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# 3D TRAJECTORY TRACKING FOR A FIXED WING UNMANNED AERIAL VEHICLE USING DYNAMIC INVERSION

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**Abstract:** In this paper we present a dynamic inversion based control technique for an autonomous aerial vehicle for the trajectory tracking problem. Our approach takes account of the model nonlinearities. The adopted scheme subdivides the vehicle dynamics into two scales of time: fast and slow. This is justified by the difference between the dynamic of the angular velocity and that of UAV airspeed and attitude (pitch, roll and yaw). Our controller uses two control loops: stabilization-tracking loop which controls the angular velocity the sideslip, pitch and roll angles. And a guidance loop which controls the vehicle altitude and heading. This approach was validated on IBISC UAV model developed in our laboratory. *Copyright © 2007 IFAC*

**Keywords:** Autonomous vehicles, Aircraft control,, Nonlinear control, Feedback linearization, Inverse dynamics control.

## 1. INTRODUCTION

Since decades the automatic control occupies increasingly significant place in aeronautic, in particular with the advent of the UAV (Unmanned Aerial Vehicle) systems and their potential civilian and military applications. Many works were made treating various aspects related to the control systems automation. Among these aspects:

- The UAV control system must answer some performance and stability criteria. These specifications are generally based on linear model for different stages of flight (Uy-Loi, 1997, Stevens *et al.*, 2003).
- It must offer a minimal margin of stability and performances of the closed loop system. Indeed, the mathematical model used during the control laws design is nominal model, its reliability is always questioned. Moreover, physical systems change behaviour in function of their configurations in the space; this is translated by a parametric variation in the dynamic model. Finally, External disturbances and the measurements noises can deteriorate the controller efficiency.
- Take account of nonlinearities of dynamical model, and compensate them instead of neglect them (Stevens *et al.*, 2003, Megretski, 2003). The principal idea of dynamic inversion techniques rests, in part on

the fact that the principal obstacle in the control laws design is the nonlinearities of the dynamical model. And in other part, the important number of linear analysis and synthesis tools of control laws. If we can, by using adequate nonlinear transformations, replacing the nonlinear model by a linear equivalent one, the use of linear techniques becomes possible (Slotine *et al.*, 1991, Isidori *et al.*, 1992). In the case of state feedback all the dynamical equations are linearised by a coordinates transformation and static state feedback. However, this linearization can be applied only to a restricted class of physical systems, because it not always possible that all the state variables are accessible, from where the idea to realise a partial dynamic linearization, more precisely, the equations describing the In/Out transfer (Guardabassi *et al.*, 2001). The residual dynamics, which do not depend explicitly of the input, are not linearsables, there are called internal dynamics. The problem with this partial state linearization comes of the fact that the internal dynamics (non observables) can be unstable; this is the case of Non Minimum Phase systems (Kanter *et al.*, 2002).

In this paper we propose an input/output feedback linearization based approach. This idea is inspired from the works of (Bugajski *et al.*, 1992, Van soest *et al.*, 2006, Singh *et al.*, 1995, Wang, 2005 and Schumacher *et al.*, 1998a, b). The dynamic inversion

is realised on two times scales (Bugajski *et al.*, 1992). A first feedback (inner loop) is applied on the angular velocity dynamic. A second one (outer loop) is applied on the sideslip, the pitch and the roll angles. This approach is validated on a UAV model. This paper is organized as follows:

In the first time we present the IBISC UAV Project developed in our laboratory. Then we present a nonlinear control approach based on the dynamic inversion for trajectory tracking. Finally, in order to validate this approach we present some simulation results.

## 2. IBISC UAV PROJECT

IBISC UAV is a project in our laboratory which consists of designing an autonomous microlight aircraft, intended for supervising missions by means of cameras (forests fires, littoral pollution, borders, flooded areas...).



Fig. 1. IBISC UAV CAO view.

It can cover a perimeter of 50 km about the take off/landing area. I twill fly with a cruise speed of 50m/s at an altitude of 2400 m. Its range is 50 km. It will be propelled by a single engine piston with a propeller; the engine power is 20 CV. The vehicle total mass is estimated at 120 kg. Table I recapitulates the vehicle characteristics.

<b>Dimensions</b> : length : 4,27 m ; Wing-span : 4,8 m ; Mean chord : 0,51 m ; Wing area : 2,5 m <sup>2</sup>
<b>Masses</b> : Total mass : 120 kg ; Payload : 30 kg
<b>Inertia en kg.m<sup>2</sup></b> : $I_{xx}=252,72$ ; $I_{yy}=519,49$ ; $I_{zz}=701,01$ ; $I_{xz}=28,13$ ; $I_{xy}=I_{yz}=0$
<b>Aerodynamic coefficients</b>
<b>Lift</b> : $C_{L0}=0,59$ ; $C_{L\alpha}=4,28$ ; $C_{L\dot{\alpha}}=-2,43$ ; $C_{Lq}=6,83$ ; $C_{L\delta_r}=0,33$
<b>Pitching moment</b> : $C_{m0}=0,194$ ; $C_{m\alpha}=-0,55$ ; $C_{mq}=-30,47$ ; $C_{m\dot{\alpha}}=-10,86$ ; $C_{m\delta_r}=-1,48$
<b>Drag</b> : $C_{D0}=0,06$ ; $C_{D\alpha}=0,2$ ; $C_{Dq}=0$ ; $C_{D\delta_r}=0$
<b>Rolling moment</b> : $C_{l\beta}=-0,03$ ; $C_{lp}=-0,3$ ; $C_{lr}=0,15$ ; $C_{l\delta_a}=-0,12$ ; $C_{l\delta_r}=0,004$
<b>Lateral force</b> : $C_{Y\beta}=-0,43$ ; $C_{Y\delta_r}=0,217$ ; $C_{Yp}=-0,14$ ; $C_{Yr}=0,29$
<b>Yawing moment</b> : $C_{n\beta}=0,2$ ; $C_{np}=-0,06$ ; $C_{nr}=-0,137$

$C_{n\delta_a}=0,008$ ; $C_{n\delta_r}=0,1$
<b>Performances</b> : Stalling speed : 22m/s; Cruise speed : 50 m/s; Cruise altitude:2400 m; Engine power :20 CV; Range:50 Km.

Table. 1. IBISC UAV characteristics.

## 3. DYNAMIC INVERSION FOR IBISC UAV

In this section we present a input/output feedback linearization based approach. The dynamic inversion is realised on two times scales (Bugajski *et al.*, 1992). A first feedback (inner loop) is applied on the angular velocity dynamic ( $p$ ,  $q$  and  $r$ ). A second one (outer loop) is applied on three variables which represent the sideslip  $\beta$ , the pitch  $\theta$  and the roll  $\phi$  angles. We will not present the principle of the standard dynamic inversion or feedback linearization. The reader can find this technique detailed in (Slotine *et al.*, 1991, Isidori *et al.*, 1992).

The controller structure is represented in Fig. 3. It constituted from two feedbacks. The inner loop uses the three aerodynamic control surfaces to controlling the fast dynamic variables ( $p$ ,  $q$  and  $r$ ). The outer loop uses the angular velocity like a control input to control the slow dynamic ( $\beta$ ,  $\theta$  and  $\phi$ ). In this loop we have neglected the directly influence of control surfaces in the slow variables, this simplification is justified by the significantly difference observed in open loop simulation between the slow and fast dynamics.

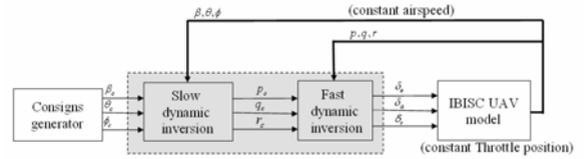


Fig. 2. Controller structure.

The variables  $p$ ,  $q$  and  $r$  are considered like fast variables because the control surfaces have a more significant and direct effect on their derivatives ( $\dot{p}$ ,  $\dot{q}$  and  $\dot{r}$ ). The vocation of control surfaces is to control the UAV rotation about its three axes. Because their locations far from the UAV gravity centre, they generate important moments with weak forces. The UAV nonlinear dynamics are described by a system of eight differential equations. The spatial dependency of dynamic is not considered in this model. Thus, the position (x, y and z) and the heading dynamics are decoupled from the remainder of the equations. The choice of the variables to be controlled is dictated on a side by the constraints related to the used technique (unstable internal dynamics), and another side by the aim of the control. This last consists to maintain the UAV on a reference trajectory in the space by controlling its altitude and heading angle. We have supposed here that the UAV

airspeed is controlled by the throttle (its original vocation) and maintained at constant value during each trajectory. The longitudinal attitude (pitch angle) will be used to control the UAV altitude, the UAV heading (yaw angle) will be controlled by the lateral attitude (roll angle). In our case the internal dynamic is that of the angle of attack, which we need to study its stability. It will be the subject of the section (4.3.2).

### 3.1 Dynamic inversion inner loop

This feedback allows to track the reference values of the angular velocity  $(p_c, q_c, r_c)$ . The control inputs are the control surfaces  $(\delta_e, \delta_a, \delta_r)$ . The controller structure is represented in Fig. 2. The consign values are generated by the outer loop of slow dynamic.

The functions  $(w_p, w_q, w_r)$  are low-pass filters with bandwidth  $w_p, w_q$  and  $w_r$  respectively. For choosing these frequencies (8 rad/s), we must take account the natural modes of the structure and the actuators modes (30 rad/s).

The dynamical equations of  $p, q$  and  $r$  can be rewritten under the following form:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_p(x) \\ f_q(x) \\ f_r(x) \end{bmatrix} + B \cdot \begin{bmatrix} \delta_e \\ \delta_a \\ \delta_r \end{bmatrix} \quad (1)$$

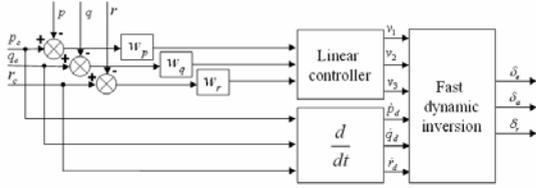


Fig. 3. Inner loop: fast dynamic.

Where:

$$f_p(x) = \left( \frac{I_z \cdot (I_y - I_z) q r + I_z \cdot I_{xz} \cdot p q + I_z \cdot \hat{L} + I_{xz} \cdot (I_x - I_y) q \cdot p - I_{xz} \cdot r \cdot q + \hat{N}}{I_x I_z - I_{xz}^2} \right)$$

$$f_q(x) = \left( \frac{I_{xz} \cdot (r^2 + p^2) + (I_z - I_x) p r + \hat{M}}{I_y} \right)$$

$$f_r(x) = \left( \frac{I_x \cdot (I_x - I_y) q \cdot p + I_x \cdot I_{xz} \cdot r \cdot q + I_x \cdot \hat{N} + I_{xz} \cdot (I_y - I_z) q r - I_{xz} \cdot p \cdot q + \hat{L}}{I_x I_z - I_{xz}^2} \right)$$

With:  $x = [V \ \alpha \ \beta \ p \ q \ r \ \phi \ \theta]^T$ ,

$$\hat{L} = \bar{q} \cdot S \cdot b \cdot (C_{l\beta} \cdot \beta + C_{lp} \cdot p + C_{lr} \cdot r) \quad (2)$$

$$\hat{M} = \bar{q} \cdot S \cdot \bar{c} \cdot (C_{m0} + C_{m\alpha} \cdot \alpha + C_{mq} \cdot q) \quad (3)$$

$$\hat{N} = \bar{q} \cdot S \cdot b \cdot (C_{n\beta} \cdot \beta + C_{np} \cdot p + C_{nr} \cdot r) \quad (4)$$

The aerodynamic coefficients in the moment' expressions (2-4) are constants. The system is input affine, the matrix B is given by:

$$B = \begin{bmatrix} 0 & \frac{I_z \cdot \bar{q} \cdot S \cdot b \cdot C_{l\beta} + I_{xz} \cdot \bar{q} \cdot S \cdot b \cdot C_{n\beta}}{I_x I_z - I_{xz}^2} & \frac{I_z \cdot \bar{q} \cdot S \cdot b \cdot C_{l\beta} + I_{xz} \cdot \bar{q} \cdot S \cdot b \cdot C_{n\beta}}{I_x I_z - I_{xz}^2} \\ \frac{\bar{q} \cdot S \cdot \bar{c} \cdot C_{m\beta}}{I_y} & 0 & 0 \\ 0 & \frac{I_x \cdot \bar{q} \cdot S \cdot b \cdot C_{n\beta} + I_{xz} \cdot \bar{q} \cdot S \cdot b \cdot C_{l\beta}}{I_x I_z - I_{xz}^2} & \frac{I_x \cdot \bar{q} \cdot S \cdot b \cdot C_{n\beta} + I_{xz} \cdot \bar{q} \cdot S \cdot b \cdot C_{l\beta}}{I_x I_z - I_{xz}^2} \end{bmatrix}$$

From B we observe the control inputs appear explicitly in dynamical equations. The output vector being  $y = [p \ q \ r]^T$ , we can deduce the relative degree vector  $r = [1 \ 1 \ 1]$ .

We choice the control law as follows:

$$\begin{bmatrix} \delta_e \\ \delta_a \\ \delta_r \end{bmatrix} = B^{-1} \cdot \left( \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} - \begin{bmatrix} f_p(x) \\ f_q(x) \\ f_r(x) \end{bmatrix} \right) \quad (5)$$

With:

$$B^{-1} = \begin{bmatrix} 0 & \frac{I_y}{\bar{q} \cdot S \cdot \bar{c} \cdot C_{m\beta}} & 0 \\ \frac{I_x \cdot C_{n\beta} + I_{xz} \cdot C_{l\beta}}{\bar{q} \cdot S \cdot b \cdot (C_{n\beta} \cdot C_{l\beta} - C_{n\beta} \cdot C_{l\beta})} & 0 & \frac{I_z \cdot C_{l\beta} + I_{xz} \cdot C_{n\beta}}{\bar{q} \cdot S \cdot b \cdot (C_{n\beta} \cdot C_{l\beta} - C_{n\beta} \cdot C_{l\beta})} \\ \frac{I_x \cdot C_{n\beta} + I_{xz} \cdot C_{l\beta}}{\bar{q} \cdot S \cdot b \cdot (C_{n\beta} \cdot C_{l\beta} - C_{n\beta} \cdot C_{l\beta})} & 0 & \frac{I_z \cdot C_{l\beta} + I_{xz} \cdot C_{n\beta}}{\bar{q} \cdot S \cdot b \cdot (C_{n\beta} \cdot C_{l\beta} - C_{n\beta} \cdot C_{l\beta})} \end{bmatrix}$$

$v = [v_1 \ v_2 \ v_3]^T$  is called auxiliary input vector. By replacing the control vector by (5) in (1) we obtain:

$$\dot{y} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = v \quad (6)$$

For each of three outputs of the equivalent linear system is a simple integrator. The control law will be designed according to problem to be solved: regulation or tracking problem. Below these two cases for any relative degree  $r$ .

Regulation problem: The aim is to maintain the output  $y(t)$  with constant value  $y_c$ . To be done we can choice a control law:

$$v(t) = -a_0 \cdot (h(x) - y_c) - a_1 \cdot L_f h(x) - a_2 \cdot L_f^2 h(x) - \dots - a_{r-1} \cdot L_f^{r-1} h(x)$$

The coefficients  $a_i$  ( $i = 0 \dots r-1$ ) must be chosen such as the eigvalues of the polynomial:

$$s^r + a_{r-1} \cdot s^{r-1} + a_{r-2} \cdot s^{r-2} + \dots + a_1 \cdot s + a_0$$

are stable (with negative real part).

For our case the relative degree is 1. we can choice a control  $v(t)$  as :  $v(t) = -a_0 \cdot (y - y_c)$  with the corresponding polynomial  $s + a_0$ . Thus, it's enough to choice  $a_0 > 0$  so that the control  $v(t)$  maintain the output  $y(t)$  with the value  $y_c$ .

Tracking problem: The aim is to tracking the reference trajectory  $y_c(t)$ . We define the error dynamic:  $e(t) = y(t) - y_c(t)$

We can easily deduce:

$$\begin{aligned}\dot{e}(t) &= \dot{y}(t) - \dot{y}_c(t) \\ \ddot{e}(t) &= \ddot{y}(t) - \ddot{y}_c(t) \\ &\vdots \\ e^{(r)}(t) &= y^{(r)}(t) - y_c^{(r)}(t) = v - y_c^{(r)}(t)\end{aligned}$$

It's enough to choice the control  $v(t)$  as:

$$\begin{aligned}v(t) &= y_c^{(r)}(t) - a_0 \cdot (h(x) - y_c) - a_1 \cdot L_f h(x) \\ &\quad - a_2 \cdot L_f^2 h(x) - \dots - a_{r-1} \cdot L_f^{r-1} h(x)\end{aligned}$$

The coefficients  $a_i$  ( $i=0 \dots r-1$ ) must be chosen such as the polynomial:

$$s^r + a_{r-1} \cdot s^{r-1} + a_{r-2} \cdot s^{r-2} + \dots + a_1 \cdot s + a_0$$

Are stables (with negative real part).

For our case the relative degree is 1. The error dynamic derivative is given by:

$$\dot{e}(t) = \dot{y}(t) - \dot{y}_c(t) = v - \dot{y}_c(t)$$

The control  $v(t)$  can be chosen as:

$$v(t) = \dot{y}_c(t) - a_0 \cdot (y - y_c)$$

With the corresponding polynomial  $s + a_0$ . For well tracking of the reference trajectory  $y_c(t)$  it's enough to take  $a_0 > 0$ .

### 3.2 Dynamic inversion outer loop ( $\beta, \theta, \phi$ )

The slow inverted dynamics are those of the sideslip  $\beta$ , the pitch  $\theta$  and the roll  $\phi$  angles. In fact, in this loop we neglect the direct effect of the control surfaces on these variables. Moreover, we suppose a perfect reference following of fast variables. By examining the dynamical equations of the slow variables ( $\beta, \theta, \phi$ ) we can observe that these variables depend mainly to the angular velocity (fast variables), from which the idea to using the angular velocity like a control input for the slow dynamic. The scheme is represented in Fig. 4.

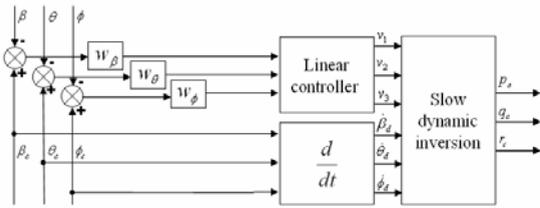


Fig. 4. Outer loop : slow dynamic.

We note that the airspeed dynamic is constant during the all the flight trajectory, and that is controlled by the Throttle input (which the position is supposed fixed). Thus, the slow dynamic is represented by four state variables: the aerodynamic angles ( $\alpha$  and  $\beta$ ) the UAV attitude ( $\theta$  and  $\phi$ ). The output vector is  $y = [\beta \ \theta \ \phi]^T$ . The angle of attack dynamic  $\alpha$  represents the internal dynamic which the stability must be ensured.

The dynamical equations of ( $\beta, \theta, \phi$ ) can be rewritten as:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} f_\beta(x_1) \\ f_\theta(x_1) \\ f_\phi(x_1) \end{bmatrix} + B_1 \begin{bmatrix} p \\ q \\ r \end{bmatrix} + B_2 \begin{bmatrix} \delta_c \\ \delta_a \\ \delta_r \end{bmatrix} \quad (7)$$

$$f_\beta(x_1) = \frac{1}{m \cdot V} \left( m \cdot g \cdot (s \theta \cdot c \alpha \cdot s \beta + c \theta \cdot s \phi \cdot c \beta - c \theta \cdot c \phi \cdot s \alpha \cdot s \beta) \right. \\ \left. + \bar{q} \cdot S \cdot (C_{Y\beta} \cdot \beta \cdot c \beta + (C_{D0} + C_{D\alpha} \cdot \alpha) \cdot s \beta) \right)$$

$$f_\theta(x_1) = f_\phi(x_1) = 0$$

with  $x_1$  is the slow variables vector :  $x_1 = [V \ \alpha \ \beta \ \phi \ \theta]^T$

$$B_1 = \begin{bmatrix} \sin \alpha & 0 & -\cos \alpha \\ 0 & \cos \phi & -\sin \phi \\ 1 & \sin \phi \cdot \tan \theta & \cos \phi \cdot \tan \theta \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \frac{\bar{q} \cdot S \cdot C_{D\delta_c} \cdot \sin \beta}{m \cdot V} & 0 & \frac{\bar{q} \cdot S \cdot C_{Y\delta_c} \cdot \cos \beta}{m \cdot V} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The bandwidths  $w_\beta$ ,  $w_\theta$  et  $w_\phi$  are chosen (2 rad/s) such that they are sufficiently lower than that of the fast dynamics, in order to avoid the coupling between the two loops. The terms of the matrix  $B_1$  are kinematics relations; it is an invertible except for  $\theta = \pm \pi / 2$ . The matrix  $B_2$  contains only two terms different from zero, but their values are negligible, this is justified by the weak values of the coefficients  $C_{D\delta_c} = 0.015$  and  $C_{Y\delta_c} = 0.19$ .

Moreover, we consider that the control surfaces dynamic is very fast so that it has a direct effect on the slow variables. The system (7) thus becomes:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} f_\beta(x_1) \\ f_\theta(x_1) \\ f_\phi(x_1) \end{bmatrix} + B_1 \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (8)$$

In the same way that for the inner loop, by analysing the matrix  $B_1$  we can observe that the control inputs appear explicitly in the first derivative output vector  $y = [\beta \ \theta \ \phi]^T$ . We deduce the relative degree vector:  $r = [1 \ 1 \ 1]^T$ .

We chose the control law as:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = B_1^{-1} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} - \begin{bmatrix} f_\beta(x) \\ f_\theta(x) \\ f_\phi(x) \end{bmatrix} \quad (9)$$

with:

$$B_1^{-1} = \frac{1}{c \alpha \cdot c \phi \cdot c \theta + s \alpha \cdot s \theta} \begin{bmatrix} s \theta & -c \alpha \cdot s \phi \cdot s \theta & c \alpha \cdot c \phi \cdot c \theta \\ -s \phi \cdot c \theta & s \alpha \cdot c \phi \cdot s \theta + c \alpha \cdot c \theta & s \alpha \cdot s \phi \cdot c \theta \\ -c \phi \cdot c \theta & -s \alpha \cdot s \phi \cdot s \theta & s \alpha \cdot c \phi \cdot c \theta \end{bmatrix}$$

by replacing the control vector by (9) in (8) we obtain:

$$\dot{y} = \begin{bmatrix} \dot{\beta} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = v \quad (10)$$

For each of the three outputs of the equivalent linear systems a simple integrator. In order to ensure a will tracking of  $y_c(t)$ , it's enough to choose  $v(t)$  as:

$$v(t) = \dot{y}_c(t) - a_0 \cdot (y - y_c)$$

With the coefficient  $a_0$  strictly positive.

Now, we return to internal dynamic in order to check its stability. The airspeed being constant the unobservable dynamic is represented by the angle of attack dynamic:

$$\dot{\alpha} = f_\alpha(x_1) + B_\alpha \cdot [p \quad q \quad r]^T \quad (11)$$

with:

$$f_\alpha(x_1) = \frac{1}{m \cdot V \cdot \cos(\beta)} \left( g \cdot m \cdot (\cos(\theta) \cdot \cos(\phi) \cdot \cos(\alpha)) + \sin(\theta) \cdot \sin(\alpha) - \bar{q} \cdot S \cdot (C_{L\alpha} + C_{L\alpha} \cdot \alpha) \right)$$

$$B_\alpha = \begin{bmatrix} \frac{-\cos(\alpha) \cdot \sin(\beta)}{\cos(\beta)} & \frac{-m \cdot V \cdot \cos(\beta) + \bar{q} \cdot S \cdot C_{Lq}}{m \cdot V \cdot \cos(\beta)} & \frac{-\sin(\alpha) \cdot \sin(\beta)}{\cos(\beta)} \end{bmatrix}$$

We replace the control vector in (11) by its expressions (9) then we study the stability of  $\alpha$  in the equilibrium i.e. with an auxiliary control vector equal to zero  $v = [0 \ 0 \ 0]^T$  and the outputs:

$$\beta = \beta_0 \quad \theta = \theta_0 \quad \text{and} \quad \phi = \phi_0.$$

For simplifying the study we consider the case of longitudinal (symmetrical) flight ( $\beta_0 = \phi_0 = 0$ ) with a constant flight path angle  $\gamma_0$ . Thus, the equation (11) becomes:

$$\dot{\alpha} = \frac{1}{m \cdot V} \cdot (\cos(\gamma_0) - \bar{q} \cdot S \cdot (C_{L\alpha} + C_{L\alpha} \cdot \alpha)) \quad (12)$$

The equilibrium condition is given:

$$\alpha_0 = \frac{\cos(\gamma_0) - \bar{q} \cdot S \cdot C_{L\alpha}}{\bar{q} \cdot S \cdot C_{L\alpha}}$$

With the constants  $\bar{q}$ ,  $S$  and  $C_{L\alpha}$  are positives. We deduce easily that this equilibrium is asymptotically stable, and this whatever the flight path angle  $\gamma_0$ . Thus, the stability of the internal dynamic is checked.

#### 4. SIMULATION RESULTS

In this section we present some simulations results of the application of our approach on the model of IBISC UAV. The controller which we have designed allows to controlling the sideslip, the pitch and the roll angles. The trajectory generator provides the altitude and heading reference values. The injection of these values in the controller loop is made via an

external loop (guidance loop). The idea consists to use the pitch and roll angles like inputs for controlling respectively the altitude and the yaw angle. The guidance loop is made of two PI controllers. It generates the reference values of the altitude and the yaw angle as is shown in Fig. 6.

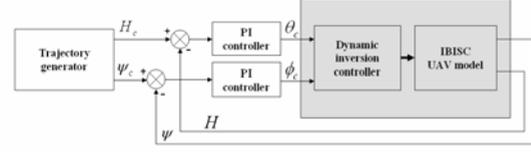


Fig. 6. Guidance loop ensuring the inter connexion between the generator and the controller.

We present here an example of mission with two waypoints. The scenario consists to supervising a geographical area which is to be flown over at cruise altitude (2.400 meters). This area is represented by a target point (15km North et 25 km East). Moreover, the UAV must be forward by two waypoints before reaching the target area which is same with that of a first example. The waypoints coordinates are respectively 15 km North and 5 km West (-5 km East) and 5 km North and 10 km East. Fig. 7 shows the trajectory in three dimensions. The curve in blue presents the reference trajectory and the red one the trajectory traversed by the UAV.

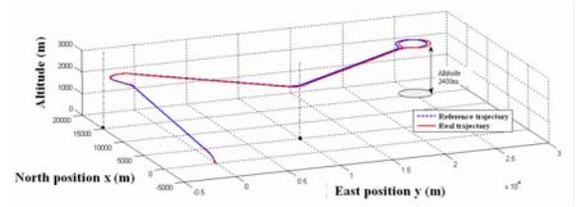


Fig. 7. Example of scenario with two waypoints.

We note that because the UAV airspeed is constant, the altitude and heading control would be enough to ensuring a good tracking of the UAV horizontal position.

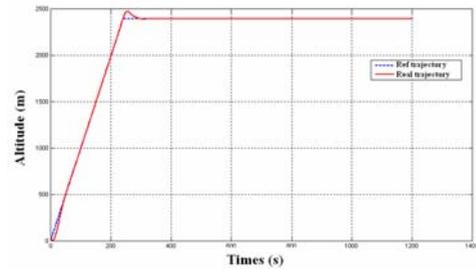


Fig. 8. Example of scenario with two waypoints : Altitude tracking.

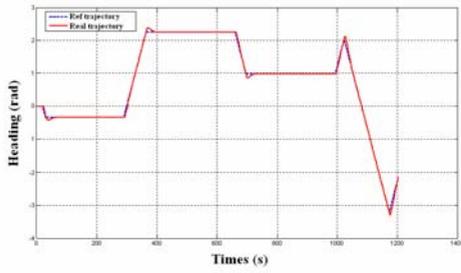


Fig. 9. Example of scenario with two waypoints: Heading tracking

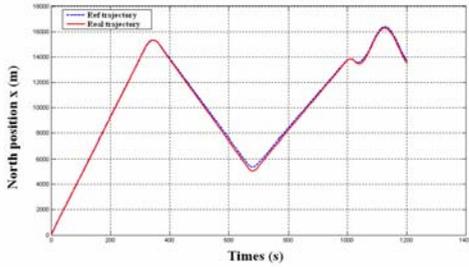


Fig. 10. Example of scenario with two waypoints: North position (x).

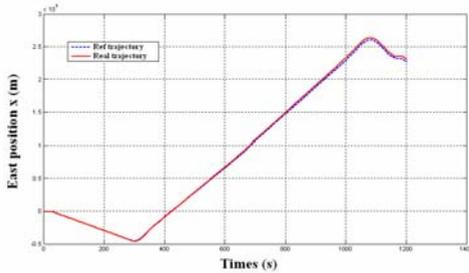


Fig. 11. Example of scenario with two waypoints: East position (y).

Fig. 12 presents the angle of attack variation during the mission. As we saw previously, the angle of attack dynamic represents the internal dynamic which is unobservable. We have analyzed its stability in the case of a longitudinal flight. We observe here that its dynamic is stable even for the case of turn.

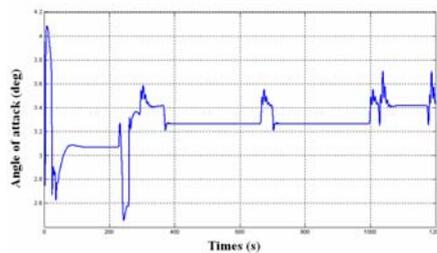


Fig. 12. Example of scenario with two waypoints: Angle of attack.

In Fig. 13 the three first figures present the slow variables, the three figures of the second line present the fast variables. And those of the third line present the control surfaces deflections. The sideslip angle stabilization is ensured by the rudder via the yaw rate

(r). The rudder deflection does not exceed 3 degrees, this justifies this original vocation like a stabilizer. The pitch angle is controlled the elevator via the pitch rate (q). We notice that during this stage the elevator deflection is positive. We can explain this by the setting-off of a moment which tends to pulling up the UAV (the gravity centre being behind the aerodynamic centre).

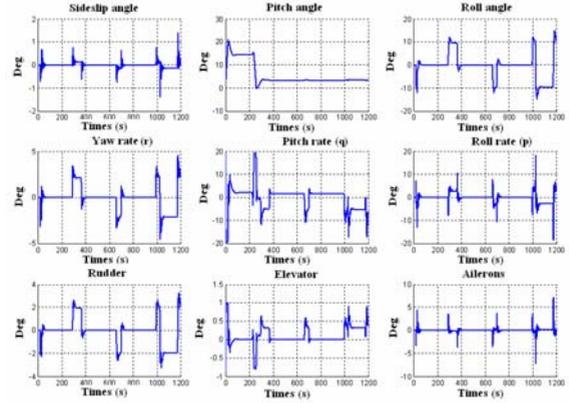


Fig. 13. Example of scenario with two waypoints: Slow and fast variables and control surfaces deflections.

In the case where the UAV executes a turn flight the elevator deflection is negative what generates positive moment to compensate the loss in lift due to the banked (roll) angle of UAV. The roll angle is used to controlling the heading. The roll angle is controlled by the ailerons via roll rate (p).

## 5. CONCLUSION

In this study we synthesised a dynamic inversion based control technique. The advantage of this technique is that takes account the model nonlinearities. The adopted scheme subdivides the UAV dynamic into two scales of time: fast and slow. This is justified by the difference between the behaviour in open loop between the dynamic of the angular velocity and that of UAV airspeed and attitude (pitch, roll and yaw). In our approach we have used two control loops: stabilization-tracking loop which controls the angular velocity the sideslip, pitch and roll angles, and a guidance loop which controls the UAV altitude and heading. The first results obtained are promising. However, improvements can be brought to this approach. This last is validated in simulation with a simplified model. The robustness of control laws to the model uncertainties and the external disturbances is not addressed in this study. Finally, in this approach we consider instantaneous actuators responses. An improvement could be brought by introducing the actuators dynamics.

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