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# Investigation on time response to impact test of a 2D shape memory structure

M. COLLET, M. OUISSE, E. FOLTETE, C. LEXCELLENT

**Abstract:** A numerical implementation of the RL model for shape memory alloys (SMA) is presented in the paper, adapted to high strain rate loading. The objective is to predict the time response of a 2D SMA structure subjected to an impulse force (duration 50 ms) and following free vibration with a decreasing amplitude. The choice of the material mechanical properties has been done in order to have phases transformations during the oscillations. The damping due to these phase changes is clearly identified when the results are compared with a linear model without induced martensite. The thermomechanical constitutive relation of the SMA has been fulfilled to be able to take into account time reaction when the strain rate is very high. The full model has been implemented in a finite element code and tested on a 2D sample.

**Keywords:** SMA, impact response.

The SMA description which has been implemented is based on the *RL* model [1], built from thermodynamics of irreversible process. A Helmholtz free energy is defined for the two-phase reference volume element containing the volume fraction  $(1 - \xi)$  of austenite phase and  $\xi$  of martensite phase:

$$\phi(\underline{\varepsilon}, T, \xi) = u_0^{(A)} - T s_0^{(A)} - \xi \pi_0^f(T) + \frac{1}{2} (\underline{\varepsilon} - \underline{\varepsilon}^{tr}) : \underline{\underline{L}} : (\underline{\varepsilon} - \underline{\varepsilon}^{tr}) + \xi(1 - \xi) \phi_{it} \quad (1)$$

$u_0^{(A)}$  and  $s_0^{(A)}$  are the internal energy and entropy of austenite phase,  $T$  is the temperature,  $\pi_0^f(T)$  is the driving force of the stress free state,  $\underline{\varepsilon}$  is the strain tensor,  $\underline{\underline{L}}$  is the Hooke tensor,  $\phi_{it}$  is a coefficient of internal interaction, and  $\underline{\varepsilon}^{tr}$  is the phase transformation strain tensor:

$$\underline{\varepsilon}^{tr} = \frac{3}{2} \frac{dev(\underline{\sigma})}{\sigma_{vm}} \gamma \xi \quad (2)$$

$\sigma_{vm}$  is the Von Mises equivalent stress and  $dev(\underline{\sigma})$  the deviator stress tensor. The driving force of phase transformation is:

$$\pi^f(\underline{\sigma}, \xi, T) = \frac{\gamma \sigma_{vm}}{\rho} + \pi_0^f(T) - (1 - 2\xi)\phi_{it} \quad (3)$$

The Clausius Duhem inequality leads to the kinetic laws of the phase transformations:

$$\dot{\xi} = f(\sigma_{vm}, \pi_f, \xi) \quad (4)$$

The function  $f$  is zero except for austenite to martensite transformation ( $0 < \xi < 1$  and  $\dot{\sigma}_{vm} > 0$  and  $\pi_f(T) > 0$ ):

$$f(\sigma_{vm}, \pi_f, \xi) = \frac{\gamma \dot{\sigma}_{vm}}{\rho \left( \frac{A_1}{1-\xi} - 2\phi_{it} \right)} \quad (5)$$

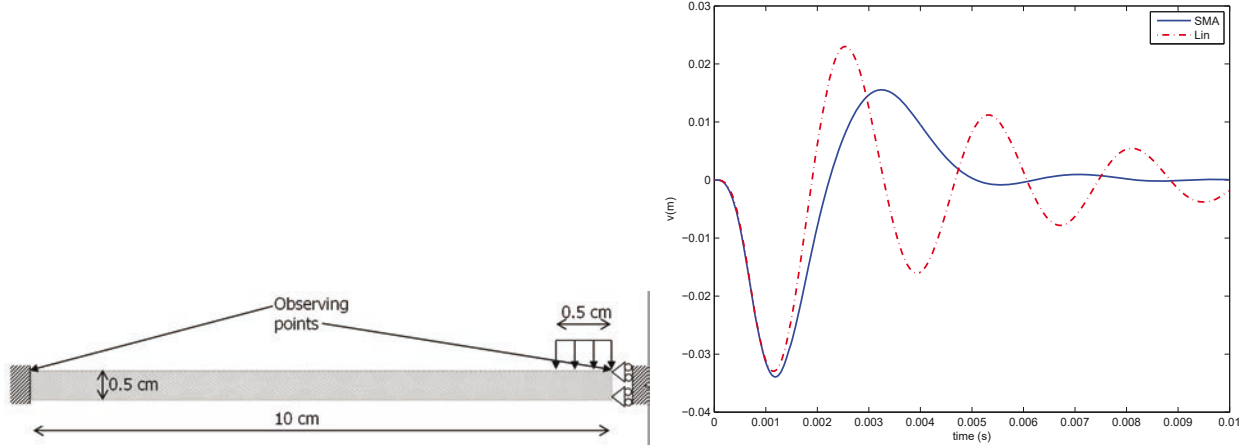
and for martensite to austenite transformation ( $0 < \xi < 1$  and  $\dot{\sigma}_{vm} \leq 0$  and  $\pi_f(T) \leq 0$ ):

$$f(\sigma_{vm}, \pi_f, \xi) = \frac{\gamma \dot{\sigma}_{vm}}{\rho \left( \frac{A_2}{\xi} - 2\phi_{it} \right)} \quad (6)$$

The model has been implemented in COMSOL Multiphysics in a finite elements context, in the same way as the one described in details in ref. [2] in the context of classical structural dynamics analysis. An implicit time-stepping scheme is used for the time-dependent solver algorithm, based on DASPK solver [3] (variable-order and variable-stepsize backward differentiation). The nonlinear system of equations at each time step is solved using a Newton-Raphson scheme. The tangent unsymmetric sparse linear system is finally solved using direct UMFPACK solver [4]. In the context of transient dynamics, such as responses to impacts, some numerical tests have shown that the convergence of the solvers was quite bad, inducing very large computation times or even not converged results. One of the main reasons for this is undoubtedly the strong nonlinearity of the kinetic law 4 whose expression is conducted by the value of the thermodynamic force  $\pi_f$  and of the equivalent Von Mises Stress  $\sigma_{vm}$ . The way which has been chosen in this case for smoothing the numerical difficulties induced by the kinetic law, is to add an inertial term in it. In a physical point of view, the austenite to martensite transformation is not instantaneous, and it has been observed that the speed of transformation is linked to the speed of sound in the alloy. This term could have a significant impact when structural solicitations induce speeds which are comparable to the sound velocity, while being negligible for quasi-static or "low-frequency range" applications. The kinetic law 4 has then been modified to :

$$t_m(\xi)\ddot{\xi} + \dot{\xi} = f(\sigma_{vm}, \pi_f, \xi) \quad (7)$$

in which  $t_m(\xi)$  is a relaxation function which can be chosen depending on the speed of sound in the alloy. This term acts as a time delay in the response function and regularizes the phase transformation behavior. Its own dependance on the martensite volume fraction  $\xi$  insures Lyapunov stability of this equation and also prevents overbounded values of  $\xi$  superior to 1 or inferior to 0. In the numerical tests,  $t_m(\xi)$  is a function which is close to  $10^{-3}$  when  $\xi$  belongs to  $[0, 1]$  and zero otherwise.



**Figure 1.** a) 2D-structure; b) Vertical displacement of observing point

The structure which has been used in the numerical simulations is a 2D plane stress component, shown on figure 1a. Its dimensions are 0.5 cm by 10 cm, and the beam is clamped in  $x = 0$  and vertically guided in  $x = 0.1$ m. Two observing points are shown in the figure. The mesh has 2048 quadratic TRI6 elements for the mechanical solver and 2048 linear TRI3 elements for the martensite rate distribution solver, resulting in a total of 19115 degrees of freedom (displacement for the structure and martensite rate).

As a first observation from the numerical tests, one can observe that the phase transformation has a significant effect on the global damping. On figure 1b, two results are shown : the first one is related to the SMA calculation corresponding to the above description, and the second one corresponding to a linear equivalent material (same resolution, with an imposed zero martensite rate). The curve shows the vertical displacement of right observing point, and one can clearly observe that the SMA decrease is faster than the linear one. A very simple 1-dof equivalent model can be used to evaluate the added damping, using a log-decrement approach. The time response of the linear model has a frequency of 360 Hz associated to a damping ratio of 0.11, while the time response of the SMA model has a frequency of 243 Hz and a damping ratio of 0.24. In both cases, the first

mode is the main component of the response. This 1-dof equivalent linear model is clearly not useful for system modeling since there is a strong non-linearity of the response versus amplitude of excitation force, but it can be used to obtain qualitative information about added damping. The other physical parameters (martensite rate, thermodynamic force and equivalent Von Mises Stress) have some coherent physical behaviors, that will be confirmed using comparisons with experiments. Moreover, the added term in the kinetic laws has a great positive effect on convergence, without affecting the results given by the original model.

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