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# A kinetic scheme for unsteady pressurised flows in closed water pipes

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## Abstract

The aim of this paper is to present a kinetic numerical scheme for the computations of transient pressurised flows in closed water pipes. Firstly, we detail the mathematical model written as a conservative hyperbolic partial differential system of equations, and then we recall how to obtain the corresponding kinetic formulation. Then we build the kinetic scheme ensuring an upwinding of the source term due to the topography performed in a close manner described by Perthame et al. [8, 1] using an energetic balance at microscopic level for the Shallow Water equations. The validation is lastly performed in the case of a water hammer in a uniform pipe: we compare the numerical results provided by an industrial code used at EDF-CIH (France), which solves the Allievi equation (the commonly used equation for pressurised flows in pipes) by the method of characteristics, with those of the kinetic scheme. It appears that they are in a very good agreement.

## 1 Introduction

The work presented in this article is the first step in a more general project: the modelisation of unsteady mixed water flows in open channels and in pipes, its kinetic formulation and its numerical resolution by a kinetic scheme.

Since we are interested in flows occurring in closed pipes, it may happen that some parts of the flow are free-surface (this means that only a part of the cross-section of the pipe is filled) and other parts are pressurised (this means that all the cross-section of the pipe is filled). The Shallow Water equations, which are written in a conservative form, are usually used to describe free surface flows of water in open channels. They are also used in the context of mixed flows using the artifice of the Preissman slot [10],[6]: Cunge and Wegner [7] studied the pressurised flow in the pipe as if it were a free-surface flow by assuming a narrow slot to exist in the upper part of the pipe, the width of the slot being calculated to provide the correct sonic speed. This approach has been credited to Preissmann. Implementing the Preissmann slot technique has the advantage of using only one flow type (free-surface flow) throughout the whole pipe and of being able to easily quantify the pressure head when the pipe pressurises. Nevertheless, as pointed out by several authors (see [9] for instance) the pressurising phenomenon is a dynamic shock requiring a full dynamic treatment even if inflows and other boundary conditions change very slowly. In addition, the Preissmann slot technique is unable to take into account the depressurisation phenomenon which occurs during a waterhammer.

The model used in this article to describe pressurised flows in closed water pipe is very close to the Shallow Water equations, and has been established by the authors in [4]. A second order well-balanced finite volume scheme was therein presented. We will recall in section 2 the main features of this previous work.

Another approach for the numerical resolution of Shallow Water equations is to use a kinetic formulation [8, 1]. The corresponding scheme appears to have interesting theoretical properties: the scheme preserves the still water steady state and involves a conservative in-cell entropy inequality. Moreover,

this type of numerical approximation leads to an easy implementation. The present modelisation of pressurised flows is formally very close to the Shallow Water equations and it may be very interesting to propose a kinetic formulation and thus to construct a kinetic scheme.

The model for the unsteady mixed water flows in closed water pipes and a finite volume discretisation has been previously studied by the authors [3] and a kinetic formulation has been proposed in [5]. We will recall in section 2 the main results and the properties of this kinetic formulation that will be useful to show the properties of the numerical kinetic scheme such as the preservation of the steady state water at rest, and the positivity of the wetted area.

Section 3 is devoted to the construction of the kinetic scheme. The upwinding of the source term due to the topography is performed in a close manner described by Perthame et al. [8] using an energetic balance at microscopic level for the Shallow Water equations.

Finally, we present in section 4 a numerical validation of this study by the comparison between the resolution of this model and the resolution of the Allievi equation solved by the research code `belier` used at Center in Hydraulics Engineering of Electricité De France (EDF) [11] for the case of critical waterhammer tests.

## 2 The mathematical model and the kinetic formulation

We derived a conservative model for pressurised flows from the 3D system of compressible Euler equations by integration over sections orthogonal to the flow axis.

### 2.1 The mathematical model : a “Shallow Water like” system of equations

The equation for conservation of mass and the first equation for the conservation of momentum are:

$$\partial_t \rho + \operatorname{div}(\rho \vec{U}) = 0 \quad (1)$$

$$\partial_t(\rho u) + \operatorname{div}(\rho u \vec{U}) = F_x - \partial_x P \quad (2)$$

with the speed vector  $\vec{U} = u\vec{i} + v\vec{j} + w\vec{k} = u\vec{i} + \vec{V}$ , where the unit vector  $\vec{i}$  is along the main axis,  $\rho$  is the density of the water. We use the Boussinesq linearised pressure law (see [10]):

$$P = P_a + \frac{1}{\beta} \left( \frac{\rho}{\rho_0} - 1 \right),$$

where  $\rho_0$  is the density at the atmospheric pressure  $P_a$  and  $\beta$  the coefficient of compressibility of the water. Exterior strengths  $\vec{F}$  are the gravity  $\vec{g}$  and the friction term  $S_f$  which is assumed to be given by the Manning-Strickler law (see [10]):

$$S_f = K u |u| \quad \text{with} \quad K = \frac{1}{K_s^2 R_h^{4/3}} \quad (3)$$

where  $K_s > 0$  is the Strickler coefficient, depending on the material, and  $R_h$  is the so called hydraulic radius given by  $R_h = \frac{S}{P_m}$ .  $S$  represents the cross-section area of the pipe whereas  $P_m$  is the perimeter of the section. Then Equations (1)-(2) become:

$$\begin{aligned} \partial_t \rho + \partial_x(\rho u) + \operatorname{div}_{(y,z)}(\rho \vec{V}) &= 0 \\ \partial_t(\rho u) + \partial_x(\rho u^2) + \operatorname{div}_{(y,z)}(\rho u \vec{V}) &= -\rho g(\partial_x Z + S_f) - \frac{\partial_x \rho}{\beta \rho_0} . \end{aligned}$$

Assuming that the pipe is infinitely rigid and has a uniform constant cross-section  $S$ , and taking mean values in sections orthogonal to the main flow axis, we get the following system written in a conservative

form for the unknowns  $M = \rho S$ ,  $D = \rho S u$ :

$$\partial_t(M) + \partial_x(D) = 0 \quad (4)$$

$$\partial_t(M) + \partial_x\left(\frac{D^2}{M} + c^2 \rho M\right) = -\rho g M(\partial_x Z + S_f) \quad (5)$$

where  $c = \frac{1}{\sqrt{\beta \rho_0}}$  is the speed of sound. A complete derivation of this model, taking into account the deformations of the pipe, contracting or expanding sections, and a spatial second order Roe-like finite volume method in a linearly implicit version is presented in [4] (see [2] for the first order implicit scheme). This system of partial differential equation is formally close to the Shallow Water equations where the conservative variables are the wet area and the discharge, thus we define an ‘‘FS-equivalent’’ wet area (FS for Free Surface)  $A$  and a ‘‘FS-equivalent discharge’’  $Q$  through the relations:

$$M = \rho S = \rho_0 A \quad \text{and} \quad D = \rho S u = \rho_0 Q \quad .$$

Dividing (4)-(5) by  $\rho_0$  we can write this system under the conservative form:

$$\partial_t U + \partial_x F(U) = G(x, U) \quad (6)$$

where the unknown state is  $U = (A, Q)^t$ , the flux vector is  $F(U) = (Q, \frac{Q^2}{A} + c^2 A)^t$  and the source term writeso  $G(x, U) = (0, -gA(\partial_x Z + S_f))^t$ . This new set of variables allows a more natural treatment of mixed flows (see [4]). In the sequel, we will suppose that the friction term vanishes. Let us now recall the main properties of the system (6) whose proofs can be found in [5].

**Theorem 1** *The system (6) is strictly hyperbolic. It admits a mathematical entropy:*

$$E(A, Q, Z) = \frac{Q^2}{2A} + gAZ + c^2 A \ln A \quad (7)$$

which satisfies the entropy inequality:

$$\partial_t E + \partial_x [u(E + c^2 \ln A)] \leq 0 \quad .$$

Also, the system (6) admits a family of smooth steady states characterized by the relations:

$$Q = Au = C_1 ,$$

$$\frac{u^2}{2} + gZ + c^2 \ln A = C_2 ,$$

where  $C_1$  and  $C_2$  are two arbitrary constants. The quantity  $\frac{u^2}{2} + gZ + c^2 \ln A$  is also called the total head.

Let us then remark that the still water steady state namely  $u \equiv 0$  satisfies:  $gZ + c^2 \ln A = C_2$ .

## 2.2 The kinetic approach

We present in this section the kinetic formulation for pressurised flows in closed water pipes modelised by the preceding system of partial differential equations (see [5] for more details and properties). Let us consider a smooth real function  $\chi$  which has the following properties:

$$\chi(\omega) = \chi(-\omega) \geq 0 , \quad \int_{\mathbb{R}} \chi(\omega) d\omega = 1, \quad \int_{\mathbb{R}} \omega^2 \chi(\omega) d\omega = 1 . \quad (8)$$

We then define the density of particles  $\mathcal{M}(t, x, \xi)$  by the so-called *Gibbs equilibrium*:

$$\mathcal{M}(t, x, \xi) = \frac{A(t, x)}{c} \chi\left(\frac{\xi - u(t, x)}{c}\right) \quad .$$

These definitions allow to obtain a kinetic representation of the system (6) by the following result (see [5] for the proof).

**Theorem 2** *The couple of functions  $(A, Q)$  is a strong solution of the system (6) if and only if  $\mathcal{M}$  satisfies the kinetic equation:*

$$\frac{\partial}{\partial t} \mathcal{M} + \xi \cdot \frac{\partial}{\partial x} \mathcal{M} - g \frac{\partial}{\partial x} Z \cdot \frac{\partial}{\partial \xi} \mathcal{M} = K(t, x, \xi) \quad (9)$$

for some collision term  $K(t, x, \xi)$  which satisfies for a.e.  $(t, x)$

$$\int_{\mathbb{R}} K d\xi = 0, \quad \int_{\mathbb{R}} \xi K d\xi = 0 \quad .$$

This result is a consequence of the following relations verified by the microscopic equilibrium:

$$A = \int_{\mathbb{R}} \mathcal{M}(\xi) d\xi, \quad (10)$$

$$Q = \int_{\mathbb{R}} \xi \mathcal{M}(\xi) d\xi, \quad (11)$$

$$\frac{Q^2}{A} + c^2 A = \int_{\mathbb{R}} \xi^2 \mathcal{M}(\xi) d\xi. \quad (12)$$

This theorem produces a very useful consequence: the nonlinear system (6) can be viewed as a simple linear equation on a nonlinear quantity  $\mathcal{M}$  for which it is easier to find simple numerical schemes with good theoretical properties: it is this feature which will be exploited to construct a kinetic scheme.

**Theorem 3** *Let  $A(x, t) > 0$  and  $Q(x, t)$  be two given functions.*

1. *The minimum of the energy:*

$$\mathcal{E}(f) = \int_{\mathbb{R}} \left( \frac{\xi^2}{2} f(\xi) + c^2 f \ln(f) + gZ f(\xi) + c^2 \ln(c\sqrt{2\pi}) f(\xi) \right) d\xi, \quad ,$$

under the constraints:

$$f \geq 0, \quad \int_{\mathbb{R}} f(\xi) d\xi = A, \quad \int_{\mathbb{R}} \xi f(\xi) d\xi = Q \quad ,$$

is attained by the function:

$$\mathcal{M}(t, x, \xi) = \frac{A}{c} \chi \left( \frac{\xi - u(t, x)}{c} \right)$$

where  $\chi$  is defined by:

$$\chi(\omega) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{\omega^2}{2} \right) \quad . \quad (13)$$

2. *Moreover, the function  $\chi$  defined by (13) ensures us to have the relation*

$$\mathcal{E}(\mathcal{M}) = E(A, Q, Z)$$

if  $A$  and  $Q$  are solution of the pressurised flow equations (6) and the entropy  $E$  is defined by (7).

3. *The Gibbs equilibrium  $\mathcal{M}$  satisfies the still water steady state equation.*

### 3 The kinetic scheme

The spatial domain is a pipe of length  $L$ . The main axis of the pipe is divided in  $N$  meshes  $m_i = ]x_{i-1/2}, x_{i+1/2}[$ ,  $1 \leq i \leq N$ , of length  $h_i$  and center  $x_i$ . We denote  $\Delta x = \min_{1 \leq i \leq N} h_i$ .  $\Delta t$  denotes the timestep at time  $t_n$  and we set  $t_{n+1} = t_n + \Delta t$ .

The discrete macroscopic unknowns are  $U_i^n = \begin{pmatrix} A_i^n \\ Q_i^n \end{pmatrix}$  with  $1 \leq i \leq N$  and  $0 \leq n \leq n_{max}$ . They represent the mean value of  $U$  on the cell  $m_i$  at time  $t_n$ .

If  $Z(x)$  is the function describing the bottom elevation, its piecewise constant representation is given by  $\bar{Z}(x) = Z_i \mathbb{1}_{m_i}(x)$  with  $Z_i = Z(x_i)$  for example.

Replacing  $Z$  by  $\bar{Z}$  and neglecting the collision term  $K(t, x, \xi)$  in a first step, the Equation (9) in the cell  $m_i$  writes:

$$\frac{\partial}{\partial t} \mathcal{M} + \xi \cdot \frac{\partial}{\partial x} \mathcal{M} = 0 \quad \text{for } x \in m_i \quad . \quad (14)$$

This equation is a linear transport equation whose explicit discretisation may be done directly by the following way. Denoting for  $x \in m_i$ ,  $f(t_n, x, \xi) = \mathcal{M}_i^n(\xi)$  the maxwellian state associated to  $A_i^n$ , and  $Q_i^n$ , the usual finite volume discretisation of the Equation (14) leads to:

$$f_i^{n+1}(\xi) = \mathcal{M}_i^n(\xi) + \frac{\Delta t}{h_i} \xi \left( \mathcal{M}_{i+\frac{1}{2}}^-(\xi) - \mathcal{M}_{i-\frac{1}{2}}^+(\xi) \right) \quad (15)$$

where the fluxes  $\mathcal{M}_{i+\frac{1}{2}}^\pm$  have to take into account the discontinuity of the altitude  $\bar{Z}$  at the cell interface  $x_{i+1/2}$ . Indeed, noticing that the fluxes can also be written as:

$$\mathcal{M}_{i+\frac{1}{2}}^-(\xi) = \mathcal{M}_{i+\frac{1}{2}} + \left( \mathcal{M}_{i+\frac{1}{2}}^- - \mathcal{M}_{i+\frac{1}{2}} \right)$$

the quantity  $\delta \mathcal{M}_{i+\frac{1}{2}}^- = \mathcal{M}_{i+\frac{1}{2}}^- - \mathcal{M}_{i+\frac{1}{2}}$  holds for the discrete contribution of the source term  $gA\partial_x Z$  in the system for negative velocities  $\xi \leq 0$  due to the upwinding of the source term. Thus  $\delta \mathcal{M}_{i+\frac{1}{2}}^-$  has to vanish for positive velocity  $\xi > 0$ , as proposed by the choice of the interface fluxes below.

Let us now detail our choice for the fluxes  $\mathcal{M}_{i+\frac{1}{2}}^\pm$  at the interface. It can be justified by using a generalised characteristic method for the Equation (9) (without the collision kernel) but we give instead a presentation based on some physical energetic balance. Let us denote  $\Delta^- Z_{i+\frac{1}{2}} = Z_{i+1} - Z_i$  and  $\Delta^+ Z_{i+\frac{1}{2}} = Z_i - Z_{i+1}$ . In order to take into account the neighboring cells by means of a natural interpretation of the microscopic features of the system, we formulate a peculiar discretisation for the fluxes in (15), computed by the following upwinded formulas:

$$\begin{aligned} \mathcal{M}_{i+\frac{1}{2}}^-(\xi) = \mathcal{M}_i^n(\xi) \mathbb{1}_{\xi \geq 0} &+ \overbrace{\mathcal{M}_i^n(-\xi) \mathbb{1}_{\xi^2 \leq 2g\Delta^- Z_{i+\frac{1}{2}}} \mathbb{1}_{\xi \leq 0}}^{\text{reflection}} \\ &+ \underbrace{\mathcal{M}_{i+1}^n \left( -\sqrt{\xi^2 - 2g\Delta^- Z_{i+\frac{1}{2}}} \right) \mathbb{1}_{\xi^2 \geq 2g\Delta^- Z_{i+\frac{1}{2}}} \mathbb{1}_{\xi \leq 0}}_{\text{transmission}} \end{aligned} \quad (16)$$

$$\begin{aligned} \mathcal{M}_{i+\frac{1}{2}}^+(\xi) = \mathcal{M}_{i+1}^n(\xi) \mathbb{1}_{\xi \leq 0} &+ \overbrace{\mathcal{M}_{i+1}^n(-\xi) \mathbb{1}_{\xi^2 \leq 2g\Delta^+ Z_{i+\frac{1}{2}}} \mathbb{1}_{\xi \geq 0}}^{\text{reflection}} \\ &+ \underbrace{\mathcal{M}_i^n \left( \sqrt{\xi^2 - 2g\Delta^+ Z_{i+\frac{1}{2}}} \right) \mathbb{1}_{\xi^2 \geq 2g\Delta^+ Z_{i+\frac{1}{2}}} \mathbb{1}_{\xi \geq 0}}_{\text{transmission}} \end{aligned} \quad (17)$$

The effect of the source term is made explicit by treating it as a physical potential. The choices (16)-(17) are thus a mathematical formalization to describe the physical microscopic behaviour of the system. The contribution of the interface  $x_{i+1/2}$  to  $f_i^{n+1}$  is given by:

- the particles in the cell  $m_i$  at time  $t_n$  with non negative velocities  $\xi$  through the term  $\mathcal{M}_i^n(\xi) \mathbb{1}_{\xi \geq 0}$  and those of them that are reflected (thus taken into account with velocity  $-\xi$ ) if their kinetic energy is not large enough to overpass the potential difference i.e.  $\xi^2 \leq 2g\Delta^\pm Z_{i+\frac{1}{2}}$ : see Figure 2.
- the particles in the cell  $m_{i+1}$  at time  $t_n$  with a kinetic energy enough to overpass the potential difference ( $\xi^2 \geq 2g\Delta^\pm Z_{i+\frac{1}{2}}$ ) and speed up or down according to this potential jump. It is the transmission phenomenon in classical mechanics as shown in Figure 1.

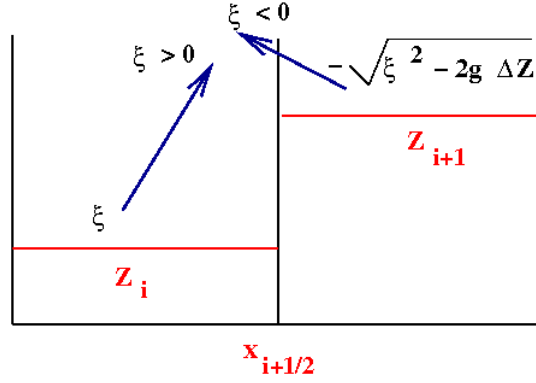


Figure 1: Transmission

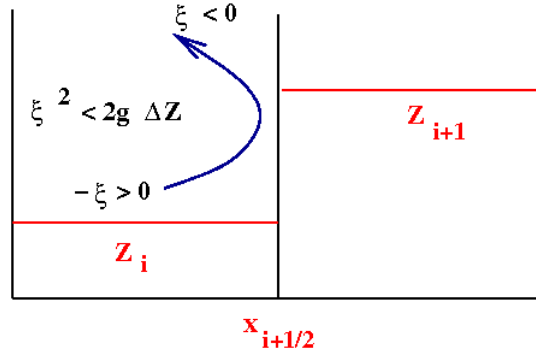


Figure 2: Reflection

Since we neglected the collision term, it is clear that  $f^{n+1}$  computed by the discretised kinetic equation (15) is no more a Gibbs equilibrium. Therefore, to recover the macroscopic variables  $A$  and  $Q$ , according to the identities (10)-(11), we set:

$$U_i^{n+1} = \begin{pmatrix} A_i^{n+1} \\ Q_i^{n+1} \end{pmatrix} \stackrel{def}{=} \int_{\mathbb{R}} \begin{pmatrix} 1 \\ \xi \end{pmatrix} f_i^{n+1} d\xi \quad (18)$$

Now, we can integrate the discretised kinetic equation (15) against 1 and  $\xi$  to obtain the macroscopic kinetic scheme:

$$U_i^{n+1} = U_i^n + \frac{\Delta t}{h_i} \left( F_{i+\frac{1}{2}}^- - F_{i-\frac{1}{2}}^+ \right) \quad (19)$$

The numerical fluxes are thus defined by the kinetic fluxes as follows:

$$F_{i+\frac{1}{2}}^\pm \stackrel{def}{=} \int_{\mathbb{R}} \xi \begin{pmatrix} 1 \\ \xi \end{pmatrix} \mathcal{M}_{i+\frac{1}{2}}^\pm(\xi) d\xi \quad (20)$$

**Remark 1**

- We see immediately that the kinetic scheme (19)-(20) is wetted area conservative. Indeed, let us denote the first component of the discrete fluxes (20)  $(F_A)_{i+\frac{1}{2}}^\pm$ :

$$(F_A)_{i+\frac{1}{2}}^\pm \stackrel{def}{=} \int_{\mathbb{R}} \xi \mathcal{M}_{i+\frac{1}{2}}^\pm(\xi) d\xi$$

An easy computation using the change of variables  $\mu = |\xi|^2 - 2g\Delta^+ Z_{i+\frac{1}{2}}$  in the formulas (16)-(17) defining the kinetic fluxes  $\mathcal{M}_{i+\frac{1}{2}}^\pm$  allows us to show that:

$$(F_A)_{i+\frac{1}{2}}^+ = (F_A)_{i+\frac{1}{2}}^-$$

- Computing the macroscopic state  $U$  by the formula (18) or the fluxes by the formula (20) is not easy if the function  $\chi$  verifying the properties (8) is not compactly supported. We use instead the function defined by  $\chi(\omega) = \frac{1}{2\sqrt{3}} \mathbb{1}_{[-\sqrt{3}, \sqrt{3}]}(\omega)$ . We get  $\mathcal{M}_i^n(\xi) = \frac{A_i^n}{2c\sqrt{3}} \mathbb{1}_{[u_i^n - c\sqrt{3}, u_i^n + c\sqrt{3}]}(\xi)$ .
- In the case where the friction  $S_f$  defined by (3) is present, from the Equation (18) defining the state  $A_i^{n+1}$  and  $Q_i^{n+1}$ , it is easy to construct it.

We are now able to state the main properties of the kinetic scheme.

**Theorem 4** *We assume the CFL condition*

$$\Delta t \max_{1 \leq i \leq N} (|u_i^n| + c\sqrt{3}) \leq \Delta x. \quad (21)$$

*Then*

- (i) *the kinetic scheme (19)-(20) keeps the pseudo wetted area  $A_i^n$  positive.*
- (ii) *the kinetic scheme (19)-(20) preserves the still water steady state,*

$$u_i^n = 0, \quad g Z_i + c^2 \ln A_i = K$$

**Proof of theorem 4** Since  $A_i = \int_{\mathbb{R}} f_i^{n+1} d\xi$ , it is sufficient to prove that  $f_i^{n+1} \geq 0$ . Writing the microscopic scheme (15), (16), (17), using the CFL condition (21), and the fact that the function  $\chi$  that we have chosen is compactly supported, one may see that if we suppose that  $A_i^n \geq 0$ , then  $f_i^{n+1}$  is a sum of non-negative quantities. For the second point, setting  $u_i^n = 0$ , we prove easily that in the discretised kinetic equation (15), we have

$$\mathcal{M}_{i+\frac{1}{2}}^-(\xi) = \mathcal{M}_{i-\frac{1}{2}}^+(\xi).$$

This implies  $f_i^{n+1} = \mathcal{M}_i^n(\xi)$ , which ensures by definition  $A_i^{n+1} = A_i^n$  and  $Q_i^{n+1} = Q_i^n$ . Thus we obtain  $u_i^{n+1} = 0$ .

## 4 Numerical validation: comparison with the solution of Allievi equations

We present now numerical results of a water hammer test. The pipe of circular cross-section of 2 m<sup>2</sup> and thickness 20 cm is 2000 m long. The altitude of the upstream end of the pipe is 250 m and the slope is 5°. The Young modulus is 23 10<sup>9</sup> Pa since the pipe is supposed to be built in concrete. The total



upstream head is 300 m. The initial downstream discharge is 10 m<sup>3</sup>/s and we cut the flow in 5 seconds. Let us define the piezometric line by:

$$piezo = z + \delta + p \quad \text{with } p = \frac{c^2 (\rho - \rho_0)}{\rho_0 g}. \quad (22)$$

We present a validation of the proposed scheme by comparing numerical results of the proposed model solved by the kinetic scheme with the ones obtained by solving Allievi equations by the method of characteristics with the so-called `belier` code: an industrial code used by the engineers of the Center in Hydraulics Engineering of Electricité De France (EDF) [11]. Our code is written in Fortran 77 and runs during a few seconds on Linux, Windows and MacIntosh operating system.

A simulation of the water hammer test was done for a CFL coefficient equal to 0.8 (i.e.  $CFL = 0.8$ ) and a spatial discretisation of 1000 mesh points (the mesh size is equal to 2 m). In the Figure 3, we present a comparison between the results obtained by our kinetic scheme and the ones obtained by the “belier” code at the middle of the pipe: the behavior of the piezometric line, defined by Equation (22), and the discharge at the middle of the pipe. One can observe that the results for the proposed model are in very good agreement with the solution of Allievi equations. A little smoothing effect and absorption may be probably due to the first order discretisation type. A second order scheme may be implemented naturally and will produce a better approximation.

## References

- [1] R. Botchorishvili, B. Perthame, and A. Vasseur. Equilibrium schemes for scalar conservation laws with stiff sources. *Math. Comput.*, 72(241):131–157, 2003.
- [2] C. Bourdarias and S. Gerbi. An implicit finite volumes scheme for unsteady flows in deformable pipe-lines. In R. Herbin and D. Kröner, editors, *Finite Volumes for Complex Applications III: Problems and Perspectives*, pages 463–470. HERMES Science Publications, 2002.
- [3] C. Bourdarias and S. Gerbi. A finite volume scheme for a model coupling free surface and pressurised flows in pipes. *J. Comp. Appl. Math.*, 209(1):109–131, 2007.
- [4] C. Bourdarias and S. Gerbi. A conservative model for unsteady flows in deformable closed pipes and its implicit second order finite volume discretisation. *Computers & Fluids*, 37:1225–1237, 2008.
- [5] C. Bourdarias, S. Gerbi, and M. Gisclon. A kinetic formulation for a model coupling free surface and pressurised flows in closed pipes. *J. Comp. Appl. Math.*, 218(2):522–531, 2008.
- [6] H. Capart, X. Sillen, and Y. Zech. Numerical and experimental water transients in sewer pipes. *Journal of Hydraulic Research*, 35(5):659–672, 1997.
- [7] J.A. Cunge and M. Wegner. Intégration numérique des équations d’écoulement de Barré de Saint Venant par un schéma implicite de différences finies. *La Houille Blanche*, (1):33–39, 1964.
- [8] B. Perthame and C. Simeoni. A kinetic scheme for the Saint-Venant system with a source term. *Calcolo*, 38(4):201–231, 2001.
- [9] C.S.S. Song, J.A. Cardle, and K.S. Leung. Transient mixed-flow models for storm sewers. *Journal of Hydraulic Engineering, ASCE*, 109(11):1487–1503, 1983.
- [10] V.L. Streeter, E.B. Wylie, and K.W. Bedford. *Fluid Mechanics*. McGraw-Hill, 1998.
- [11] V. Winckler. Logiciel belier4.0. Notes de principes. Technical report, EDF-CIH, Le Bourget du Lac, France, 1993.

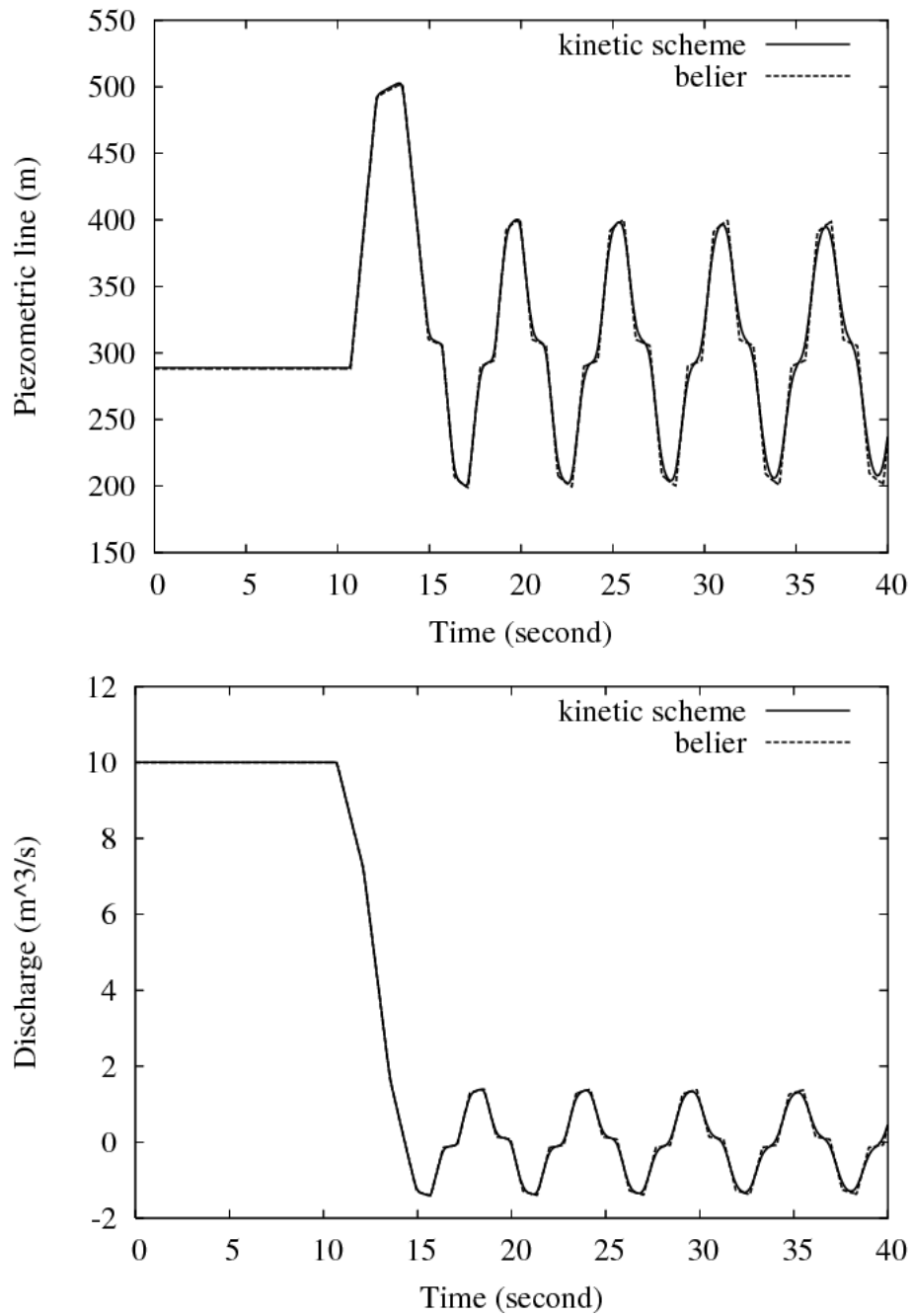


Figure 3: Comparison between the kinetic scheme and the industrial code belier  
Piezometric line (top) and discharge (bottom) at the middle of the pipe