An efficient modeling of a flexible airships

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Abstract

This paper presents an efficient modelling of airships with small deformations moving in an ideal fluid. The formalism is based on the Updated Lagrangian Method (U.L.M.).This formalism proposes to take into account the coupling between the rigid body motion and the deformation as well as the interaction with the surrounding fluid. The resolution of the equations of motion is incremental. The behaviour of the airship is defined relatively to a virtual non-deformed reference configuration moving with the body. The flexibility is represented by a deformation modes issued from a Finite Elements Method analysis. The increment of rigid body motion is represented similarly by rigid modes. A modal synthesis is used to solve the general system equations of motion. Time constant matrices appears (i.e. mass and structural stiffness matrices), and we show a convenient technique to actualise the time dependant matrices.

Keywords: flexible airships, small deformations, incremental scheme, modal synthesis.

1. Introduction

The interest for the modelling and control of airships increases significantly in the last years.
The complexity and capability of airships are expanding rapidly and the range of missions they designed to support is growing. However in order for airships to reach this potential, significant technical issue must be overcome. One main point of this challenge is the modelling of the structural flexibility. It is important to note that several kinds of airships, usually called blimps, are mainly constituted of a balloon filled of gas. The only solid parts are the careen and the tail fins. The integration of the structural flexibility in the dynamic analysis is then useful, but it is now in an embryonic state and is only just emerging. Several researches were done using the assumption of rigid body behaviour for airships [1, 2]. The flexibility effects are sometimes modelled as a perturbation. However in other flying objects, such as light aircrafts, the introduction of the flexibility in the dynamic model becomes essential [3, 4]. We try by this study to contribute to the analysis of the deformation of the airships by introducing the effect of flexibility as non controlled supplementary degrees of freedom. The deformation of the blimp is not considered as a perturbation but rather acting on the motion of the airship.

The influence of structural flexibility on dynamics of mechanical systems has become increasingly important in classical robotics [5, 6] and recently in flying robots (i.e. Airships, UAVs…).

Several approaches, to study the problems of flexible bodies, have been proposed in the literature. These approaches can be classified into two groups. The first group uses the Newton-Euler description [7] which is an interesting method in regard of the time computation. However it is not easy to use it in the case of the flexible body and it is sometimes sensitive to the numerical simulation. In the other hand the total Lagrangian method [8], this consists to define the motion relatively to a fixed reference frame, leads also to complex relations when describing stresses and strains in the flexible body.

An Updated Lagrangian Method (U.L.M.) was proposed by Bathe & al. [9] and developed for deformable bodies that undergo large translational and rotational displacements. The resolution of the dynamic problem is incremental. The configuration and the motion of the body are identified using a moving reference configuration representing the position of the deformable body in the preceding step. Azouz & al. [10] propose as reference configuration a rigid body configuration which follows the motion of the body without coinciding with it. This approach is convenient for a body with small deformations. Using this approach, we develop an efficient formalism to describe the behaviour of airships with small deformations. The motion is given by coupled sets of rigid and elastic variables. The nonlinear equations are formulated in terms of a set of time invariant matrices expressed in a reference configuration (i.e. mass and stiffness matrices). Time-variant quantities appear in the nonlinear terms that represent the dynamic coupling between the rigid body modes and the elastic deformation. We show in this study a suitable technique to actualise these terms using matrix partitioning and canonical decomposition. The dynamic system is reduced and solved through a modal synthesis. The elastic deformation is represented by a set of generalised variables and natural modes issued from a Finite Element method (FEM) discretization.

Airships are also governed by the aerodynamic forces that have to be modelled. The basis to analyse the motion of a rigid body in a perfect fluid has been established in the 19th century and has been described by Lamb [11]. In his work, Lamb considered the case of simple displacement in a big infinite mass of fluid and where the movement of this last is entirely due to the motion of a solid, and it is irrotational and acyclic. He proved that the kinetic energy of the fluid can be expressed as a quadratic shape of the six velocities of translation and rotation of the vehicle. The derivations given by Lamb will be used in the description of the airship, in a stationary uniform atmosphere. The terms depending on the acceleration or the added masses come from the fact that the fluid considered perfect is accelerated. When an ellipsoid body moves in an incompressible and infinite inviscid fluid so that the external flow is everywhere irrotational and continuous, the kinetic energy of the fluid produces an effect equivalent to an important increase of the mass and of the moments of inertia of the body [12, 13]. In our model, the fluid effects have been introduced into the dynamical system through a modal synthesis to build the global dynamic system of the flexible airship**.**

2. Dynamic model of the airship

The dynamics of the airship with small deformation is highly non-linear. The non linearities are essentially due to the large rotations of the body and the interaction between rigid body motion and the deformation.

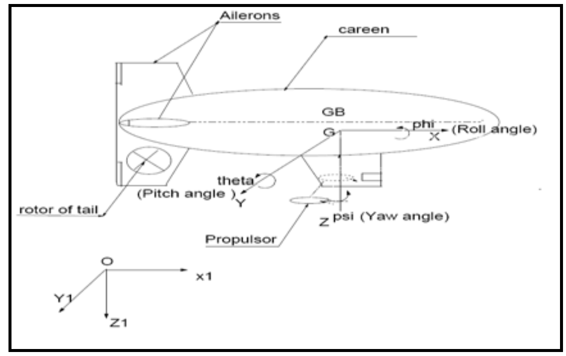

Figure 1. Presentation of the different frames

In our formulation, we use an Update Lagrangian Method. The description of the airship behaviour is made relatively to a reference configuration which changes in the time accordingly to the airship configuration. This method is chosen because it takes into account the buckling between the rigid body motion and the deformation, and could be coupled with results of classical structural dynamics. In fact, many dynamic parameters will be kept unchanged during the simulation. In the case of a small deformation, the configuration at the time t **(**$\mathbf{C_t}$) is not so different from the one obtained by the transformation of a rigid body in the initial configuration. We choose as a reference configuration, the configuration of a virtual rigid body following the motion. Hence, we can linearize the airship displacement around this configuration. We consider two frames in the derivation of the motion equations. These frames are: the fixed frame related to the earth $\mathbf{R_f}$ and the moving frame $\mathbf{R_m}$ related to the reference configuration **(**$\mathbf{C_{ref}}$). The position and the orientation of the vehicle should be described relatively to the reference frame. The origin **O'** of $\mathbf{R_m}$ coincides with the inertia centre of the undeformed vehicle in the reference frame. Its axes are the principal axes of symmetry.

2.1 Description of the reference configuration

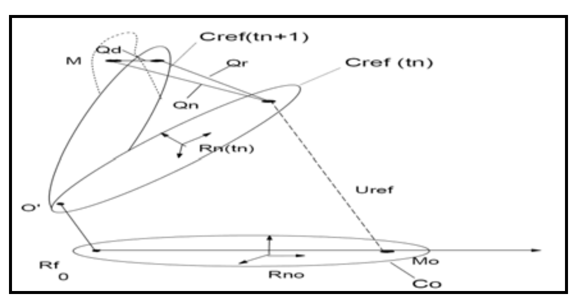

Figure 2. Incremental scheme

In this model we use an incremental scheme. The displacement of each point L is given by:

$$U(L) = U_{ref}(L) + \underbrace{\Delta U_{ref}(L) + U_d(L)}_{Q} \qquad (1)$$

Where U_d is the small displacement related to the whole deformation, ΔU_{ref} is the increment of the rigid body motion. Let us note H the rotation matrix between the initial frame and the local reference R_m such as $H^T H = H.H^T = I_d$, and let us note $\tilde{Q} = H^t Q$; from the equation (1) we can obtain the following expression:

$$\ddot{U} = \ddot{U}_{ref} + \left(H\ \ddot{\tilde{Q}} + 2\dot{H}\ \dot{\tilde{Q}} + \ddot{H}\ \tilde{Q} \right) \qquad (2)$$

$\dot{\tilde{Q}}, and\ \ddot{\tilde{Q}}$ represent the time derivative of the displacement $\tilde{Q}$.

2.2 Description of the strains

To describe the strains we use the symmetric Green-Lagrange tensor $\overline{\overline{\varepsilon}}$ which is related to the reference configuration. We note J the gradient tensor relative at the reference configuration co-ordinates $\overline{J} = \nabla \tilde{Q}$ and the Green tensor $\overline{\overline{\varepsilon}}$ can be expressed as: $\overline{\overline{\varepsilon}} = \frac{1}{2}\left[\left(\overline{J}^t + \overline{J} \right) + \overline{J^t.J} \right]$.

In this case we have small deformations, consequently $\overline{J} \ll 1$; we can then neglect the non linear terms, hence the strain tensor becomes:

$$\overline{\overline{\varepsilon}} = \frac{1}{2}\left[\overline{J}^t + \overline{J} \right].$$

When using the vector notation such as: $\underline{\varepsilon} = \left[\varepsilon_{11} \quad \varepsilon_{22} \quad \varepsilon_{33} \quad \varepsilon_{12} \quad \varepsilon_{13} \quad \varepsilon_{23} \right]^T$, we can obtain this relation:

$$\underline{\varepsilon} = D\tilde{Q} \qquad (3)$$

Where D is a differential operator, ε_{ij} are the components of the strain tensor.

2.3 Description of the stresses

For the description of the stresses applied on the airship, we use the second tensor of Piola-Kirchoff $\overline{\overline{P_k}}$ defined on the reference configuration $\overline{\overline{P_k}} = \frac{\rho^{ref}}{\rho} J^t.\overline{\overline{\sigma}}.J^{-t}$, with $\overline{\overline{\sigma}}$ is the stress tensor of Cauchy, $J = I_d + \overline{J}$, I_d is the 3x3 identity matrix, and ρ^{ref} is the mass density of the reference configuration equal to its initial value ρ^0 defined on the initial configuration (C^0).

The material of the careen is assumed elastic, homogeneous and isotropic such as we have a linear behaviour law between stresses and strains:

$$\underline{\sigma} = E\underline{\varepsilon} \qquad (4)$$

Here $\underline{\sigma}$ is the vector of the stress components and E the symmetric tensor of elastic properties of the material.

2.4 Dynamic equilibrium

Let's consider the dynamic equilibrium of an airship. According to the classical mechanics laws, the equilibrium is represented by the two following relations [14]:

$$\mathrm{div}\overline{\overline{\sigma}} + \mathrm{f_v} = \rho\,\ddot{\mathrm{U}} \quad \text{on } (C^t) \qquad (5)$$

$$\overline{\overline{\sigma}}.\vec{\mathrm{n}} = \mathrm{t_n} \qquad \text{on board}\left(\partial C^t \right) \qquad (6)$$

f_v are the volumic forces acting on the elements of volume such as gravity. t_n is the boundary stress vector, it represents the boundary surfacic forces such as the air pressure.

Now, we consider the following assumption: $V \approx V_{ref}$ and $S \approx S_{ref}$ the volume and surface of the actual configuration (C^t) are close to those of the reference configuration. This permits to confuse the Piola stress tensor $\overline{\overline{P_k}}$ with the Cauchy stress tensor $\overline{\overline{\sigma}}$.

If we use the principle of the virtual works and consider a small displacement $\delta\tilde{Q}$, the equation of equilibrium becomes:

$$\int_{V_{ref}} \rho_{ref}.\ddot{U}^T.\partial\tilde{Q}.dV_{ref} + \int_{V_{ref}} \underline{\sigma}^T.\partial\underline{\varepsilon}.dV_{ref} = \int_{V_{ref}} f_v^T.\delta\tilde{Q}.dV_{ref} + \int_{S_{ref}} t_n^T.\partial\tilde{Q}.dS_{ref} \qquad (7)$$

Consequently, the equation (7) is similar to the dynamic equation of the deformable bodies in structural mechanics.

3. Resolution of the dynamic equation

To define the exact configuration of the deformable airship, we should use an infinite number of co-ordinates in order to calculate the location of every point of the body. It has been shown that the motion of the deformable airship is governed by a set of partial differential equations that are space and time dependent. The discretization consists to subdivide the airship in a number of sub-domains with simple shapes that could be studied easily. We will use approximate techniques namely Rayleigh-Ritz method coupled with Finite Elements discretization.

3.1 Discretization of the airship

We indicate that the discretization consists to subdivide the airship in a number of sub-domains with simple shape. The displacement of the airship is then described through some interpolations functions and nodal displacement. We must note that the number of elements required for an acceptable approximation of the displacement should be high. However when using modal synthesis, this number of elements play a part only in a preliminary computation. Just a few numbers of "useful" nodes are kept.

We should note that the nacelle is represented by three points in the bottom of the airship. In the central point we apply its actual heavy, in the other point we model the effect of the two propellers.

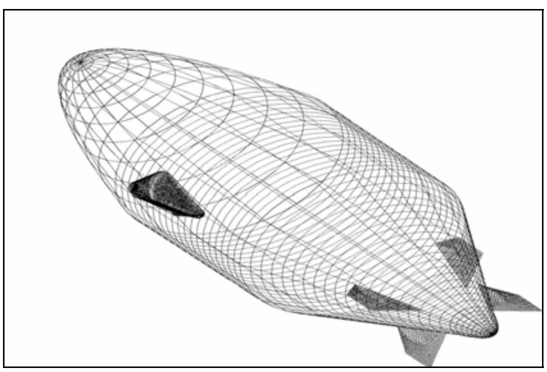

Figure 3. Discretization of the airship

Let us now consider a displacement field U^i of an element i of the body. It can be expressed in function of the column matrix $\bar{u}$ representing the displacement, relatively to the local reference frame, of all nodes as follows:

$U^i = N^i\bar{u}$ where N^i is an interpolation space function [15]. This relation is valid for any given displacement, especially for:

$$\tilde{Q} = N.\bar{q} \tag{8}$$

We note M^i and K_L^i the mass and stiffness matrices of the element i defined as follows:

$$M^i = \int_{V_i} \rho_{ref}.N^{iT}.N^i.dV^i_{ref} \tag{9}$$

$$K_L^i = \int_{V_i} N^{iT}.D^T.E^i.D^i.N^i.dV^i_{ref} \tag{10}$$

V^i and E^i are respectively the volume and the tensor of the elastic properties of the element i,
D^i is the differential operator of an element i.

The mass and stiffness matrices of the whole airship are built by an adequate assembly of elementary matrices of mass and stiffness:

$$M = \sum_{i=1}^{ne} M^i \tag{11}$$

$$K_L = \sum_{i=1}^{ne} K_L^i \tag{12}$$

ne is the number of elements of the airship.

3.2 Equation of motion of the discretized airship

Lets us now analyse the different terms of the equilibrium equation (7).
According to the equation (2), the first left hand term could be expressed as:

$$\int_{V_{ref}} \rho_{ref}.\ddot{U}^T.\delta\tilde{Q}dV_{ref} = \int_{V_{ref}} \rho_{ref}.\ddot{U}^T_{ref}.\delta\tilde{Q}dV_{ref} + \int_{V_{ref}} \rho_{ref}.\left(H\ddot{\tilde{Q}}+2\dot{H}\dot{\tilde{Q}}+\ddot{H}\tilde{Q}\right)^T.\delta\tilde{Q}dV_{ref} \tag{13}$$

We just add here that, in the incremental scheme the reference acceleration $\ddot{U}_{ref}$ related to the reference configuration is known.

Using the equations (3) and (7), we can write the second left hand term as:

$$\int_{V_{ref}} \underline{\sigma}^T.\partial\underline{\varepsilon}.dV_{ref} = \int_{V_{ref}} \bar{q}^T.N^T.D^T.E.(D.N).\partial\bar{q}.dV_{ref} \tag{14}$$

Relations (3; 8-13) give :

$$K.\bar{q} + B.\dot{\bar{q}} + M.\ddot{\bar{q}} = \tilde{F}_e + \tilde{F}_f - \tilde{F}_r \tag{15}$$

We define here the diagonal bloc matrix of rotation $\bar{H}$ which concerns the rotation of all the nodes:

$$\left[\overline{\mathrm{H}}\right]=\begin{pmatrix}[\mathrm{H}] & & & & & \\ & [\mathrm{H}] & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & [\mathrm{H}]\end{pmatrix} \tag{16}$$

The stiffness matrix K is constituted by:

K_L the structural stiffness, and $K_D = M\,\overline{H}^T\ddot{\overline{H}}$ the dynamic stiffness due to the Coriolis effect. $B = C_s + C_g$ with $C_g = 2M\,\overline{H}^T\dot{\overline{H}}$ the anti-symmetric matrix of gyroscopic terms; C_s is a matrix of structural damping. We suppose in the following that we can neglect this matrix is front of C_g. K_D and B are consistent matrices which take into account the inertial coupling between the overall motion and the deformation. $\tilde{F}_e = \overline{H}^T F_e$, $\tilde{F}_f = \overline{H}^T F_f$ are respectively the external (volumic and boundary forces) and fluid forces; $\tilde{F}_r = M\,\overline{H}^T \ddot{\overline{u}}_{ref}$ is the column matrix of residual inertia terms due to the dynamics of the reference configuration.

The structural stiffness and mass matrices are defined relatively to a moving reference frame $\mathbf{R_m}$ in the “rigid” reference configuration. They are then constant. The dynamic equation (15) has a huge size and the number of degree of freedom (d.o.f) is tremendous, especially if we choose to model the airship with small elements; this leads to a very high computational cost and many problems to establish control laws. For this reason, we tried to solve this dynamic equation using a modal synthesis.

3.3 Modal synthesis

We decompose the increment $\tilde{Q}$ into a rigid contribution $\tilde{Q}_r$ and a deformable part $\tilde{Q}_d$ such as in its discretized form we have:

$$\overline{q}_r = \sum_{i=1}^{6} Y_{ri}\, X_{ri} \tag{17}$$

Y_{ri} being the modal amplitude of the rigid mode i. Hence for a 3D motion, we have six variables $\left(Y_{trax}, Y_{tray}, Y_{traz}, Y_{rotx}, Y_{roty}, Y_{rotz}\right)$.

$$\mathbf{X}_{rig} = \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ I_{d_{3x3}} & \widehat{O'L} \\ 0 & I_{d_{3x3}} \\ \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \tag{18}$$

$$\underbrace{}_{X_{tra}} \quad \underbrace{}_{X_{rot}}$$

X_{tra} : are three modes of the translation motion.
X_{rot} : are the three modes of rotation.
The displacement due to the deformation of the airship is:

$$\overline{q}_d = \sum_{i=1}^{nd} Y_{di} X_{di} \tag{19}$$

nd: is the number of significant deformable modes kept for the study, Y_{di} is a time dependent variable of the deformable mode i, X_{di} is a free-free natural mode of rank i. The choice of these boundary conditions is in harmony with the real configuration of the airship. O’ is the origin of $\mathbf{R_m}$, L is a given nodal point, $\widehat{O'L}$ is the skew matrix associated with the vector $\overrightarrow{O'L}$.

The deformable airship could be considered as low frequency structure. We can model its vibration accurately with the first modes of lower frequencies.
The projection of dynamical equation of motion (15) in the modal basis composed of rigid and flexible modes permits to reduce the number of d.o.f. The dynamic equation can then be written as:

$$\mathbf{M}.\ddot{Y} + \mathbf{B}.\dot{Y} + \mathbf{K_D}\,Y + \mathbf{K_L}.Y = \overline{\mathbf{F}}_e + \overline{\mathbf{F}}_f - \overline{\mathbf{F}}_R \tag{20}$$

$\mathbf{M}$ and $\mathbf{K_L}$ are the constant mass and stiffness matrices projected in the modal basis ($\mathbf{M} = \mathbf{X}^T$.M.$\mathbf{X}$), $\mathbf{X}$ is the matrix of the modal basis issued from the collection of all rigid and flexible modes. The other bold terms are terms projected in the modal basis:

$$\mathbf{M} = \begin{bmatrix} M_{TT} & M_{TR} & M_{Td} \\ & M_{RR} & M_{Rd} \\ Sym & & M_{dd} \end{bmatrix} \quad \text{and}$$

$$\mathbf{K_L} = \begin{bmatrix} 0 & 0 & 0 \\ & 0 & 0 \\ Sym & & K_{dd} \end{bmatrix}$$

The mass matrix is defined in the “rigid” reference configuration. We have then as sub-matrix of translation (M_{TT}) a diagonal matrix corresponding to the mass of the airship. M_{RR} is the inertia matrix of rotation; and according that the centre of gravity coincides with the origin of the reference frame, we have $M_{TR} = 0$. The choice of free modes of deformation allows to say that $M_{Td} = M_{Rd} = 0$.

The use of constant mass and stiffness matrices is important for the reduction of the computational time. In the other hand, we obtain two time-variant matrices ($\mathbf{B}$ and $\mathbf{K_D}$) that we should actualise at each step. However we develop a computational technique which makes easier this task. For example a given matrix A can be written as:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \sum_{ij}\sum_{ij} a_{ij} b_{ij} = a_{11}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + a_{12}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \ldots$$

This decomposition is useful for our block-diagonal matrices such as:

$$[\bar{H}] = \begin{pmatrix} [H] & & & & & \\ & [H] & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & [H] \end{pmatrix} = \sum_{i=1}^{3}\sum_{j=1}^{3}\left[H_{ij}\right].A_{IJ} \qquad (21)$$

Here $A_{IJ} = \vec{e}_i \otimes \vec{e}_j$ are canonical matrices. The symbol $\otimes$ represents the tensorial product.

For example:

$$A_{12} = \begin{bmatrix} 0 & 1 & 0 & & & & & & \\ 0 & 0 & 0 & & & & & & \\ 0 & 0 & 0 & & & & & & \\ & & & 0 & 1 & 0 & & & \\ & & & 0 & 0 & 0 & & & \\ & & & 0 & 0 & 0 & & & \\ & & & & & & \cdot & & \\ & & & & & & & \cdot & \\ & & & & & & & & \cdot \end{bmatrix}$$

The matrix $\mathbf{K_D}$ can then be computed by this way:

$$\mathbf{K_D} = 2\mathbf{X}^T M.\bar{H}^T\ddot{\bar{H}}.\mathbf{X} = 2\sum_{i=1}^{3}\sum_{j=1}^{3}\left[H^T\ddot{H}\right]_{ij} Z_{IJ} \qquad (22)$$

where $Z_{IJ} = \mathbf{X}^T M A_{IJ} \mathbf{X}$ are constant matrices defined before the beginning of the time varying simulation of motion.

4. Interaction fluid-airship

Consider a simple dynamical model of the action of a fluid on a body. To present this model, we assume that the flow is quasi-steady, i.e. the distribution of the velocities of particles of the medium coincides with the distribution corresponding to the steady motion of the body. Thus the medium responds only to the current motion of the body and forgets its initial conditions. Therefore, within the framework of this hypothesis, the resultant force and torque acting on the body can be represented in the form of a function of the instantaneous distribution of velocities in this body. Thus we arrive at the statement of the problem of the motion of a body in a dragging medium as a problem of classical dynamics.

4.1 Flow representation

To take into account the interaction of the airship with the surrounding fluid medium, a model of the flow is needed. Here, we rely on the potential flow theory corresponding to the following hypothesis:

a. the air can be considered as a perfect fluid with uniform density ρ_{air} , i.e. an incompressible gas with vanishing viscosity,
b. the flow is irrotational

c. Only the flow outside the airship contributes significantly to the aerodynamics forces.

Denoting by **v** the velocity field in the fluid domain Ω_{air}, the incompressibility and irrotational assumptions leads to:

$$\nabla.\mathrm{v} = 0 \quad ; \quad \nabla \wedge \mathrm{v} = 0 \qquad (23)$$

$\wedge$ is the vectorial product of two vectors. ∇ is the gradient symbol, and the flow field may be described in terms of a potential F such that:

$$\mathrm{v} = \nabla \Phi \qquad (24)$$

From the incompressibility constraint, it is easy to show that the potential obeys to the homogeneous Laplace equation:

$$\nabla^2 \Phi = 0 \quad \text{in } \Omega_{air} \qquad (25)$$

with Newman boundary conditions:

$$\nabla\Phi.\mathrm{n} = -\dot{\mathrm{q}}.\mathrm{n} = -([\mathbf{X}]\dot{\mathrm{Y}}).\mathrm{n} \qquad \text{on} \left(\partial C^t\right) \qquad (26)$$

[$\mathbf{X}$] is the matrix of all the modes (rigid and deformable modes), Y is the column matrix of all the time dependent variables defined in §3.4, **n** is a unit vector, normal to $\left(\partial C^t\right)$.

Thus, one of the important characteristic of this representation is that **v** only depends on the current boundary conditions, and not on the history of the flow: the model is quasi-steady. To solve the potential equation, we use the boundary integral representation of the Laplace equation, together with standard boundary element method. It consists in the determination of a piecewise constant distribution of singularities over $\left(\partial C^t\right)$ (see [16] for details on the numerical treatment).

4.2 Fluid forces

For this assumption, the pressure at any point in the fluid domain (including $\left(\partial C^t\right)$) is given by Bernoulli theorem:

$$P + \rho_{air}\left[\frac{1}{2}\mathrm{v}.\mathrm{v} + \frac{\partial \Phi}{\partial t}\right] = P_\infty + \rho_{air}.\frac{1}{2}\mathrm{v}_\infty.\mathrm{v}_\infty \qquad (27)$$

The subscript ∞ denotes the undisturbed conditions far from the airship. This pressure distribution over the airship surface can be integrated to compute the resulting forces and torques which in turn are projected on the modal basis. At the end, and with the linear property of the Laplace equation, the generalised fluid forces vector can be rewritten as:

$$F_f = -M_{ad}\ddot{\mathrm{Y}} - B_f\dot{\mathrm{Y}} \qquad (28)$$

where M_{ad} is the matrix of the added masses (virtual masses), and Y is the column matrix of all Y_i, B_f is a modal damping due to the flexibility of the hull.

4.3 Modal projection

Taking into account the previous developments, the dynamic equation (20) becomes:

$$\mathbf{M'}.\ddot{\mathrm{Y}} + \mathbf{B'}.\dot{\mathrm{Y}} + \mathbf{K_d}\mathrm{Y} + \mathbf{K}.\mathrm{Y} = \bar{\mathbf{F}}_e - \bar{\mathbf{F}}_R \qquad (29)$$

we note $\mathbf{M'} = \mathbf{M} + M_{ad}$, $\mathbf{B'} = \mathbf{B} + B_f$. The effect of the fluid on the structure is then represented mainly by the adjunction of the added masses matrix $\mathbf{M_{ad}}$ to the mass matrix of the structure.

Where

$$M_{ad} \simeq \begin{pmatrix} -k_1 m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_2 m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_2 m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_3 I_y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_3 I_y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

m is the mass of the body; k_1, k_2 and k_3 are constants depending of the shape of the airship, I_x, I_y and I_z are moments of the inertia's body.

For a quasi-ellipsoid airship the extra-diagonal terms of $\mathbf{M_{ad}}$ can be neglected (for more details about the constitutive terms of $\mathbf{M_{ad}}$, the reader can see [11, 1]).

5. Simulation results

To illustrate this incremental formalism we study the blimp belonging to the L.S.C having the following characteristics:

-The envelope:
Length: 6.25 m.
Diameter: 1.52 m.
Volume: 7.48 m^3.
-Mass of the airship: 5.8 Kg.
- Payload: 1.58 Kg.

The blimp is thrusted by two contrarotating propellers.

Figure 4. The LSC airship

To illustrate our incremental formalism, two kinds of motions are tested for this airship.

In the first step we apply to the blimp two opposite forces in the propellers to generate a yaw motion (Fig 7).The flexible airship should rotate about 90° around the z-axis. A P.I.D controller will impose this task, and we study the behaviour of the airship during this manoeuvre. The number of deformable modes kept is nd = 2. This number seems to give an acceptable approximation of the flexible behaviour. We were guided for that by the modal masses of these two modes which represent roughly 65% of the total mass of the airship.
In this simulation one applied around the yaw angle a torque consigns with:

$$f_{eg} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ k_d(\psi_d - \psi) + k_v(\dot{\psi}_d - \dot{\psi}) \end{pmatrix}$$

$K_d = 800$, $K_v = 50$, ψ_d : desired angular position and ψ : angular position of the airship.

One notices in figure 5 that the amplitude of deviation of the yaw angle decreases significantly in few seconds, it stabilizes oneself while merging with the instruction.

We show the position of the tip of the airship in figure 6.

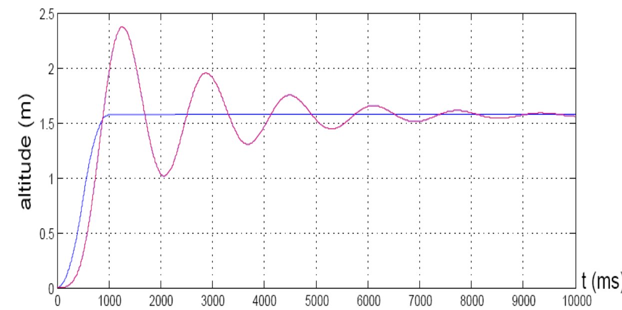

Figure 5. The superposition of trajectories (yaw angle and desired).

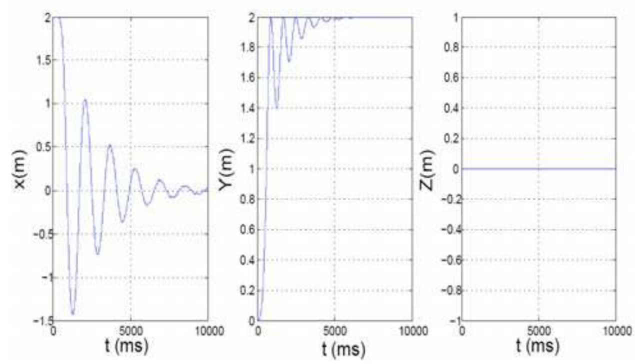

Figure 6. Position of the tip of the airship

In figure 7 one superimposes total displacement along the X-axis of the rigid airship and the flexible device. It is noticed that the flexible device continues to oscillate what proves the impact of flexibility on displacement. For the rigid behaviour we eliminate the two deformable modes and kept only the six rigid modes.

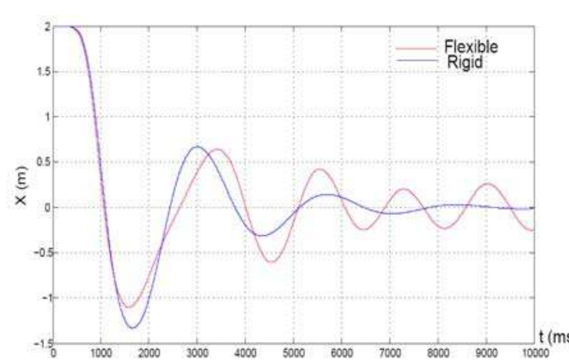

Figure 7. Superposition of the motions of the flexible and rigid airship

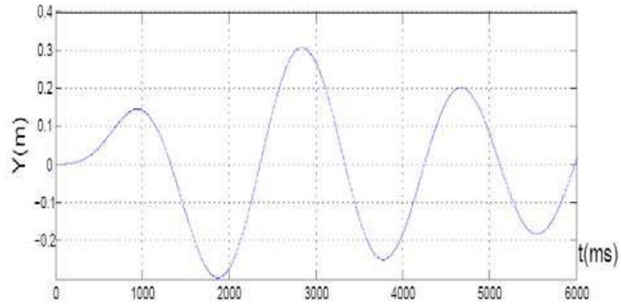

Figure 8. The deformation's displacement

The deformations (in figure 8) are about 0.15m what is more or less significant considering one has small deformations. In this simulation one visualized the tip of the airship, and one sees well the oscillation of the latter with non-negligible amplitude.One classical motion of an airship is the helicoidally motion. This usually represents the optimal trim trajectories.

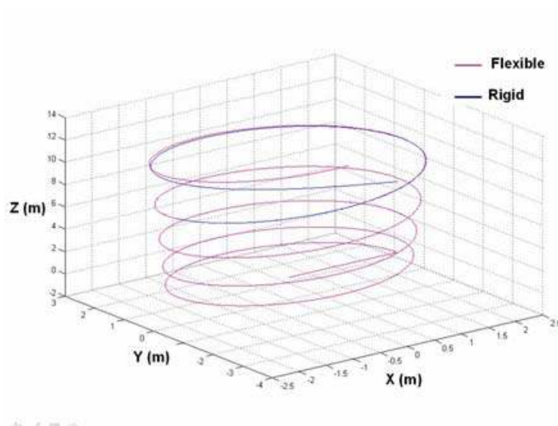

Figure 9. Rise of the airship.

This represents a complete aerial motion of the airship. The propellers were oriented adequately to assure a combined motion along the moving x-axis and z-axis and the tails were oriented to give a homogeneous yaw motion.

One can see (in figure 9) the influence of the deformation on the overall motion of the airship. This proves that in our model, the deformation is not a sample perturbation around the main rigid motion, but it acts on this motion.
Experimental data will be available in few months to validate the results of our incremental scheme.

Finally we show the influence of the added masses on the total displacement of the airship.

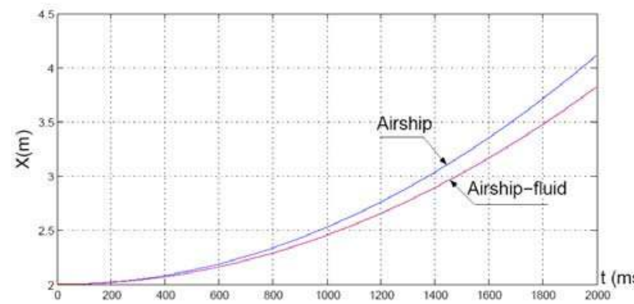

Figure 10. Superposition of the displacement of the airship with and without air environing

It is noticed (in figure 10) that the immersed airship has a delay compared to the body alone and this is because of the addition of the virtual masses representing the influence of the displacement of the mass of air around the airship when this last accelerates.

6. Conclusion

The study of the flexible structures in the space has different performance requirements which may lead to troublesome calculation and inextricable problems of controllability.

We introduced in this paper the flexibility of the airship as an extension of the classical structural dynamics taking into account the coupling between the rigid body motion and the deformation.

The Updated Lagrangian Method (U.L.M) applied on rigid reference configuration and the uses of an updated modal synthesis permit to diminish the dynamic equations of the flexible airship to a reduced set of degrees of freedom.

Simulation results prove that the integration of the flexibility in the dynamic system of the airship is very important and could not be neglected. The influence of surrounding air is also taken into account in our global model by the mean of added masses and through the same modal algorithm.

The effect of surrounding air with the flexibility of the hull will be treated in future works.

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