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# Trajectory Generation and Tracking of a Mini-Rotorcraft\*

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**Abstract**— We present in this paper the stabilization (tracking) with motion planning of the six independent configurations of a mini unmanned aerial vehicle equipped with four streamlined rotors. Naturally, the yaw-dynamic can be stabilized without difficulties and independently of other motions. The remaining dynamics are linearly approximated around a small roll and pitch angles. It will be shown that the system presents a flat output that is likely to be useful in the motion generation problem. The tracking feedback controller is based on receding horizon point to point steering. The resulting controller involves the lift (collective) time derivative for what flatness and feedback linearization are used. Simulation tests are performed to progress in a region with approximatively ten-meter-buildings.

**Index Terms**— Mini-UAV, tracking control, flatness, motion planning.

## I. INTRODUCTION

Unmanned aerial Vehicles (UAV) terrain mission control is a matter of both interest and controversy for scientific research and engineering design. A large class of industrial and military control constraints consist in planning and following predefined trajectories. Examples range from unmanned and remotely piloted airplanes and submarines performing surveillance and inspection, mobile robots moving on factory floors and multi-fingered robot hands performing inspection and manipulation inside the human body under a surgery control. All these systems are highly nonlinear and require accurate performance.

Modelling and controlling aerial vehicles (blimp [2], mini rotorcraft [3]) are the principal preoccupation of the *Lsc*, *Lim*- groups. In [2] authors are proposed a time-varying feedback control law for the blimp. The stabilization of the X4 mini-flyer is studied in [3]. In this paper we stretch our results to point to point control with motion planning. Further we consider the dynamic of the rotors and the stabilisation of their angular velocities. Within this optic, a mini-UAV was constructed by the *Lsc*-group taking into account the industrial constraints. The aerial flying engine couldn't exceed  $2kg$  in mass with a wingspan of  $50cm$  and a  $30mn$  autonomy flying-time. So it can be held that our system belongs to a family of mini-UAV. A mini-flyer with streamlined rotors and blades was envisaged by the

group. It is an autonomous hovering system, capable of vertical take-off, landing and quasi-stationary (hover or near hover) flight conditions. Compared to helicopters, named quad-rotor, [8], [12], [1] the four-rotor rotorcraft has some advantages [11], [6]: given that two motors rotate counter clockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend, in trimmed flight, to cancel. An X4-flyer operates as an omnidirectional UAV. Vertical motion is controlled by collectively increasing or decreasing the power for all motors. Lateral motion, in  $x$ -direction or in  $y$ -direction, is achieved by differentially controlling the motors generating a pitching/rolling motion of the airframe that inclines the collective thrust (producing horizontal forces) and leads to lateral accelerations.

A model for the dynamic and configuration stabilization of quasi-stationary flight conditions of a four rotor vertical take-off and landing (VTOL) was studied by Hamel [6] where the dynamic motor effects are incorporating and a bound of perturbing errors was obtained for the coupled system. The stabilization problem of a four rotor rotorcraft is also studied and tested by Castillo [4] where the nested saturation algorithm is used. The idea is to guarantee a bound of the roll and pitch angles with a fixed bounded in control inputs. With the intent to stabilize aircrafts that are able to take-off vertically as helicopters, the control problem was solved for the planar vertical take-off and landing (PVTOL) with the input/output linearization procedure [7] and theory of flat systems [5], [9], [10].

The paper presents as follows: the translational and rotational motions, described by the Newton-Euler formalism, are detailed in section II. Section III deals with the flatness of the system and the way the reference motion is scheduled. The stabilization of the relative equilibrium is addressed in section IV where the stability of altitude/attitude motion is accomplished. A strategy to solve the tracking problem through point to point steering is shown in section V; incorporating the real time control and the trajectory realization. Finally, simulation tests, results and comments are put at work.

## II. CONFIGURATION DESCRIPTION AND MODELLING

The X4-flyer is a system consisting of four individual electrical fans attached to a rigid cross frame. It is an omnidirectional (vertical take-off and landing) VTOL vehicle ideally suited to stationary and quasi-stationary flight conditions. We consider a local reference airframe

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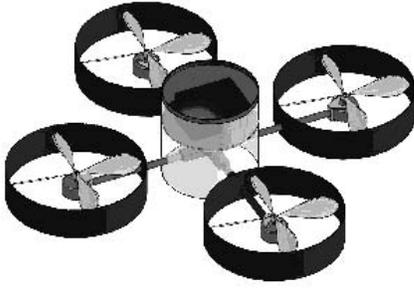


Fig. 1. General view of the four rotors rotorcraft.

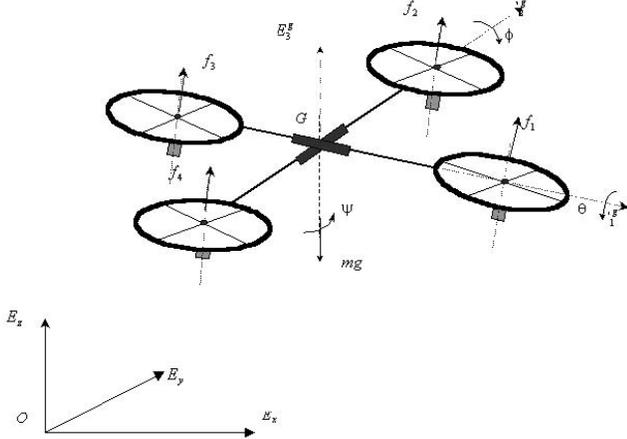


Fig. 2. Frames attached to the four rotor rotorcraft.

$\mathfrak{R}_G = \{G, E_1^g, E_2^g, E_3^g\}$  attached to the center of mass  $G$  of the vehicle. The center of mass is located at the intersection of the two rigid bars, each of which supports two motors. Equipment (controller cartes, sensors, etc.) onboard are placed not far from  $G$ . The inertial frame is denoted by  $\mathfrak{R}_o = \{O, E_x, E_y, E_z\}$  such that the vertical direction  $E_z$  is upwards. Let the vector  $\xi = (x, y, z)$  denote the position of the center of mass of the airframe in the frame  $\mathfrak{R}_o$ . While the rotation of the rigid body is determined by a rotation  $R: \mathfrak{R}_G \rightarrow \mathfrak{R}_o$ , where  $R \in SO(3)$  is an orthogonal rotation matrix. This matrix is defined by the three Euler angles,  $\theta$ (pitch),  $\phi$ (roll) and  $\psi$ (yaw) which are regrouped in  $\eta = (\theta, \phi, \psi)$ . A sketch of the X4-flyer is given in Fig.1 and Fig.2.

### A. Vehicle Dynamics

We consider the translation motion of  $\mathfrak{R}_G$  with respect to (wrt)  $\mathfrak{R}_o$ . The position of the center of mass wrt  $\mathfrak{R}_o$  is defined by  $\overline{OG} = (x \ y \ z)^t$ , the second time derivative permits to define the acceleration:  $\frac{d^2 \overline{OG}}{dt^2} = (\ddot{x} \ \ddot{y} \ \ddot{z})^t$ . The translational and rotational motions are summarized

as follow (further details are given in [3])

$$\begin{aligned} m\ddot{x} &= -us_\theta \\ m\ddot{y} &= uc_\theta s_\phi \\ m\ddot{z} &= uc_\theta c_\phi - mg \\ \ddot{\theta} &= \tilde{\tau}_\theta \\ \ddot{\phi} &= \tilde{\tau}_\phi \\ \ddot{\psi} &= \tilde{\tau}_\psi \end{aligned} \quad (1)$$

which is a set of nonlinear differential equation with drift.  $m$  denotes the vehicle total mass. The input  $u$  combines the principal non conservative forces applied to the X4-flyer airframe including forces generated by the motors (Fig. 2) and drag terms. Drag forces and gyroscopic due to motors effects will be not considered in this work. Then the lift (collective) force  $u$  is the sum of four forces

$$u = \sum_{i=1}^4 f_i \quad (2)$$

with  $f_i = k_i \omega_i^2$ .  $k_i > 0$  is a given constant (we consider  $k_i = k$ ) and  $\omega_i$  is the angular speed resulting of motor  $i$ . The system'input is such that  $u > 0$ .

The rotational motion are decoupled and linearized through the form given in (1). The initial torques are function of  $f_i$  [2]. Let  $l$  denotes the distance from  $G$  to motor  $i$ , then

$$\begin{aligned} \tau_\theta &= l(f_2 - f_4) \\ \tau_\phi &= l(f_3 - f_1) \\ \tau_\psi &= \kappa(f_1 + f_3 - f_2 - f_4) \end{aligned} \quad (3)$$

$\kappa$  is the yaw-torque appropriate coefficient.

The four inputs  $u$ ,  $\tilde{\tau}_\theta$ ,  $\tilde{\tau}_\phi$  and  $\tilde{\tau}_\psi$  will be calculated to success the point to point stabilization with motion planning. It is clear that the device belongs to families of underactuated systems. Compared to blimps [2], the model (1) is attractive for control objectives.

Note that in (1), the appropriate choice of  $\tilde{\tau}_\psi$  permits to stabilize  $\psi$  at any desired value  $\psi_d$  modulo  $2\pi$ . As well as for its first and second time derivatives. While  $\theta$  and  $\phi$  variables are limited to an open set defined by  $\pm \frac{\pi}{2}$ .

### III. MOTION PLANNING AND TRACKING

A system is flat if we can find a set of outputs (equal in number to the number of inputs) such that all states and inputs can be determined from these outputs without integration (see [10] and references there in).

*Proposition 1:* The X4-flyer described by the dynamic (1) is flat with  $\xi = (x, y, z)$  is its flat output.

*Proof:* First, we define the state by  $X = (x, y, z, \theta, \phi, \dot{x}, \dot{y}, \dot{z}, \dot{\theta}, \dot{\phi})$ , we denote  $\dot{X}$  its time derivative, and the input vector is regrouped in  $U = (u, \tilde{\tau}_\theta, \tilde{\tau}_\phi)$ , then the system can be written as

$$\dot{X} = f(X, U) \quad (4)$$

To prove that the state and the control vector are function of the flat output and their derivatives, for any given trajectory  $(x(t), y(t), z(t))$  smooth enough, we get ( $u > 0$ )

$$\begin{aligned} u &= m (\ddot{x}^2 + \ddot{y}^2 + (\ddot{z} + g)^2)^{\frac{1}{2}} \\ \phi &= \arctg \left( \frac{\ddot{y}}{\ddot{z} + g} \right) \\ \theta &= -\arctg \left( \frac{c_\phi \ddot{x}}{\ddot{z} + g} \right) \end{aligned} \quad (5)$$

Indeed,  $u, \theta, \phi, \dot{u}, \dot{\theta}$  and  $\dot{\phi}$  are function of  $\xi, \xi^{(2)}, \xi^{(3)}$ . So, it is straightforward to verify that  $X = \varphi(\xi, \xi^{(2)}, \xi^{(3)})$ . Moreover, we can derive  $\theta(t), \phi(t)$  and prove the  $\xi$ -dependence of  $\tilde{\tau}_\theta = \alpha_{\tilde{\tau}_\theta}(\xi^{(2)}, \xi^{(3)}, \xi^{(4)})$  and  $\tilde{\tau}_\phi = \alpha_{\tilde{\tau}_\phi}(\xi^{(2)}, \xi^{(3)}, \xi^{(4)})$ . ■

The relative equilibrium of the flying vehicle is subject of  $\ddot{x} = \ddot{y} = \ddot{z} = 0$  and  $\ddot{\theta} = \ddot{\phi} = \ddot{\psi} = 0$ . The explicit form given by equation (5) permits to write  $\theta = \phi = 0, u = mg$  and  $\tilde{\tau}_\theta = \tilde{\tau}_\phi = 0$ . Integrating motion planning, in the following we stabilize an equilibrium of the form  $(x_d, y_d, z_d, 0, 0, \psi_d)$ . The flatness property of the system will serve for the trajectory planning between the given initial flat output  $(\xi_i, t_i)$  and the final one  $(\xi_f, t_f)$  where  $t_i$  and  $t_f$  are the initial and final time, respectively. As we have demonstrated in section III, we can write all trajectories  $(X(t), U(t))$  satisfying the differential equation type (4) in terms of the flat output and its derivatives. In what following, we will see that time derivatives at fourth order of the flat output will be needed. In the simple stabilization control problem, *i.e.* without motion planning, time derivatives of the reference flat output are equal to zero. In our case, these derivations appear. Thus, our investigation can be viewed like case of tracking problem.

At first, we assume that  $(\theta, \phi) \in (0, 0)$  such that (1) can be transformed to

$$\begin{aligned} m\ddot{x} &= -u\theta \\ m\ddot{y} &= u\phi \\ m\ddot{z} &= u - mg \\ \ddot{\theta} &= \tilde{\tau}_\theta \\ \ddot{\phi} &= \tilde{\tau}_\phi \\ \ddot{\psi} &= \tilde{\tau}_\psi \end{aligned} \quad (6)$$

#### A. Altitude $z$ -stabilization and $\psi$ -control

The control of the vertical position (altitude) can be obtained considering the following control input

$$u = mg + m\ddot{z}_d - mk_1^z(\dot{z} - \dot{z}_d) - mk_2^z(z - z_d) \quad (7)$$

where  $k_1^z, k_2^z$  are the coefficients of stable polynomial and  $z_d$  is the desired altitude.

The yaw attitude can be stabilized to a desired value with the following tracking feedback control

$$\tilde{\tau}_\psi = \ddot{\psi}_d - k_1^\psi(\dot{\psi} - \dot{\psi}_d) - k_2^\psi(\psi - \psi_d) \quad (8)$$

where  $k_1^\psi, k_2^\psi$  are stable coefficients.

Indeed, introducing (7) into (6), we obtain

$$\begin{aligned} \ddot{x} &= -(g + f(z, z_d))\theta \\ \ddot{y} &= (g + f(z, z_d))\phi \\ \ddot{z} &= f(z, z_d) \\ \ddot{\theta} &= \tilde{\tau}_\theta \\ \ddot{\phi} &= \tilde{\tau}_\phi \\ \ddot{\psi} &= \ddot{\psi}_d - k_1^\psi(\dot{\psi} - \dot{\psi}_d) - k_2^\psi(\psi - \psi_d) \end{aligned} \quad (9)$$

where the function  $f(z, z_d) = \ddot{z}_d - k_1^z(\dot{z} - \dot{z}_d) - k_2^z(z - z_d)$  is assumed to be regular wrt to their arguments.

#### B. $x$ -stabilization and $\theta$ -control

As the output  $x$  is flat, then its dynamic is transformed in order to make appear the control  $\tilde{\tau}_\theta$ . Recall that

$$\ddot{x} = -(g + f(z, z_d))\theta \quad (10)$$

When one derive twice this expression, we get

$$x^{(4)} = -\ddot{f}(z, z_d)\theta - 2\dot{f}(z, z_d)\dot{\theta} - (g + f(z, z_d))\tilde{\tau}_\theta \quad (11)$$

*Proposition 2:* By the fact that  $g + f(z, z_d) = \frac{1}{m}u$ , which is by hypothesis positif definite as  $u > 0$  (see equation (2)), the asymptotic stability of  $x$ , consequently of  $\theta$  is asserted by (property of the flat output)

$$\tilde{\tau}_\theta = -\frac{1}{g + f(z, z_d)}(\nu_x + \ddot{f}(z, z_d)\theta + 2\dot{f}(z, z_d)\dot{\theta}) \quad (12)$$

with

$$\begin{aligned} \nu_x &= x_d^{(4)} - k_1^x(x^{(3)} - x_d^{(3)}) - k_2^x(\ddot{x} - \ddot{x}_d) - \\ &\quad - k_3^x(\dot{x} - \dot{x}_d) - k_4^x(x - x_d) \end{aligned} \quad (13)$$

$k_1^x, k_2^x, k_3^x, k_4^x$  are positives and stable coefficients.

*Proof:* Incorporating (12) into (11), it leads to the decoupled  $x$ -motion

$$x^{(4)} = \nu_x \quad (14)$$

further, taking  $\nu_x$  as given in (13),  $x$  and their successive time derivatives are asymptotically stable. It means, by virtue of the original system (1),  $\theta$  reaches its equilibrium ( $\theta(t_f) = 0$ ). ■

#### C. $y$ -stabilization and $\phi$ -control

As detailed above,  $\phi$  denotes the roll angle. This attitude has the same behavior like for  $\theta$ . Roll allure is necessary to the *X4-flyer* to correct motion in the  $y$ -direction. These variables are related by the cascade system

$$\begin{aligned} \ddot{y} &= (g + f(z, z_d))\phi \\ \ddot{\phi} &= \tilde{\tau}_\phi \end{aligned} \quad (15)$$

As before, we will proceed by four derivatives of the flat output  $y$  with respect to time

$$y^{(4)} = \ddot{f}(z, z_d)\phi + 2\dot{f}(z, z_d)\dot{\phi} + (g + f(z, z_d))\tilde{\tau}_\phi \quad (16)$$

*Proposition 3:* By the fact that  $g + f(z, z_d) = \frac{1}{m}u$ , which is by hypothesis positive definite as  $u > 0$ , the asymptotic stability of  $y$ , consequently of  $\phi$  is such that

$$\tilde{\tau}_\phi = -\frac{1}{g + f(z, z_d)}(\nu_y + \ddot{f}(z, z_d)\phi + 2\dot{f}(z, z_d)\dot{\phi}) \quad (17)$$

with

$$\begin{aligned} \nu_y = & y_d^{(4)} - k_1^y(y^{(3)} - y_d^{(3)}) - k_2^y(\dot{y} - \dot{y}_d) - \\ & - k_3^y(\dot{y} - \dot{y}_d) - k_4^y(y - y_d) \end{aligned} \quad (18)$$

$k_1^y, k_2^y, k_3^y, k_4^y$  are positives and stable coefficients.

*Proof:* Incorporate (17) into (16), it leads to a decoupled  $y$ -motion

$$y^{(4)} = \nu_y \quad (19)$$

further, taking  $\nu_y$  as given in (18),  $y$  and their successive time derivatives are asymptotically stable. It means, by virtue of the original system (1),  $\phi$  reaches its equilibrium ( $\phi(t_f) = 0$ ).

*Remark 1:* The proposed stabilizing controllers  $\tilde{\tau}_\theta$  and  $\tilde{\tau}_\phi$  involve the first and second time derivatives of  $f(z, z_d)$ . We can easily calculate it from (7) and (1). Therefore,  $\dot{f}(z, z_d) = z_d^{(3)} + ((k_1^z)^2 - k_2^z)\dot{e}_z + k_1^z k_2^z e_z$  and  $\ddot{f}(z, z_d) = z_d^{(4)} - ((k_1^z)^3 - 2k_1^z k_2^z)\dot{e}_z - ((k_1^z)^2 k_2^z - (k_2^z)^3)e_z$ .

#### IV. TRAJECTORY GENERATION AND POINT TO POINT STEERING

Due to the structure limit of the *X4-flyer*: motion can be asserted only in straight line along the  $x$ ,  $y$  and  $z$  directions. In our case, that is sufficient to navigate in a region. Otherwise, an other version of the engine is under study by the group. The version flyer is to make easy maneuvers in corners with arc of circle. In the following, we solve the tracking problem as point to point steering one over a finite interval of time. Then we take each ending point with its final time as a new starting point. Figure 4 illustrates the reference trajectory along the  $x$ ,  $y$  and  $z$  directions. As we see, the *X4-flyer* will fly in the  $z$ -direction followed by the  $x$ -motion and the  $y$ -motion. The reference trajectory is parameterized as

$$z^r(t) = h_d \frac{t^5}{t^5 + (T_f^1 - t)^5} \quad (20)$$

where  $h_d$  is the desired altitude and  $T_f^1$  the final time. In order to solve the point to point steering control, the end point of the trajectory (20) can be adopted as an initial point to move along  $x$ , then we have

$$x^r(t) = h_d \frac{(t - T_f^1)^5}{(t - T_f^1)^5 + (T_f^2 - (t - T_f^1))^5} \quad (21)$$

As soon as for  $y^r(t)$

$$y^r(t) = h_d \frac{(t - T_f^2)^5}{(t - T_f^2)^5 + (T_f^3 - (t - T_f^2))^5} \quad (22)$$

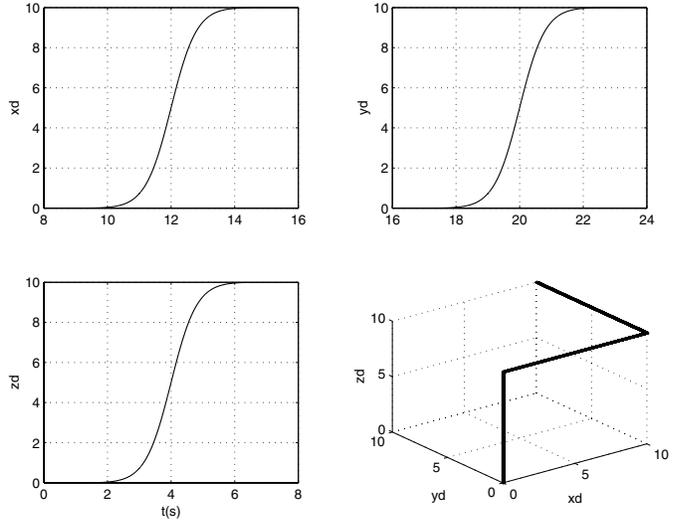


Fig. 3. Motion planning with  $h_d = 10m$ .

Constraints to perform these trajectories are such that

$$\begin{aligned} z^r(0) = x^r(T_f^1) = y^r(T_f^2) &= 0 \\ z^r(T_f^1) = x^r(T_f^2) = y^r(T_f^3) &= h_d \\ \dot{z}^r(0) = \dot{x}^r(T_f^1) = \dot{y}^r(T_f^2) &= 0 \\ \dot{z}^r(T_f^1) = \dot{x}^r(T_f^2) = \dot{y}^r(T_f^3) &= 0 \\ \ddot{z}^r(T_f^1) = \ddot{x}^r(T_f^2) = \ddot{y}^r(T_f^3) &= 0 \\ \ddot{z}^r(0) = \ddot{x}^r(T_f^1) = \ddot{y}^r(T_f^2) &= 0 \end{aligned} \quad (23)$$

minimizing the time of displacement implies that the *X4-flyer* accelerates at the beginning and decelerates at the arrival.

#### A. Real time control and velocities investigation

From experimental point of view, we consider a flystick with six degrees of freedom to animate the *X4-flyer*. This material is interpreted as a commanded position/orientation signal helping the user to progress in a hostile environment. The user will be informed about the vehicle positions by a visual feedback. The study of visual feedback involves image based visual servo control. This investigation would be the subject of future work. In this paragraph, we incorporate relations between torques, motor velocities and the command referenced positioning. Recall that the *X4-flyer* equipped with four brushless dc-motors which are commanded in voltages (currents) and not directly in torques. Brushless motors deliver high rate, largely boarded on mini flying machines. The variation of current permits to adjust speeds and forces given by relations (2) and (3). Feasible trajectories are subject of tests on limits and constraints related by (5). In order to interpret the control in term of velocities, recall that (2) and (3) permit to write

$$\begin{pmatrix} u \\ \tau_\theta \\ \tau_\phi \\ \tau_\psi \end{pmatrix} = k \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ 1 & -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix} \triangleq k \Upsilon \omega^2 \quad (24)$$

The proposed control law in  $(u, \tau_\theta, \tau_\phi, \tau_\psi)$  gives an  $\bar{\omega}^2 = (\bar{\omega}_1^2, \bar{\omega}_2^2, \bar{\omega}_3^2, \bar{\omega}_4^2)$  different of  $\omega_i^2$  ( $i = 1, 4$ ) developed by the actuators. Differences are due to the presence of motor dynamics which should be integrated to the system equations. Such an idea allows to control the system in velocities. To do so, let it be born in mind that the motor shaft dynamic is connected to the rigid body dynamics via the velocity component ( $i = 1, \dots, 4$ )

$$I_r \dot{\omega}_i = \tau_{mi} - k \bar{\omega}_i^2 \quad (25)$$

where  $k \bar{\omega}_i^2 = \sum_{j=1}^4 \Upsilon_{ij}^{-1} \Gamma_j$  and we assume that  $\ddot{\bar{\omega}}_i = \dot{\omega}_i$ . The constant  $I_r$  represents the shaft inertia and  $\tau_{mi}$  is the torque transmitted by the shaft (assumed to be rigid). Perturbation due to frictions and/or backlash can be easily incorporated in the model. The following analysis shows that such an undesirable phenomenon influences accuracy in motion. Then it should be compensated by the control  $\tau_m$ .

Given the reference flat output with their derivatives  $(\xi_d, \dot{\xi}_d, \ddot{\xi}_d, \xi_d^3, \xi_d^4, \xi_d^5)$ , the reference velocities obtained from (24) ( $l, k > 0$ ) verify

$$\begin{pmatrix} \bar{\omega}_{1d}^2 \\ \bar{\omega}_{2d}^2 \\ \bar{\omega}_{3d}^2 \\ \bar{\omega}_{4d}^2 \end{pmatrix} = k^{-1} \Upsilon^{-1} \begin{pmatrix} u^d \\ \tau_\theta^d \\ \tau_\phi^d \\ \tau_\psi^d \end{pmatrix} \triangleq k^{-1} \Upsilon^{-1} \Gamma_d \quad (26)$$

where  $u^d$  is given by (5) with  $\xi$  is replaced by  $\xi_d$ . The other elements of  $\Gamma_d$  are in (1) where we substitute current states by the reference ones. Identically in (25); the shaft reference velocity should verify ( $\dot{\bar{\omega}}_{id} = \dot{\omega}_{id}$ )

$$I_r \dot{\omega}_{id} = \tau_{mi}^d - k \bar{\omega}_{id}^2 \quad (27)$$

Therefore, we have

$$\begin{aligned} I_r (\dot{\omega} - \dot{\omega}_d) &= \tau_m - \tau_m^d - k (\bar{\omega}^2 - \bar{\omega}_d^2) \\ &= \tau_m - \tau_m^d - \Upsilon^{-1} (\Gamma - \Gamma_d) \end{aligned}$$

With the proposed input  $\tau_m$

$$\tau_m = \tau_m^d + \Upsilon^{-1} (\Gamma - \Gamma_d) - k_\omega (\omega - \omega_d) \quad (28)$$

such that  $k_\omega > 0$ , we can assert the convergence of  $w$  to  $w_d$ . Moreover  $\tau_m \rightarrow \tau_m^d$  and  $\Gamma \rightarrow \Gamma_d$ .

## V. SIMULATIONS

Recall that the objectives consist in testing the point to point stabilizing configuration. What we need to compare is the tracking problem with and without motion planning. Motion generation is described here by an important and limited acceleration in ascent following with an important deceleration which permits to reach at  $t_f$  the desired point. The generated motion could satisfy  $\xi_d(t_i) = \dot{\xi}_d(t_i) = 0$  and  $\xi_d(t_f) = \xi_d$ ,  $\dot{\xi}_d(t_f) = 0$ . The final time  $t_f$  shouldn't be reduced enough to limit an excessive reference acceleration. Without motion planning  $\xi_d$  can't be more then  $1m$ ,

otherwise the system diverges. Tests have been effectuated as follows: for  $\xi_d = \xi_d(t_f) = 1m$  ( $t_f = 4s$ ) (with and without motion planning) and  $\xi_d(t_f) = 10m$  ( $t_f = 8s$ ) (only with motion planning). All control parameters are  $k_1^z = 8$ ,  $k_2^z = 16$ ,  $k_1^x = k_1^y = 20$ ,  $k_2^x = k_2^y = 150$ ,  $k_3^z = k_3^y = 500$  and  $k_4^x = k_4^y = 625$ . The masse is  $m = 2kg$ .

### A. Results and comments

Let the desired configuration  $\xi_d(x_d, y_d, z_d) = 10m$ . We show in Fig.4 the behavior of the tracking errors. Without motion planning, the controller amplitudes are important (Fig. 5), chattering dominates the behavior of inputs when the system leaves its initial configuration. With a predefined path a minimum of energy is asserted which is requested for flying vehicles. Motion in different directions  $z$ ,  $x$  and  $y$  is also tested and shown by Fig.7. With motion planning, we can assert a good behavior of the X4-flyer even in presence of drag forces. Drag forces  $(0.5\dot{x}, 0.5\dot{y}, 0.5\dot{z})$  influence motion along the  $x$  and  $y$  directions (Fig.8), but with a good allure of motion. Simulation tests are accomplished with Fig.6 where we prove the well tracking of shafts velocities. Recall that these velocities are subject of motion planning integrating constraints of the system. The simulated parameters, and used in Fig.6, are  $I_r = k = 1$  and  $k_\omega = 1000$ .

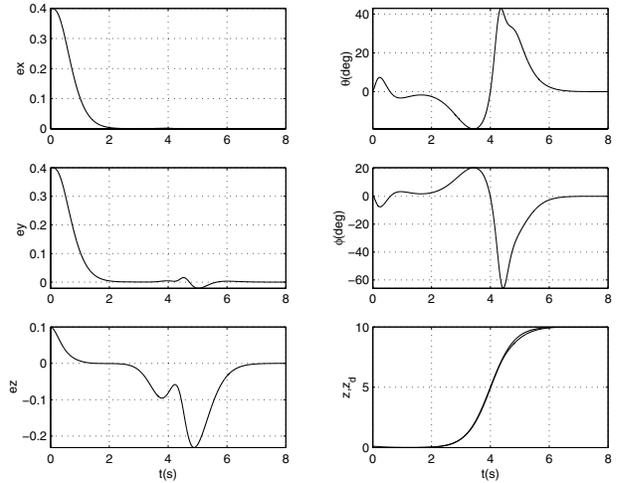


Fig. 4. Stabilization errors with motion planning ( $h_d = 10m$ ).

## VI. CONCLUSIONS

We have presented a rotorcraft with streamlined four rotors. The dynamic model involves four control inputs used to stabilize the engine with predefined paths. The

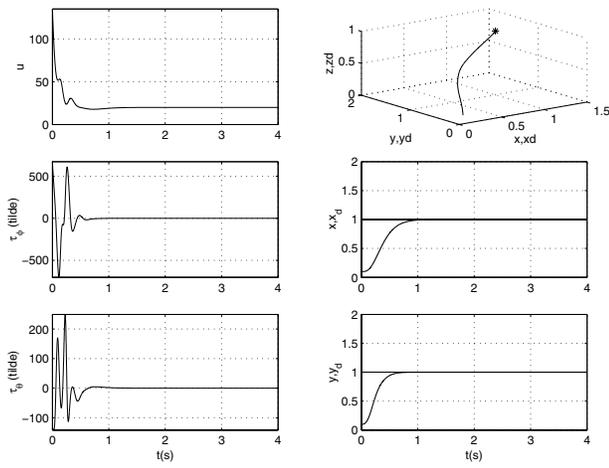


Fig. 5. Necessary inputs to stabilize ( $h_d = 1m$ ) without motion planning.

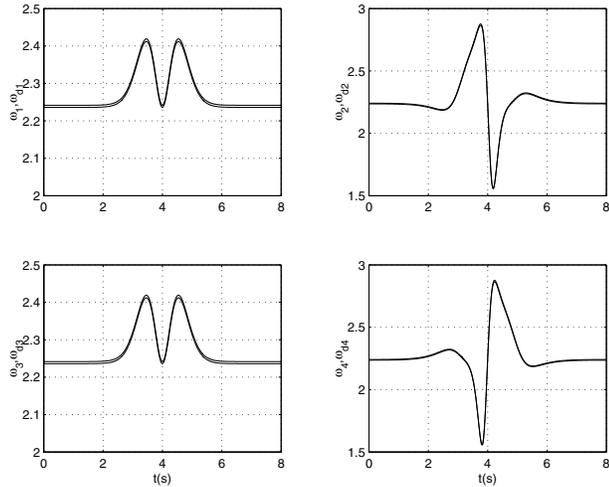


Fig. 6. Behaviour of rotors velocities compared to reference velocities (motion within the  $x$ -direction).

system presents a flat output which was efficiency exploited in motion planning, in point to point stabilization and in tracking control with respect to a region with ten-meter-buildings. It was shown that the algorithm is sensible to the necessary final-time of the reference trajectory. Due to limits in autonomy of batteries in fly, acceleration/deceleration of the vehicle in motion is justified. The proposed control law is extended to the actuator dynamics which permits to control the shaft and the blade velocities. With the proposed motion planning, actuator saturations can be overcome, consequently economy in energy of batteries can be asserted during the fly. This work will be extended to systems with delay and flatness based-visual feedback control.

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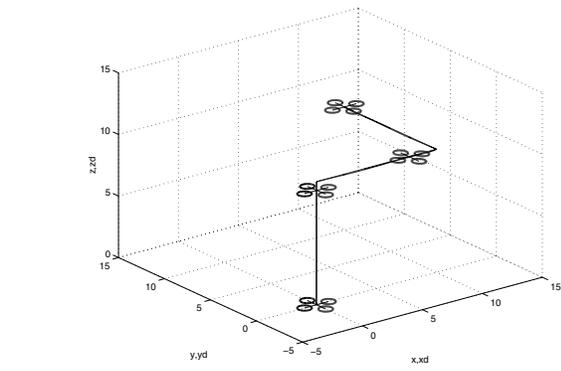


Fig. 7. Point to point steering control with motion planning and without perturbed model.

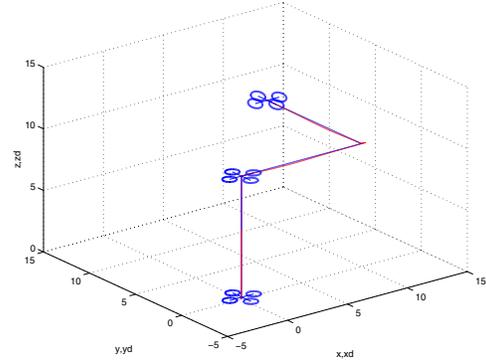


Fig. 8. Point to point steering control with motion planning and perturbed model.

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