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CORDON PRICING IN THE MONOCENTRIC CITY  
MODEL: THEORY AND APPLICATION TO  
ILE-DE-FRANCE

André de Palma  
Moez Kilani  
Michel De Lara  
Serge Piperno

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DEPARTEMENT D'ECONOMIE

Route de Saclay

91128 PALAISEAU CEDEX

(33) 1 69333033

<http://www.enseignement.polytechnique.fr/economie/>  
<mailto:chantal.poujouly@polytechnique.edu>

# Cordon pricing in the monocentric city model: theory and application to Île-de-France

André de Palma<sup>\*†</sup>      Moez Kilani<sup>‡</sup>      Michel De Lara<sup>‡</sup>  
Serge Piperno<sup>‡</sup>

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## Abstract

We propose a method to compute an equilibrium solution for the monocentric city model with traffic congestion, and to quantify the impact of cordon tolls on social surplus. The focus of this paper is on the comparison of road pricing of one and two cordons, with the no toll and first-best situations as benchmarks. We find that a one-cordon toll yields a social efficiency of 63% with respect to first-best, and that an optimal two-cordon toll increases the efficiency to 73%. Both policies have a positive impact on CO<sub>2</sub> emissions because they reduce the average length of trips and reduce the road size.

**JEL code:** R21; R41; R48

**Keywords:** Monocentric model; Cordon toll; Acceptability of road pricing

## 1 Introduction

From a transportation economics perspective, it is well established that congestion pricing is necessary to adequately allocate the usage of transportation infrastructure as an economic good. From a broader perspective of urban economics, congestion pricing is required to correctly allocate land between housing and transportation. Pricing removes market distortions introduced by transportation externalities among users and induces an improvement in the social welfare.

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<sup>\*</sup>ENS Cachan and École Polytechnique, 61 avenue du Président Wilson, Cachan 94230, France.

<sup>†</sup>Corresponding author. Phone: +33 1 47 40 55 75. Fax: +33 1 47 40 24 60. Email: andre.depalma@ens-cachan.fr.

<sup>‡</sup>Université Paris-Est, CERMICS, École des Ponts ParisTech, 6-8 avenue Blaise Pascal, 77455 Marne-la-Vallée Cedex 2, France. Emails : M. Kilani (kilanim@cermics.enpc.fr); M. De Lara (delara@cermics.enpc.fr); S. Piperno (piperno@cermics.enpc.fr)

In the presence of a cordon toll, a driver pays a toll when he crosses the cordon.<sup>1</sup> When demand is inelastic (and fixed to home-to-work trips) the toll paid by a household depends on where she/he lives and where she/he works. In the monocentric city model, where all economic activities are located in the city center, there are two types of households: Those who live outside the cordon and pay the toll, and those who live inside the cordon and do not pay the toll. Households outside the cordon typically enjoy larger housing area, due to smaller competition on land. With two cordons there are three groups of households: Those who live inside the two cordons and do not pay any toll; those who live between the two cordons and pay a moderate toll; and those who live outside the two cordons and pay a high toll. As we move away from the city center, the households incur a larger generalized travel cost but occupy a larger area.

In recent years, an important literature on cordon tolls has emerged. Maruyama & Sumalee (2007) discuss equity issues and compare cordon and area-based tolls. Cordon tolls are found to be better in terms of equity. De Palma & Lindsey (2006) conduct a case study on Paris on the basis of a dynamic simulation (Metropolis model). A discussion of cordon toll in Edinburgh with smaller value than the socially optimal one is evaluated in Laird et al. (2007) on the basis of the MARS model.<sup>2</sup> Santos (2002) measures the impact of cordon toll on travel cost for a set of cities in UK. The author compares single and double cordon schemes and finds that the latter induces an appreciable gain by comparison to the former. The scope of these studies remains limited to short-term effects. Indeed, they do not take into account the impact of road pricing on the urban form (the origin/destination pair remains unchanged, for all travellers, before and after pricing is implemented).

Cordon tolls have been discussed under the framework of the monocentric city model in Mun et al. (2003) and Verhoef (2005), who found that cordon tolls reach a relatively high efficiency level. Both papers, however, consider a restrictive versions of the monocentric city model. In particular, the trade-off between land devoted to housing and land devoted to transportation has not been considered (the land allocated to transport is fixed). Safirova et al. (2005) uses a numerical model to evaluate the impact of cordon tolls for California. This paper considers this trade-off and compares first-best and cordon tolls. A simple area-based toll has been implemented in London<sup>3</sup> and related reports (cf. Santos & Fraser 2006, Leape 2006) show that it yields benefits, both on the level of congestion and on the level of emissions, even if the environmental objective has not been mentioned at the implementation stage of the project. Extensions of this experience are being under consideration and the need to better understand the positive and the negative impacts of multiple cordon is urgent (cf. Transport for London 2006). This is precisely the scope of this paper.

We extend De Lara et al. (2008), henceforth DDKP, by adding a two-cordon regime and compare with the no-toll, one cordon and first-best regimes. In

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<sup>1</sup>By contrast, with a zone toll drivers pay the toll as they drive inside the zone.

<sup>2</sup>This issue has been motivated by a referendum in which the cordon toll project for the city of Edinburgh has been rejected.

<sup>3</sup>The so-called "London congestion charge".

DDKP, the linear toll has been characterized by a high level of efficiency (93%) with respect to the first-best toll. A linear toll is proportional to travel distance, and can be implemented as gasoline tax. But, to reach an acceptable level of efficiency, an excessive level of toll is required (about five times higher than the actual vehicle operating cost). Under the present situation it seems difficult to convince road users by an additional (and systematic) increase in energy cost. Decision makers do not seem to consider this issue seriously. Cordon tolls have the advantage to depend on the trip characteristics, namely the origin/destination pair. In this sense, they are proposed as tools that control at the same time fuel consumption and congestion. Positive echos from the experience of London may contribute to facilitate acceptability of cordon tolls among other European capital cities.

We wish to provide answers to the following question: What is the benefit of a two-cordon toll by comparison to a one-cordon toll? Indeed, an optimal urban form implies smaller city and higher concentration of households around the Central Business District (CBD) (cf. Kanemoto 1977, Pines & Sadka 1985, De Lara et al. 2008). With one cordon we reach an efficiency level of 63%, and intuition suggests that with two cordons it is easier to address both objectives (reduce urban sprawl and increase households' density near the city center) more accurately. Theoretically, first-best toll may be (arbitrarily close) approximated by a sequence of cordon tolls covering the urban area. In practice, decision makers are generally interested in simple schemes involving one or two-cordon tolls.

To implement the two-cordon toll, we first extend the solution approach proposed in DDKP and search for the optimal locations and values of the tolls. Let us call "first cordon" the outer cordon and "second cordon" the inner cordon.<sup>4</sup> We find that in Île-de-France (IDF) it is optimal to set the first and second cordons at distances of 31km and 14km from the city center, respectively. Optimal toll levels are 16€ at the first toll and 13€ at the second cordon. With this pricing rule, we reach an efficiency of 73%. So, the marginal benefit of the second cordon with respect to the first cordon is 10%. Concerning CO<sub>2</sub> emissions, which is assumed here to be proportional to the travel distance, it involves a decrease of only 3%. For the two cordon scheme, the city becomes more dense and its radius decreases by 30% by comparison to the no toll (which is the reference situation).

Our solution approach replaces the optimality condition of the monocentric city model by a set of two backward differential equations. A standard numerical approach is then used to efficiently compute the solution. At the empirical stage, we calibrate the model so that its output matches observations from IDF.

The paper is organized as follows. The next section recalls the monocentric city model and the solution approach we propose (may be skipped in a first reading). Section 3 describes the calibration undertaken on Île-de-France and describes the no toll equilibrium (corresponding to actual situation). In Section

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<sup>4</sup>In the text we make usage of "second toll" to refer to a comparison with the case of one cordon toll.

4 we discuss the impact of pricing and focus on the two-cordon scheme. We conclude in Section 5.

## 2 Formulation and solution procedure

The analysis is carried out under the classical framework of a monocentric city. We adopt the formulation of Fujita (1989) and denote the model by  $HS_T$ .<sup>5</sup> All households are identical and spend their revenue on housing and a composite good. Their utility level is increasing in the quantities of both goods. The transport sector claims for land to allocate to roads. Congestion externalities are taken into account. The radius of the city, the distribution of households on the residential area and the amount of land devoted to roads are determined at equilibrium. Without a congestion toll, the city is too large and there are too many roads. There exists an optimal toll that removes market distortions and yields a first-best optimum. An efficient equilibrium can be reached with a toll that endogenizes external costs.

The number of households living in the city is fixed and equal to  $\bar{N}$  (closed city). The variable  $r$  denotes the distance from the center of the city. Each household makes daily trips from his location, at distance  $r$  from the center of the city, to the Central Business District (CBD) that extends to distance  $r_c$  from the center of the city. Inside the CBD, we assume that transportation is costless. The radius of the city is denoted by  $r_f$ .  $N(r)$  is the number of households located further than distance  $r$  from the city center.  $L(r)$  is the amount of land available for housing or transportation at  $r$ .  $L_T(r)$  is the amount of land allocated for transportation at  $r$ . Each household consumes two goods, housing  $s$  and a composite good  $z$ , and gets a utility  $U(z, s)$  where  $\partial U(z, s)/\partial z > 0$  and  $\partial U(z, s)/\partial s > 0$ . All households have the same utility function and the same (pretax) revenue  $Y$ . The price of the composite good is normalized to 1 and the unitary price of land, or land rent, at distance  $r$  from the city center is  $R(r)$ . The opportunity cost of land, or the agricultural rent, is denoted by  $R_A$ .

The amount of composite good necessary to achieve utility level  $u$  when housing area is  $s$  is  $Z(s, u)$  and corresponds to the solution of  $(z, s) = u$  in  $z$ . Let  $I$  denote the revenue net of taxes. The household *bid rent function*  $\psi(I, u)$  is given by

$$\psi(I, u) := \max_{s \geq 0} \frac{I - Z(s, u)}{s}, \quad (1)$$

where the maximum is reached at the *bid-max lot size*  $S(I, u)$

$$S(I, u) := \arg \max_{s \geq 0} \frac{I - Z(s, u)}{s}. \quad (2)$$

The quantity  $\phi(R, u)$  is the aftertax revenue required by a household having utility level  $u$  and willing to pay a land rent  $R$ . The government is responsible for

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<sup>5</sup>Fujita (1989) refers to the model as the Herbert-Stevens model with traffic congestion.

providing transportation infrastructure,  $L_T(r)$ , and has the possibility to levy two kinds of taxes: a population tax that does not depend on  $r$  and is denoted by  $g$ , and a location (or congestion) tax that depends on  $r$  and is denoted by  $l(r)$ .

The *road occupancy* at  $r$  is defined by the ratio of the number  $N(r)$  of households located further away than  $r$  from the city center to the amount  $L_T(r)$  of land devoted to transport use at  $r$ . At each distance  $r$ , the transport cost depends on the road occupancy at  $r$ :  $c(N(r)/L_T(r))$ , where the function  $c$  is assumed to satisfy  $c(w) > 0$ ,  $c'(w) > 0$  and  $c''(w) > 0$  for all  $w \geq 0$ . The transport cost from distance  $r$  to the CBD is

$$\tau(r) = \int_{r_c}^r c\left(\frac{N(r)}{L_T(r)}\right) dx. \quad (3)$$

Define the *bid rent*  $\psi_T$  of the transport sector at each distance  $r$  as the marginal benefit of land for transportation at  $r$ :

$$\psi_T\left(\frac{N(r)}{L_T(r)}\right) = -\frac{\partial c(N(r)/L_T(r))}{\partial L_T(r)} N(r). \quad (4)$$

The bid rent of the transport sector  $\psi_T(N(r)/L_T(r))$  represents the cumulated gain for the  $N(r)$  commuters from a unit increase of roads at  $r$ .

Since all households are identical, it is convenient to assume that they all reach the same utility level at an optimal solution. The objective of the central planner is to maximize the total surplus in the city. Let  $n(r)$  denotes the number of households in an annulus of unit width at  $r$ . The objective function to be maximized over (nonnegative) variables  $n(r)$ ,  $s(r)$ ,  $L_T(r)$  and  $r_f$  is the following total surplus  $\mathcal{S}$ :

$$\mathcal{S} = \int_{r_c}^{r_f} \{[Y - \tau(r) - Z(s(r), u) - R_A s(r)]n(r) - R_A L_T(r)\} dr. \quad (5)$$

Any distribution  $n(r)$  of households should satisfy the following constraints. First, the total amount of land devoted to housing and transportation must be lower or equal than the amount of land available:

$$n(r)s(r) + L_T(r) \leq L(r) \quad \text{for } r_c \leq r \leq r_f. \quad (6)$$

Second, the distribution of households satisfies:

$$N(r) = \int_r^{r_f} n(r) dr \quad \text{for } r_c \leq r \leq r_f. \quad (7)$$

Finally, all households locate inside the city:

$$\bar{N} = N(r_c) = \int_{r_c}^{r_f} n(r) dr. \quad (8)$$

The bid-rent function  $\psi(I, u)$  is continuously increasing in  $I$ , so we can define  $\phi(R, u)$  by

$$\phi(R, u) := I \Leftrightarrow \psi(I, u) = R. \quad (9)$$

Each household chooses  $r$ ,  $z$  and  $s$  in order to maximize utility  $U(z, s)$  under revenue constraint  $z + R(r)s = Y - g - l(r) - \tau(r)$ , where  $\tau(r)$  denotes the transport cost incurred at  $r$ . If we replace  $I$  in (1) by<sup>6</sup>  $Y - g - l(r) - \tau(r)$ , we obtain the household bid rent at distance  $r$

$$\psi(Y - g - l(r) - \tau(r), u) = \max_s \frac{Y - g - l(r) - \tau(r) - Z(s, u)}{s}, \quad (10)$$

and the corresponding bid-max lot size  $S(Y - g - l(r) - \tau(r), u)$ .

In order to compare the optimal pricing rule with alternative policies, we introduce multiple pricing rules using the function (see DDKP for the details)

$$H(r) = \begin{cases} c'(\Psi_T^{-1}(R(r)))\Psi_T^{-1}(R(r)) & \text{(first-best)} \\ \bar{\kappa} & \text{(no toll)} \\ \xi_{r_1} \delta_{\{r_1\}}(r) + \xi_{r_2} \delta_{\{r_2\}}(r) & \text{(cordon toll),} \end{cases} \quad (11)$$

where  $\bar{\kappa}$  and  $\xi_{r_1}$  (and  $\xi_{r_2}$ ) are positive constants and  $\delta_{\{r_d\}}(r)$  is a function that takes value 1 at distance  $r_d$  from the city center and zero elsewhere. Under the no-toll regime users pay only the vehicle operating cost for the trip. Below, we use the value  $\bar{\kappa} = 0.0414/m/year$  which is suggested in DDKP. The third pricing rule in (11) reflects one and two-cordon pricing schemes. When there are two cordons, both  $r_1$  and  $r_2$  in (11) are positive and reflect the locations of the respective cordons. With one cordon,  $r_1$  is fixed to the corresponding location of the cordon and  $r_2 = 0$ .

Let  $u > 0$  be a fixed utility level. The solution of the problem that consists in maximizing (5) subject to constraints (6), (7) and (8) can be computed in the following way (see DDKP). Solve, for all positive  $r_f$  and for  $r_c \leq r \leq r_f$ , the system of backward differential equations :

$$\begin{cases} R'(r) = -\frac{H(r) + c(\Psi_T^{-1}(R(r)))}{\frac{\partial \phi}{\partial R}(R(r), u)} \\ N'(r) = \frac{\frac{N(r)}{\Psi_T^{-1}(R(r))} - L(r)}{S(\phi(R(r), u), u)}, \end{cases} \quad (12)$$

with terminal conditions  $R(r_f) = R_A$  and  $N(r_f) = 0$ . Then, find  $r_f$  such that  $N(r_c) = \bar{N}$ . From these, we compute  $L_T(r) = N(r)/\Psi_T^{-1}(R(r))$ ,  $s(r) = S(\phi(R(r), u))$ , and  $l(r) = \int_{r_c}^r c'(\Psi_T^{-1}(R(r'))) \Psi_T^{-1}(R(r')) dr'$  for  $r_c \leq r \leq r_f$ .

### 3 Calibration

In this section we calibrate the model parameters to match some target variables related to IDF region. The monocentric city model may be criticized as unrealistic. Indeed, many metropolitan regions have a polycentric structure. This

<sup>6</sup>Indeed,  $Y - g - l(r) - \tau(r)$  is the part of the income that remains for the consumption of housing ( $s$ ) and the homogeneous good ( $z$ ).



has led some authors to suggest that the main effort should focus on polycentric models instead (cf. Mieszkowski & Mills 1993, for example). The monocentric framework, however, remains very useful for three reasons, at least. First, for the case of IDF, as we discuss below, there is a high concentration of (non-industrial) activities in the CBD located inside Paris. Second, the monocentric city model is useful when we consider only a part of the economic activity and the related transportation. In particular, in IDF, most economic activities with highly skilled employees are concentrated in the CBD. Third, given that the theory underlying the monocentric city model is much more coherent and complete (many theoretical insights have been already gained), the empirical exercise can be evaluated much more accurately than if polycentric models were used. We do not intend to say that the monocentric city model is superior to polycentric models, but we argue that there are many lessons we can draw from it if we remain aware of its limitations.

Moreover, empirical observations still confirm the high concentration of economic activities in small areas. For the case of IDF, a recent report by Pottier et al. (2007) states that more than three million households (among a total of five million) are working in the twenty districts inside Paris. The ratio is even higher for highly skilled employees, who generally use private cars relatively frequently. Moreover, maps from AIRPARIF show a high concentration of emissions in the CBD and the region around. Road transportation account for 27% of (greenhouse gas) emissions. On the basis of these observations, we think that many attributes of IDF can be explored within the monocentric framework. In the remaining of this section we fit the above model with data from IDF.

### Land available

We assume

$$L(r) = \mu(r) \times 2\pi r, \quad (13)$$

where  $\mu(r)$  is the fraction of land devoted to housing and transportation at  $r$ . Data from IDF show that the ratio of land used for housing and transportation by the total land available decreases as we move away from the CBD. Furthermore, collective houses are more concentrated near the CBD and individual houses spread away from the city center. Collective houses are generally built on more than four levels, while individual houses are built on one or two levels.

We take into account these facts and approximate  $\mu(r)$  by an exponential expression, which yields

$$\mu(r) = 3.191 e^{-8.7 \times 10^{-5} r} \quad (R^2 = 0.99). \quad (14)$$

As we move away from the CBD the fraction of land available for housing and transportation decreases substantially.

## Travel speed

There are two options at least on how to compute free-flow travel speed:  $v_0$ . First, one may consider that it is constant over all the region. In this case it can be computed as the (harmonic) mean of the maximum allowed speeds over the network of three kinds of roads. On the basis of the network of IDF we get a value of about 55 km/h.

A better approach is to consider that the free-flow travel speed decreases as we get closer to the CBD. This is because a driver inside Paris uses mainly (slow) local roads, but can drive on faster roads in outer regions. To take into account the fact that the free-flow travel speed increases as we move away from the CBD, we approximate it as follows. At the city border a traveller mainly uses highways where the speed limit is 110 km/h. A household will be likely to use highways less as we get closer to the CBD. We assume<sup>7</sup> that to travel from the city center to the CBD, on average, 80% of the trip is made on highways, and 20% on main roads. A trip that starts closer to the CBD uses less highways but the same fraction of main roads. Instead, urban area roads (with speed limit of 50 km/h) substitute for highways. Denoting by  $w_h$  and  $w_n$  the respective fractions of usage of highways and main roads, the average speed is the harmonic mean

$$\left( \frac{w_h}{110} + \frac{w_n}{70} + \frac{1 - w_h - w_n}{50} \right) = \frac{1}{v_0},$$

or  $v_0 = 3\,850 / (77 - 42w_h - 22w_n)$ . As we have mentioned above  $w_n$  is fixed at 20%. Assuming a linear form of  $w_h$  and taking into account that  $w_h = .8$  at  $r_f$  and  $w_h = 0$  at  $r_c$ , we end up with the following relation between the free-flow travel speed and the distance to the city center:

$$v_0 = \frac{51\,931}{1 - 5.92 \times 10^{-6}r}.$$

So, the free-flow travel speed decreases from about 90km/h at distance 70km (entrance of the city) from the city center to 52km/h at distance 10km (where the maximum speed generally becomes small). This is more realistic and leads to better calibration than the fixed  $v_0$ .

## Households

We consider a population of drivers going to and from the city center 230 days a year,<sup>8</sup> and estimate costs over one year. The number of households used is adjusted so that it corresponds to the number of vehicles used for home-to-work trips. Since we consider a CBD of radius 3.5 km, and since we consider only households that make trips to the CBD, we remove half of the population located in the ring that extends from 0 to 7 kilometers. Accordingly, we consider a total population of  $\bar{N} = 2\,120\,493$  households.

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<sup>7</sup>Based on the authors judgement from a Google-Earth exploration.

<sup>8</sup>This is approximately: 5 days x 52 weeks - 30 days (holidays).

### Utility function

Households preferences is represented by a Cobb-Douglas utility function

$$U(z, s) = z^\alpha s^\beta, \quad (15)$$

with  $\alpha, \beta > 0$ . From the Cobb-Douglas utility functions properties, we know that the ratio  $\beta/\alpha$  is equal to the share of the available revenue spent on housing with respect to the share spent on the homogeneous good. Robson (1976) assumed a value of 50% and Kanemoto (1977) has reduced the approximation to what seems to be a more realistic 20%. In the base-case, we consider the second value which matches recent estimation by INSEE.<sup>9</sup> Thus, we have  $\alpha = 4\beta$ , so that

$$U(z, s) = (z^4 s)^\beta. \quad (16)$$

An alternative value of  $\beta$  is considered for the sake of comparison.

### Congestion term

We consider a BPR congestion function of the form

$$c(\gamma(r)) = \frac{\theta}{v_0}(1 + k\gamma(r)^\lambda), \quad (17)$$

where  $k$  and  $\lambda$  are given constants,  $v_0$  the free-flow travel speed,  $\theta$  the households' valuation of time and  $\gamma(r) = N(r)/L_T(r)$ . The value of time in IDF was estimated in 2001 to 11.6€/h for home-to-work trips (Source: Boiteux (2001)).<sup>10</sup> To take into account the increase since 2001, we take the value of 15€/h (which corresponds to a five year growth rate at 5%). So, during a year with 230 working days and an average of two trips per day, we have  $\theta = 15 \times 230 \times 2$  ( $\text{€}h^{-1}\text{year}^{-1}$ ). To obtain a convex congestion cost function in (17), we selected  $\lambda > 1$  and positive  $k$ . Both parameters are used in the calibration of the model.

### Tolling schemes

We consider four policies:<sup>11</sup>

1. no toll (NT), where  $\bar{\kappa} = 0.0414$  in (11) reflects the vehicle operating cost;
2. one-cordon toll (CT1), where a driver pays a toll when he enters inside the cordon region;

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<sup>9</sup>See INSEE (2003).

<sup>10</sup>For the sake of comparison, the average value of time for work trips reported in Small & Verhoef (2007), Chapter 3, is \$9.14/h for metropolitan areas in the US in 2003.

<sup>11</sup>In some sense, these policies discriminate between households on the basis of their location. This remains, however, different from the usual price discrimination discussed in Anderson & Renault (2005) and Pigou (1932).

3. two-cordon toll (CT2), where a driver pays a toll each time he enters one of the two cordon regions;
4. a first-best toll (FB) that internalizes the external costs.

NT may be interpreted as a small tax or, better, the *vehicle operating cost* per kilometer. On the basis of gasoline price of 1.5€ per liter, the gasoline cost per meter for an average vehicle that consumes 6 liters per 100 kilometer is 0.0207€ per meter per year. Assuming that gasoline price is half the vehicle operating cost we use  $\kappa = 0.0414$  for the NT policy.<sup>12</sup> For the cordon tolls, both the location and the value are chosen to maximize the surplus given (5). In practice, the optimization process (in particular, with CT2) is a tedious but a straightforward task. Pricing rule NT is the reference policy, since it is closer to the real situation.

### Calibration procedure

A dataset related to rings with 7km intervals is used to feed the model with data. To replicate the urban structure of IDF, we construct a loss function (denoted “*Loss*”) that depends on the four parameters  $u$ ,  $\beta$ ,  $k$  and  $\lambda$ . The loss function is equal to the weighted sum of square errors between observed data and the output of the model. We focus on the radius of the city ( $r_f$ ), the distribution of the households (pop), the travel time (tt) and the level of the urban rent (rent). The expression of the loss function is

$$\begin{aligned}
 Loss(u, \beta, k, \lambda) = & \\
 & \sum_{r \in \{7, 14, \dots, 70\}} \left\{ w_{r_f} \left( \frac{M_{r_f}^r - r_f}{r_f} \right)^2 + w_{\text{rent}} \left( \frac{M_{\text{rent}}^r - R(r)}{R(r)} \right)^2 + \right. \\
 & \left. w_{\text{tt}} \left( \frac{M_{\text{tt}}^r - \text{tt}(r)}{\text{tt}(r)} \right)^2 + w_{\text{pop}} \left( \frac{M_{\text{pop}}^r - \text{pop}(r)}{\text{pop}(r)} \right)^2 \right\}, \quad (18)
 \end{aligned}$$

where  $w_x$  denotes the weight of variable  $x$ ,  $M_x^r$  denotes the value of  $x$  predicted by the model at  $r$  ( $r$  measured in km). The four variables are not measured in the same way : “rent” is the average rent between  $r$  and  $r - \Delta r$  (we have used  $\Delta r = 7\text{km}$ ), “tt” is the average travel time for households between  $r$  and  $r - \Delta r$ , “pop” is the number of households between  $r$  and  $r - \Delta r$ . The weights are set equal (and normalized to one) by default. They may be changed to focus the calibration on a given set of variables. By construction, the function  $Loss(u, \beta, k, \lambda)$  reaches a unique minimum when observed values match the output of the model.

The model is calibrated with respect to policy NT, i.e. when each driver pays a tax that reflects the vehicle operating cost. The output of the model with parameter values  $u = 11\,976$ ,  $\beta = 0.2$ ,  $\lambda = 4.02$  and  $k = 6.6 \times 10^{-12}$  fits particularly well the distribution of households and travel time. Figure 1 shows

<sup>12</sup>Based on authors’ judgement and data in INSEE (2005)

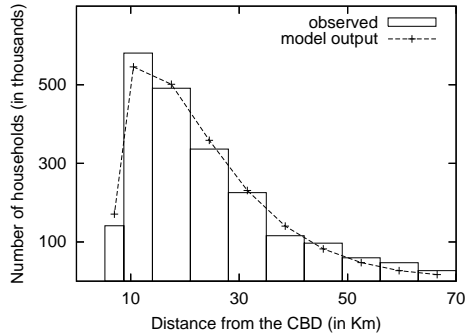


Figure 1: Distribution of households: observed and predicted ( $R^2 = 0.987$ ).

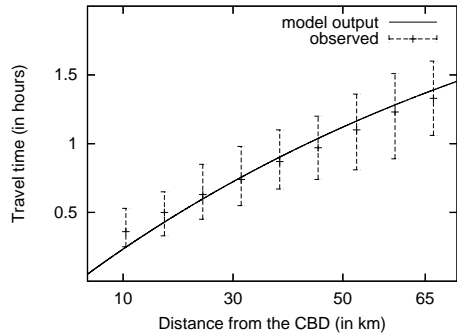


Figure 2: Travel time: observed and predicted ( $R^2 = 0.97$ ).

the observed distribution of households in IDF and the distribution produced by the model. The correlation is satisfactory. Figure 2 shows observed and predicted values for the travel time. The correlation between the two sets is high, even if the slope of the predicted values seems higher. The variable free-flow travel speed has been useful to refine the approximation of the travel time.

The only variable that does not seem to be well fitted by the model is the land rent. This fact may be explained intuitively as follows. Under the monocentric city framework, the market rent is an exclusive result of transport costs. The attractiveness of the CBD lies in the fact that we incur lower travel time. But, in reality the attractiveness of the CBD of Paris is the result of many other attributes: a richer social life, better access to many facilities, and so on. This is one of the limitations of the model used here.

## 4 Results

The no toll situation (NT), where road users face the vehicle operating cost only, is taken as the reference situation. We consider three alternative pricing rules: (1) one-cordon toll, (2) two-cordon toll, and (3) first-best toll. A cordon toll requires the specification of a location as well as a value of the toll. We use a grid search to find both values that maximize the social surplus. With two cordons, two locations and two values are required. Again, we perform a grid search to find those values that yield highest social surplus. The computation of the optimal values with four arguments is time consuming. The first-best toll endogenizes the external costs and yields the optimal form of the city.

Simulation output are summarized in Table 1. The first column of Table 1 provides location tax corresponding to  $H(r)$  in (11). The second column contains the radius of the city  $r_f$ . Column  $\bar{s}$  corresponds to the average area occupied by a household ( $\bar{s} = \int_{r_c}^{r_f} s(r) n(r) dr / \bar{N}$ ). The average (one-way) vehicle-kilometers is given in column  $VK$  and we have  $VK = \int_{r_c}^{r_f} r n(r) dr / \bar{N}$ .

	$tx$	$r_f$	$\bar{s}$	$VK$	$RD$	$TT$	$C_L$	$C_T$	$\Delta\mathcal{S}$
	Total tax	City radius	Hous. area	Trip len.	Roads' surf.	Trav. time	Land cost	Trsp. cost	Surp. varia.
	€/y	km	m <sup>2</sup>	km	10 <sup>4</sup> m <sup>2</sup>	mins	€/y	€/y	€/y
NT	769	73.42	84.24	22.08	7 539	37.7	5 750	3 469	0
CT1	2 410	55.63	84.89	19.63	6 482	33.9	5 542	3 130	181
CT2	3 541	50.85	83.64	19.13	6 237	33.2	5 426	3 072	210
FB	7 172	48.65	83.13	18.71	6 049	32.7	5 360	3 021	286

NT: no toll; CT1: one-cordon toll; CT2: two-cordon toll; FB: first-best toll

One cordon (22km, 22.5€/day)    Two cordons (31km, 16€/day):(14km, 13€/day)

Table 1: Impacts of road pricing under the four regimes.

Column  $RD$  indicates how much land is allocated to roads.  $TT$  denotes the average travel time for a trip. Column  $C_L$  reflects the opportunity cost of land per household per year. The generalized transport cost per year for and average household is given in column  $C_T$ . The last column, column  $\Delta\mathcal{S}$ , contains the average impact of pricing on the surplus per household per year. The social surplus is computed as the amount of money that is not spent by households (given they reach the target utility level  $u$ ). So, an increase in the surplus indicates that households remain at the same utility level but have larger part of their revenues unspent. An efficient city structure leads to an increase in the social surplus.

Under the FB regime, the radius of the city is reduced by 34% by comparison to the unpriced situation. The travel distance is reduced by 15%, while the average travel time drops from 38 to 33 mins (by 13%). So, congestion pricing has positive impact on the social welfare but also on the level of emissions. Assuming that CO<sub>2</sub> emissions are proportional to travel distance, an appreciable positive impact on pollution is indirectly obtained. FB toll leads to high increase of the land rent for the areas close to the CBD. As shown by Figure 4, the population becomes particularly dense around the CBD. Indeed, road pricing has two main impacts on the form of the city. First, it reduces urban sprawl and leaves more land available for alternative usage (agriculture). Second, it motivates households to locate near the CBD. The distribution of households is re-shaped with a high density near the CBD. Most pricing policies are effective in reducing the size of the city but fail to adequately motivate households to locate optimally. This may be a further argument favoring road pricing against urban boundary as a tool to combat urban sprawl (cf. Brueckner 2000, Pines & Sadka 1985).

With one-cordon the optimal location is at 21km from the city center and the optimal toll is about 22€ per household per day. An optimal one-cordon toll is 63% as efficient as the first-best toll, where efficiency is based on the increase in social surplus. The radius of the city is reduced to 57km and the average

travel distance decreases from 22.075 to 19.720km. The land rent remains flat, but the jump at the cordon location (cf. Figure 3) induces a higher rents inside cordon area. Table 1 shows that the gain in surplus comes from savings in the opportunity cost of land and in travel cost. Under the compact city, congestion decreases and the average travel for a trip drops from 38 to 34 mins (11%). From column RD in Table 1, the amount of land allocated to roads decreases by 13% by comparison to NT. Indeed, unpriced congestion leads to over-investment in roads and leaves less land for housing usage.

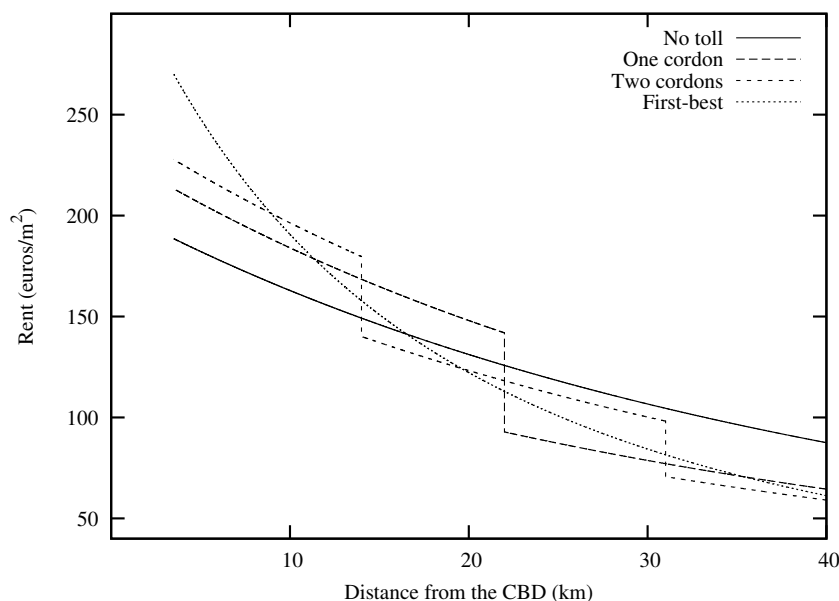


Figure 3: Land rent.

Intuitively, the two-cordon pricing would be more effective than one-cordon.<sup>13</sup> Indeed, the two locations may be chosen so as to act separately on the radius of the city (the first cordon) and on getting more households close to the CBD (the second cordon). Optimal locations are found to be at 31km and 14km from the city center, respectively. The toll levels are 16€ and 13€, respectively for the first and second cordons, which are located respectively at 31km and 14km. By comparison to first-best, two-cordon toll yields a 73% of the optimal surplus. So, the second cordon yields a marginal efficiency of 10%. The radius of the city decreases to 50km and the average travel distance becomes 19.128km. In this sense, the second toll decreases CO<sub>2</sub> emissions slightly (by 3%). The land rent remains flat by comparison with the curve obtained under first-best,

<sup>13</sup>In practice, the welfare impacts of each tolling policy will depend on how toll revenues are spent and/or redistributed. De Palma et al. (2007) discusses a number of issues in this sense.

but the upwards jumps increase the price levels particularly in the area inside the second cordon. As shown by Figure 4, there is a higher concentration of households around the CBD. The surface of land allocated to roads decreases by 17%, which is close to the 20% decrease reached under FB. As a consequence, even if the city gets smaller, the average housing area occupied by each household does not decrease too much under CT2 and FB. This area even increases under CT1. Indeed, the decrease in the radius of the city induces a relatively smaller decrease in the available land (from the expression in (13)). At the same time, roads decrease at all distances from the city center. Overall, the resulting variation in the housing area remains almost the same.

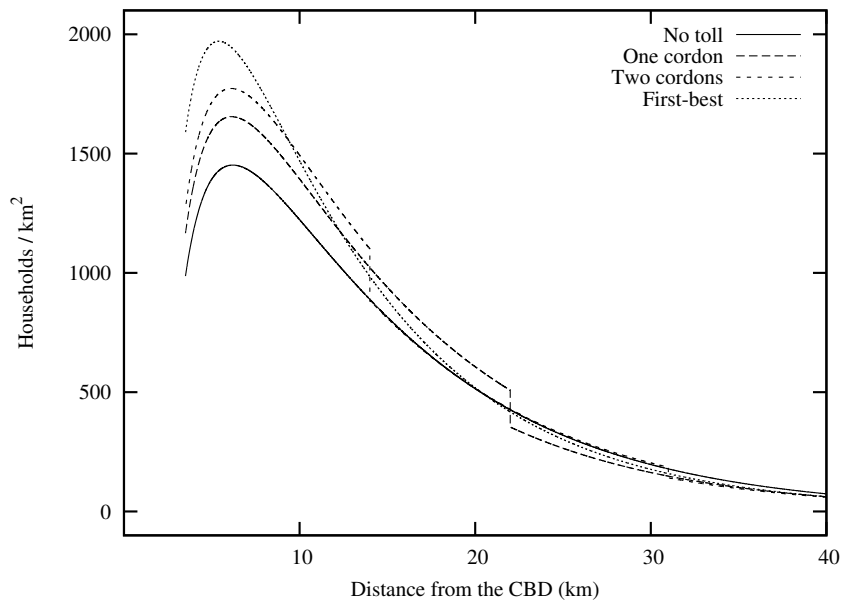


Figure 4: Distribution of the population.

In practice, the first-best toll has always been criticized as being complicated to devise. Alternative, simpler and more acceptable tolling rules have been proposed to replace first-best toll. Linear tolls (discussed in DDKP) may be implemented as a tax on gasoline. But, increasing further the gasoline price may face opposition from road users. Cordon tolls have the convenience to depend on the trip characteristics (origin and destination), and by so offer an interesting alternative to collect revenues, limit the level of emissions and congestion as well as yielding an acceptable gain in surplus.

The last point we discuss is the users' opposition to road pricing. In general, users perceive the toll as an additional tax and not as a regulation instrument. The introduction of the toll is likely to be rejected by those who are located outside the cordon region. With an inelastic demand, users inside the cordon region



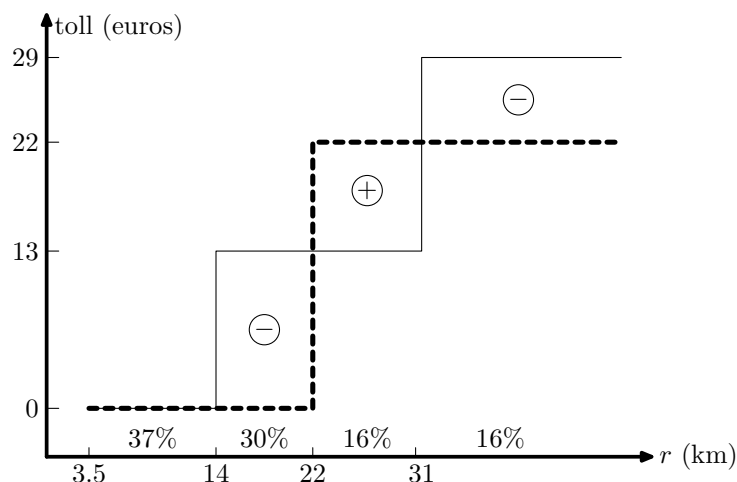


Figure 5: Voting for two cordons.

are indifferent since they do not pay the toll. Overall, there is an opposition towards the introduction of the toll.

Suppose now that the first cordon is already in place and the government wishes to introduce a second toll. Some users will be opposed but others will support the second cordon toll because they pay less under the new policy. Figure 5 shows the toll levels (with one and two cordons) as a function of the household's location. The continuous line corresponds to the toll level with two cordons and the discontinuous line corresponds to the toll level with one cordon. The numbers in percentage indicate the proportion of households living in the given location. The impact of the second toll, whether positive or negative, is indicated whenever it is not zero.

The second toll is rejected by households living between 14 and 22km and those located beyond 31km. So, 46% of the population is likely to vote against this second toll. At the same time households located between 22 and 31km will pay a lower toll and will support the introduction of the toll. In our case this second group contains only 16% and does not represent a majority. Households living inside the radius of 14km (37% of the population) are indifferent to the introduction of the cordon. Notice that the first cordon would have been rejected by 32% ( $=16\%+16\%$ ) of the population. The opposition to the second cordon, which is about 30% ( $=46\%-16\%$ ), is quite similar. These observations show that a second cordon has a smaller marginal benefit than the first cordon but is almost as unpopular. This fact may explain that most real experiences have been limited to one cordon toll.

## 5 Conclusion

This paper quantified the impact of a second cordon toll on the urban structure of Île-de-France. The intuition supporting this policy is to let a first cordon toll reduce the radius of the city and let a second concentrate households around the CBD. Indeed, these two features characterize the optimal toll which is relatively difficult to implement in reality (cf. Figure 4).

With one cordon toll the radius of the city is reduced by 23% by comparison to the no-toll situation. With the second toll we get a decrease of 30%. Social surplus (the objective function) increases from 63% to 73% with the second cordon. The impact of the second cordon is less important on congestion and travel distance, where a decrease of about 3% is obtained in both cases. A dense city is characterized by higher energy efficiency, lower levels of emissions and lower investment on roads. Given the difficulties to implement first-best tolls and the difficulty to further increase the gasoline price, cordon tolls turn out to be a useful tool to efficiently allocate land and reduce emissions for large metropolitan areas (like Île-de-France).

The algorithm presented to solve the  $HS_T$  model is flexible and extends to various pricing schemes, including cordon tolls. The computation of the equilibrium toll is typically a challenging task and a number of authors have used specific algorithms for this purpose. For example, Sumalee et al. (2005) uses a genetic algorithm optimization process to compute the optimal values of the toll. De Palma et al. (2005) and Zhang & Yang (2004) conduct an analysis based on simplified graph structures. Ho et al. (2005) use a particular formulation and compute optimal locations of multiple cordons. The numerical approach of finite element method used to approximate the solution is innovating in that it allows the treatment of two dimensional problem.<sup>14</sup> These papers, however, focus on the short-term impacts and do not consider the re-localization of households (on the long term) given the change in the travel cost introduced by road pricing. The focus of this paper is the long run impact.

As it is usually the case in the monocentric city model, we have assumed that the toll is collected in the social surplus as a lump sum. There is a number of interesting alternatives one may propose for a better usage of toll revenues (cf. De Palma et al. 2007). For example, it could be better to invest in the public transport. In this case, each household will choose either to make the daily trip using his car or using public transport, or by commuting between the two modes. The households decision, and by so the welfare impact, will depend on how the public infrastructure is introduced in the model. A second option, mainly concerned by CO<sub>2</sub> emissions, is to use toll revenues to provide incentives to vehicle producers and drivers to switch to cleaner technologies.

On a basis of a voting scheme, the implementation of a second toll is likely to face as much opposition as the implementation of the first cordon. From

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<sup>14</sup>In the standard monocentric city model, all quantities depend on the distance from the city center, so the problem is one dimensional. This assumes that the city (with the housing and transportation land) has a regular shape (generally, a circular form). When the urban shape of the city is not so symmetric the problem is no longer one dimensional.

another perspective, when acceptability is based on the value of the toll,<sup>15</sup> then the two-cordon scheme may be more attractive to most users than the one-cordon scheme. Indeed, with a two-cordon toll, most users (83%) pay only 13€ or less and only 16% pay high toll of 29€. With one cordon, 32% of the users pay 22€. Moreover, a parallel computations we have conducted show that the efficiency level of a one-cordon scheme can be reached by a two-cordon scheme, but with a 30% discount in the values of the two tolls. Under this latter policy, only 30% of drivers, those living between 14km and 22km from the city center, are worse off with the two-cordon by comparison to the one-cordon. The moderate level of the toll paid by most households may be an argument favoring the introduction of a second toll.

Finally, we'd like to highlight the parallel between the multi-cordon toll problem and multi-step tolls proposed as a simple alternative to the fine toll. They both amount to a finite approximation of an optimal toll, where the approximation is motivated by simple implementation issues. Arnott et al. (1990) discusses the case of a one fixed step and Laih (1994) extends the analysis towards two and three steps assuming a linear travel cost function. Laih concludes that an  $n$ -step toll is  $n/(n + 1)$  as efficient as the fine toll. So, from one to a two-step toll efficiency increases by 16.67% (from 50 to 66.67%). This variation is higher, but remains comparable, to the induced gain of 10% obtained by the second cordon toll in our analysis.

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<sup>15</sup>For example, the toll level of the London congestion charge is considered as too high by most road users (cf. Blow et al. 2003)

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