losses aside from those resulting from the damping mechanism are supposed to be negligible, and the oscillator is stationary, which means that a shift in time would not affect the system operation.

The main question we wish to consider is the following: Assuming that the power fed into the oscillating mechanism is strictly constant in time, does the dissipated power $P(t)$ fluctuate? Under ideal conditions and for slow variations (small Fourier frequencies), our answer is that the dissipated power does not fluctuate. The general argument is as follows. In a conservative device (i.e., with no internal loss or gain) the output power may fluctuate even if the input power is a constant. This is because mechanical or electrical energy is stored in the device. However, the law of conservation of energy entails that the output power, integrated over a sufficiently long time, must be equal to the input power, integrated over the same long time interval[1]. In the case of the grand-mother pendulum, the input power, fed in with the help of an escapement mechanism, is a constant. We prove in the next section, both analytically and through numerical calculations, that, in agreement with the above general argument, the dissipated power $P(t)$ does not fluctuate at small Fourier frequencies. Here damping events occur at times that are Poisson distributed (crudely speaking this means that the damping events are "random" in time). But some of these events absorb a large energy while others absorb a small energy. This is why the absorbed power may be regulated.

A laser diode, on the other hand, may be fed in by a constant current, generated for example by a battery in series with a large cold resistance. The electrical potentials

[1] As an example, note that in the electrical power grid where energy storage is negligible, the power generated by, say, nuclear plants must, at any instant, be equal to the consumed power, or nearly so. Because the consumed power varies quickly in time but the power generated by nuclear plants cannot be made to vary quickly, burning-gas generators are required from time to time. Of course, the law of conservation of energy tells us nothing about the power distribution among various customers.

across the laser diode and across the detector diode that convert the emitted light back into an electrical current, are nearly equal to h/Te, where h denotes the Planck constant, T the oscillation period, and e the electron charge. It follows that if a constant current is fed into a laser diode, conservation of energy alone tells us that the photo-detector, which is the only cause of loss according to our assumptions, delivers a current that does not fluctuate either at small Fourier frequencies. In the present situation, each photo-detection event carries the same energy because the electron charge is a constant. It follows that regulation of $P(t)$ requires that the detector electronic events be distributed in time in a "sub-Poissonian" manner. By that, one means that the event occurrence times are more regularly spaced than in a Poissonian distribution.

It is often argued that nonclassical features of the electromagnetic field states such as sub-Poissonian statistics are consequences of the field quantization. For sure Quantum Optics provides a framework for a complete interpretation of nonclassical light features. The first theory of "sub-Poissonian" emission by regularly driven laser diodes was given that way [1]. But we show here that accounting for the emitter-detector system as a whole quantum device produces the same interpretation only using the law of conservation of energy. The formulas obtained for the photodetection statistics thus agree in every details with the results obtained independently by quantum theorists. Some other measurable effects do indeed require light quantization. However, this is not the case for the kind of oscillators that we are considering. Our view point is that lasers, on that respect, are akin any oscillator.

2 The "grand-mother" clock

The basic element of a grand-mother clock is a weight W suspended at the end of a weightless bar of length L in the earth gravitational field g. As was first shown by Galileo the oscillation period $T = 2\pi\sqrt{L/g}$ does not depend on the oscillation amplitude as long as this amplitude remains small, a condition that we assume fulfilled. The period T does not depend either on the weight value according to the equivalence principle: inertial mass equals gravitational mass. For simplicity we suppose that the pendulum period is unity, that is $T = 1$ s. This amounts to selecting some appropriate L value, considering that $g \approx 10$ m/s^2. We also suppose that $W = 1$ so that the highest weight altitude E represents the pendulum energy since the kinetic energy then vanishes. In the following we denote by $E_k,\ k = 1, 2, ...$ the pendulum energies at successive periods of oscillation. This energy gets decremented by a random damping mechanism to be specified below, and incremented by a regular escapement mechanism.

Let us first describe the damping mechanism. The pendulum, with energy E_k, is supposed to pick up with probability $p \ll 1$ at each period a molecule of weight w at rest at the lowest level, and to release it at the highest level E_{k+1} (see Fig. 1). Because the probability $p \ll 1$, the molecule-picking events form a Poisson process. The average inter-event time for a Poisson process is known to be $1/p$ [2, 3]. Note that we are considering only time intervals much larger than the pendulum period $T = 1$. Raising a molecule of weight w from altitude 0 to altitude E_{k+1} amounts to reducing