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SURVEYING THE FRONTIER BETWEEN SIGNAL PROCESSING AND DYNAMICAL SYSTEMS

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ABSTRACT

The first and most obvious rapprochement between the areas of signal processing (SiP) and that of dynamical systems (DS) is their very fundamental interest: the analysis, prediction and control of real world physical systems, either natural or man-made. This occurred quite fast at the level of analysis and modeling tools, either research area borrowing with great ease methods and concepts from another, from elementary to the very complex levels. The main purpose of this paper is to review several such convincing examples, presenting them from a global perspective. The question of the chaotic systems, a subclass of DS, is also addressed. While already used in large bandwidth communications, their influence is shown to be extendable even further. On this line, their use in sea electromagnetic echo is presented. The article ends with conclusions, which analyze the pros and the cons, and perspectives, which are aimed at future research axis.

1. INTRODUCTION

A relatively large number of physical systems, either natural or man-made, are studied under two paradigms: either in the traditional and reductionist way, either from a holistic, systemic perspective. Usually, the classical signal processing (SiP) techniques, aimed at extracting pieces of information about the analyzed system (or signal) falls in the first category. This is not to say that the two domains are dichotomic. At the contrary, they strongly interfere, by the very nature of their object of study.

Many often encountered systems which may be analyzed and modeled under a systemic paradigm are nonlinear and a part of them are even chaotic ones. One could enumerate: the meteorological phenomena, the eco-systems, the social and economical systems, the financial markets, several electronic circuits (like oscillators), chemical reactions, and communications and urban/highway traffic etc.

Many of these may be very easily modeled by using a dynamical system (DS) perspective. On the other hand, their analysis using the classical reductionist analysis tools may become overwhelming.

Another common point of dynamical systems and signal processing techniques lies in the study of nonlinearity. The latter benefits of an increasing interest from researchers in signal processing area, allegedly being the "buzzword of the day". It is its considerable modeling power, which may be applicable to even very complex systems, which justifies this research trend, despite the underlying difficulties. On the other hand, the dynamical systems had assumed, for many years now, hypothesis of nonlinear behaviors in their studies. As such, is very appealing and it should be easy for signal processing researchers to benefit of this existing expertise.

One such example, where nonlinearity is not only assumed and intensely studied, but is also a prerequisite, is the study of chaotic systems. The chaotic systems are either discrete or continuous time dynamical (whose state is time-varying) systems which exhibits apparent disorder, similar to randomness, while still remaining completely deterministic. In fact, the proper sense of the term "chaotic" should be read as "extreme dependence of initial conditions". The figures representing such systems often have very interesting and complicated shapes, widely known as "strange attractors", most of them being, in fact, fractals.

The sensitivity to the initial conditions translates in increasing divergence in system evolution. After n steps, this divergence is assumed to bind to an exponential model:

$$f^n(x+\varepsilon) - f^n(x) = \varepsilon \exp(n\lambda) \qquad (1)$$

where λ is the Lyapunov exponent. This quantity is positive for chaotic systems.

Some chaotic models, as will be shown, already made their way into traditional signal processing areas, such as in radar noise modeling. However, they remain a relatively new concept for signal processing researchers. Good introductory works are, by example, [1] and [2].

2. APPLYING DS KNOWLEDGE TO SIP

One of the first breakthroughs of the DS theory into a domain traditionally belonging to the signal processing area is the proposed use of chaotic models for the high-resolution, low grazing angle sea clutter returns [3], in the case of coastal maritime surveillance radars. While still subject to discussions and not fully accepted by the scientific community, the subject is of great importance nonetheless.

The traditional ways of doing with sea clutter and the degradation of target detection are by use of Neyman-Pearson or Bayesian hypothesis testing criteria. They require the probability distribution of noise amplitude (and of the target echo) to be properly modeled. While for target echo a

number of models have been defined, notably the Swerling models [4], the noise was generally accepted as white an Gaussian.

The problem is that, for large bandwidth radars and for low grazing angles, the Gaussian hypothesis is far from true. This misassumption greatly penalized the detection performance of the first large bandwidth coastal radars, despite the higher resolution leading to a better SNR.

Different others probability density functions have been proposed for the sea clutter amplitude, serious candidates being Weibull [5], lognormal [6] or K-compound [7].

However, all of them failed to achieve general consensus. More, the estimation of their underlying parameters is often tedious.

In this context, supported by results obtained while analyzing the published 8 [8] radar dataset, professor Haykin made the hypothesis of an underlying purely deterministic, albeit chaotic, model of the sea clutter. His first results [3] were encouragingly, showing sea clutter exhibiting some well-known characteristics of dynamical systems, such as positive Lyapunov exponent and a fractal dimension.

However, the presented results were questioned, when a number of publications [9] shown their ambiguity. The main source of confusion lied in the very similar behavior that some particular purely random (i.e. stochastic, not chaotic) processes exhibit. It is the case of $1/f$ processes, whose spectral density is:

$$s(f) = f^{-(2H+1)} \tag{2}$$

where $H \in (0,1)$ is the Hurst parameter.

While the outcome is still debatable in terms of theoretical soundness, other scientists [10] developed Haykin's original idea, by proposing a new kind of chaotic model:

$$\xi(t) = \lim_{\Delta x(0)\to 0} \frac{\Delta x(t)}{\Delta x(0)} = \left[1+(1-q)\lambda_q t\right]^{\frac{1}{1-q}} \tag{3}$$

which is a generalization of (1).

The latter paper shown surprisingly good results in terms of naval target detection, based on the different values that $\beta = (1-q)^{-1}$ in the presence or absence of a target (see Figure 1). Values between 0.8 and 1 indicate that a target is present in the analyzed resolution cell.

Another interesting and very useful applications of methods originating in the DS field is presented in [11]. This time, one of the most central concepts of the signal processing is put under the loupe: the stationarity.

This concept, while mathematically formalized in a rigorous way, is largely interpretable in the framework of a particular application.

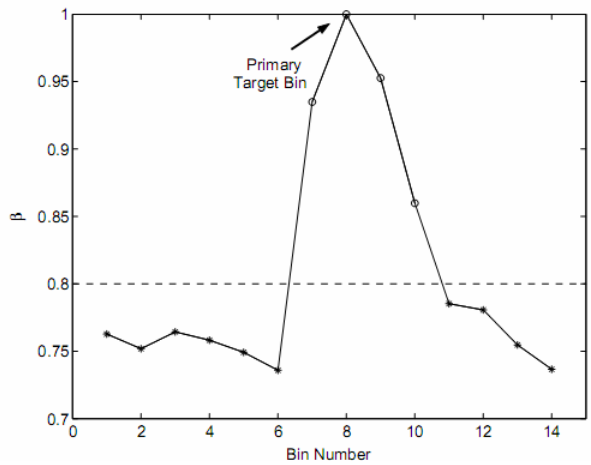

Figure 1: Values of β for 14 range resolution cells; the higher values denotes presence of target

By example, when random processes are considered, stationarity stands for temporal invariance of signal's statistics. On the other hand, when deterministic signals are studied, signal's stationarity means the invariance of its spectral properties. Finally, claiming that a signal is stationary or not is only pertinent with respect to a particular interval of time or frequencies, the pertinence of which depends on the problem at hand. This "localization" clearly contradicts with the permanence notion associated with the stationarity concept. The same signal may appear as stationary or not, function of the length of analyzing interval which, basically, defines the scale at which the signal is studied.

In an intuitive way, for a given average spectrum, a stationary signal should present no temporal structure, i.e. no predictability. This should manifest for stationarry signals only, in the phase of their spectra.

Based on this observation, a mathematical, albeit practical, definition of stationarity may be given. It uses the notion of *surrogates*, which are signals having the same power spectrum as the original signal, but are stationnarized. This is achieved by replacing the phase of original signal's spectrum by a random phase, completely destroying its temporal structure (see Figure 2). A time-frequency representation shows a completely dispersed spectral content (see Figure 3).

The concept of surrogates originates in the field of DS, being a tool for testing the nonlinearity [12] and the chaoticity [13].

As acknowledged, the surrogates are stationary signals, having the same spectrum amplitude as the original signal, but their spectrum phase is noise. The original signal is then compared against its surrogates. To compare, several tools may be used: the Kullback-Leiber and the log-spectral divergences [11], neural networks, SVM etc.

The results of this procedure are shown in ... (for amplitude-modulated noise and for a frequency sinusoidal modulated deterministic signal) on three scales: macro (upper), medium (middle) and micro (lower). The first and the third are identified as stationary, wile the middle one is not.

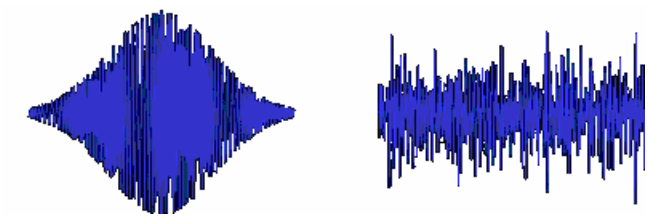

Figure 2: A signal with sinusoidal frequency modulation (left) and one of its substitutes

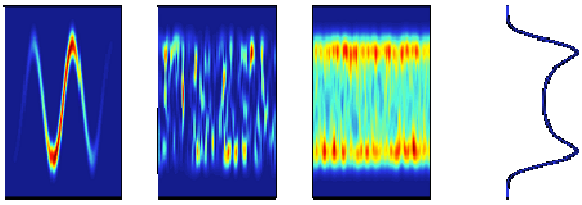

Figure 3: Spectrograms of the original signal, a surrogate, and the mean over multiple surrogates and the spectrum

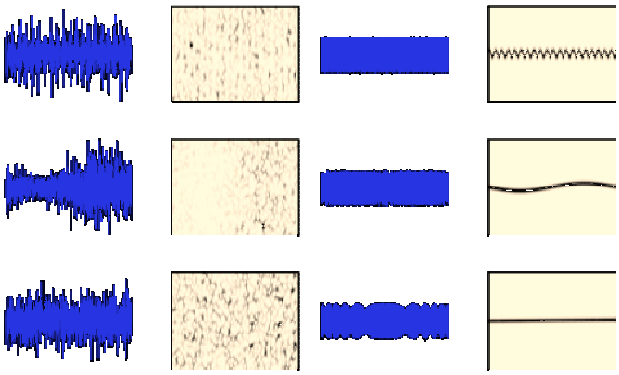

Figure 4: Samples and spectrograms of amplitude modulated noise (left) and frequency sinusoidal modulated signal

3. APPLYING SIP KNOWLEDGE TO DS

Most of the usual signal processing techniques quickly find their application in the DS domain, as they are valuable information extraction and characterization tools. At te very basic level, there are the purely mathematic ones, such as ARMA modeling, PCA, ICA etc.

On a higher level, more specific techniques are used, such as the spectral and spectrogram representation methods. An illustrative application is presented in [14], showing the analysis of a Duffing oscillator's impulse response. The author prefers these tools instead of the more traditional "bone curves" because they clearly give the information concerning the evolution of the system in time.

A remarkable contribution of time-frequency (and, generally speaking, signal processing) methods is in distinguishing between a chaotic and a random system. In fact, this remains an open issue for the scientific community and there is not any absolute reliable criterion.

A possible use of time-frequency representation has been presented recently by [15], exploiting one fundamental characteristic of deterministic chaotic systems: the approximate (but not identical) recurrence of their behavior and sequence of states.

While the recurrences and their occurrences remain unpredictable, it is possible to average their time-frequency representations, thus filtering out the measurement noise and the intrinsic instability of the system, while enhancing the common, deterministic part.

The first step is to identify these recurrences. A method specific to the DS, the *time delay embedding* [15] is used for this purpose. Its central idea is that the evolution of all parameters of a dynamical system may be approximated by a temporal embodiment of just only one known parameter. This translates in constructing the following vectors:

$$\mathbf{z}_n = \left[z_n, z_{n+\tau}, z_{n+2\tau}, \ldots, z_{n+(d-1)\tau} \right]^T \quad (4)$$

where d is the embedding dimension and τ is the delay value, and $z_0, z_1, \ldots$ are the values of the known parameter.

The neighborhood of radius ε of $\mathbf{z}_n$ is defined as:

$$\mathbf{N}_n =^{def} \left\{ \mathbf{z}_k : \left\| \mathbf{z}_k - \mathbf{z}_n \right\| < \varepsilon, 1 \le k \le L-(d-1)\tau \right\} \quad (5)$$

Figure 5 shows an example for the state x of the chaotic Lorenz attractor of parameters $(10, 28, 8/3)$. The black points are the samples between the extreme elements of vector (4), which starts, each time, in the reference point or in one of its neighbours.

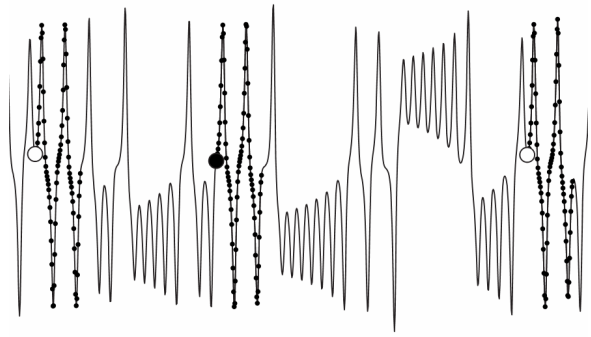

Figure 5: The reference point (black disk) and two of its neighbours (white disks), along with the associated embedded vectors (black points)

These samples (marked with black points) form the vectors $\mathbf{s}_n =^{def} \left[z_n, z_{n+1}, z_{n+2}, \ldots, z_{n+(d-1)\tau} \right]^T$.

Intuitively, the Figure 5 shows that the waveforms $\mathbf{s}_n$ occur irregularly, but they are somehow similar. For each neighborhood $\mathbf{N}_n$, the matrix $\mathbf{D}_n = \left[\mathbf{x}_{k_1}, \mathbf{x}_{k_2}, \ldots, \mathbf{x}_{k_N} \right]$ where $\mathbf{x}_{k_i} = \mathbf{s}_{k_i} - \overline{\mathbf{s}_n}$ is defined. Then, the covariance matrix is computed: $\mathbf{C}_n = N^{-1} \mathbf{D}_n \mathbf{D}_n^T$ whose eigenvalues are λ_i and corresponding eigenvectors are $\mathbf{u}_i$. The Neighbourhood-based Spectral Estimator is then computed as:

$$P_{NSE}(\omega) = \frac{1}{L_w} \sum_{i=1}^{L_w} \lambda_i \, | V_i(\omega) |^2 \qquad (6)$$

where $V_i(\omega)$ is the Fourier transform of $\mathbf{u}_i$.

When the reference point is varied in time, successive representation of spectra (6) forms the NSE time-frequency representation. An example is shown in Figure 6 (left). The black points show the highest-amplitude frequency (the spectral mode) at a given time bin. Compared with the same representation, but of a random process (right), the one corresponding to the chaotic system exhibits short, but continuous, albeit irregular, lines of local maxima.

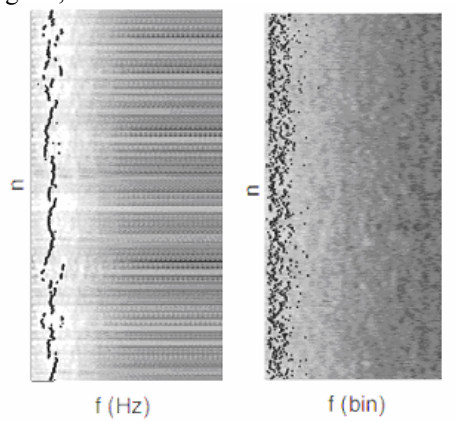

Figure 6: NSE time-frequency representations for a chaotic (left) and random (right) process, along with the mode frequencies

These characteristics may help in distinguishing between a chaotic and a random process, respectively.

4. CONCLUSIONS AND PERSPECTIVES

The techniques and the concepts exposed in the paper have only an illustrative purpose. One could easily imagine that the possible applications are almost countless and that the perspectives are widely open.

The interactions between DS and SiP domains are strong and are becoming stronger. While is not realistic to imagine the future of this research axis, some major arising opportunities are summarized below:

1. Greatly extend the range of studied systems, by including meteorological phenomena, sea clutter, chemical reactions etc.;
2. Making use of the expertise already available in DS when it comes to complex systems and their behavior, and particularly the focus on nonlinearity;
3. Taking advantage of the power of DS as a modeling tool: while simple systems ma exhibit astonishing complex and unpredictable behavior and, reversely, quite complex systems may find simple-to-interpret DS models;
4. The possibility to apply deterministic, although chaotic models, to phenomena traditionally considered as random.

5. ACKNOWLEDGMENTS

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