Receding horizon climate control in metal mine extraction rooms

Guillaume Sandou, Emmanuel Witrant, Sorin Olaru, Silviu-Iulian Niculescu

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***Abstract*— This papers proposes a novel climate control strategy for mine extraction rooms based on the receding horizon optimal control scheme. Being a model-based procedure, the development of a pertinent prediction model is one of the keystones. According to recent technological advances, we consider that distributed measurements are available and provided by a wireless network. An enhanced modeling approach, based on stratification and sigmoid description of concentrations in the extraction rooms, is then proposed and allows for an optimal use of information provided by the wireless sensor network (WSN). The complexity of the resulting model, due to the nonlinearities, different time scales and time-delays, is handled by using an on-line shape prediction, included in the design of an optimal sequence of control actions over a finite horizon. Physical and communication constraints are successfully handled at the design stage and the resulting closed-loop system is robust with respect to variations in the pollutant dynamics.**

I. INTRODUCTION

The mining ventilation control is seen as a challenging automation problem with objectives that rise several research problems of immediate actuality, such as the wireless automation and the control of complex interconnected system. Indeed, the system considered is composed of the interconnection of fans, tarpauline tubes, extraction rooms and a wireless network. The complexity arises from the different physical properties - and associated dynamics - of the subsystems. In a broader picture, all these engineering problems imply to deal with fluid models and the connection of different subsystems. Global control strategies are of prime importance to deal with such problems. Indeed, it has been established in [1] that the savings associated with global control strategies for fluid systems (pumps, fans and compressors) represent 22.20 % of the total manufacturing motor system energy savings.

A. Fluid models and control over WSN

Model-based control strategies clearly have a significant advantage, to ensure optimized performances and handle classical control problems, such as actuation and communication constraints, disturbances rejection and energy minimization. In order to provide a global control strategy for a large-scale interconnected system, we first present a simplified fluid model that makes use of the available distributed measurements. The aerodynamics in the room environment is mainly set by the gas buoyancy and resulting stratification. In the extraction rooms, the pollutant sources from trucks can be considered as forced plumes, the incoming fresh air as a jet and the gas in the room as a stratified flow. Specific simplified fluid models have been proposed to represent such flows in [2], [3], which gave rise to an active research field in fluid mechanics. The development of *Intelligent Buildings* automation, and particularly *Underfloor Air Distribution* systems [4], renewed the interest in these models, due to the associated potential costs and energy savings (buildings air conditioning currently represent 10 % of all energy use in the United States).

The novel modeling approach presented in this paper is primarily motivated by the shapes similarity of the experimental buoyancy profiles presented in [4] and related works. Indeed, fluid stratification (relative gravity) monitored for different inflow buoyancy, momentum and number of sources, always exhibit sigmoid-like profile. WSN measurements motivate further use of this property, as they can be easily associated with an appropriate estimation strategy to provide for "on-line" shape monitoring. This provides for an extra simplification suitable to the proposed global approach. Establishing the control strategy on the shape properties also has the significant advantage that, compared with classical space discretization methods, the closed-loop performances are structurally robust with respect to the time-varying localization of the measurements.

B. Receding horizon control

The intrinsic complexity of the phenomena, the coupling between the dynamics and important (time-varying) propagation delays are important difficulties that have to be taken into account in the control design. We consider in this paper the control of the pollutants concentration in the mining room. Classical control schemes present poor control performances due to the presence of delays and disturbances even if the system is open-loop stable. Two important aspects have to be taking into consideration: the distributed property of the pollutants concentrations and the constraint on the level of admissible pollution at a certain level (for example at the height of a human). The main control objective is the minimization of the ventilation energy (the power consumption is proportional to the cube of the mass flow rate) while satisfying a set of constraints and canceling the effects of delays.

We propose a receding horizon optimal control scheme that compares different control sequences with respect to a performance index. This index evaluates the energy consumption and the concentration of pollutants over a prediction window, which shifts with the time evolution. This

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receding horizon principle proved its versatility on the so-called Model Predictive Control (MPC) schemes [5].

The paper is organized as follows. First, the description and the modeling of the system is presented in section II. The receding horizon control strategy is depicted in section III. Simulation results are provided in section IV, for both unconstrained (showing the influence of the tuning parameters of the control law) and constrained cases. A comparison with a classical PI controller is also given. Results are more than satisfactory and show the robustness of the control law against the pollutant sources prediction errors. Finally, concluding remarks and forthcoming works are drawn in section V.

II. System description and modeling

We consider the underground ventilation system presented in Figure 1, where fresh air is provided from the surface by using a vertical ventilation shaft. A fan is connected to this shaft and ventilates the extraction room through a tarpaulin tube. Note that the fact that we are focusing on metal mines is associated with the use of underground fans, which are generally prohibited in coal mines (at least in the United States). Distributed wireless sensors provide for chemical measurements at different locations in the extraction room. We consider that the sensors signals are carried to the fans embedded control units over a wireless multi-hop network, to account for the fact that a wired network would be difficult to install and maintain in such environment.

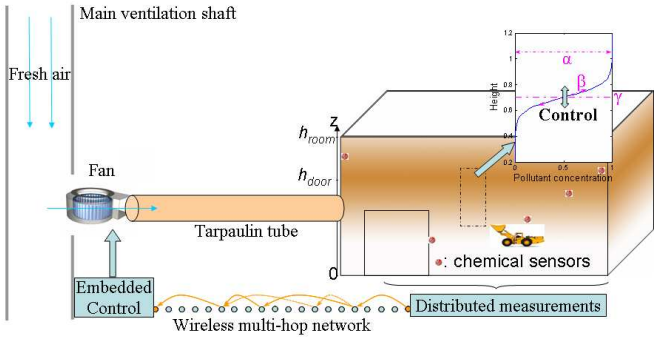

Fig. 1. Stratification and sigmoid description in extraction rooms.

A. Time-delays

Two different delays are involved in this model. The first one is a *physical delay*, associated to the airflow in the tarpaulin tube, between the fan and the extraction room (the time needed for a change of the mass flow rate due a modification in the fan actuation to reach the extraction room). This airflow is considered inviscid and incompressible, and modeled as a *time-varying* delay $\tau_{tarp}(t)$. Indeed, for a 1-dimensional Poiseuille laminar flow and the previous hypotheses, the flow speed $u(x,t)$ and temperature $T(x,t)$ are obtained from Navier-Stokes equations (see, for example, [6] or similar textbooks for details) as

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ T \end{bmatrix} + \begin{bmatrix} u & r \\ \gamma T & u \end{bmatrix} \frac{\partial}{\partial x}\begin{bmatrix} u \\ T \end{bmatrix} = 0$$

where r is the gas constant per unit of mass and γ is the ratio of specific heat coefficients. The characteristic velocities $v(x,t)$ are then the solutions of

$$\det \begin{vmatrix} -v+u & r \\ \gamma T & -v+u \end{vmatrix} = 0$$
$$\Leftrightarrow \quad v_{1,2}(x,t) = u(x,t) \pm \sqrt{r\gamma T(x,t)}$$

We are interested in the down-flow time-delay, which is approximated from the previous equation as

$$\tau_{tarp}(t) \approx \frac{L}{\bar{u}(t) + \sqrt{r\gamma \bar{T}(t)}}$$

where L is the length of the tarpaulin tube, $\bar{u}(t)$ and $\bar{T}(t)$ are the space-averaged flow speed and temperature, respectively.

The second source of delay is due to the distributed measurements and wireless transmission between the extraction room and the fan. This delay is time-varying and denoted as $\tau_{wsn}(t)$, to indicate that it is related to Wireless Sensor Network (WSN) automation. We consider that a wireless multi-hop protocol is set, as detailed in [7], to minimize the energy consumption of the wireless nodes according to specific communication quality constraints. The associated end-to-end delay writes as

$$\tau_{wsn}(t) = h(t)F + \sum_{i=1}^{h(t)} (\alpha_i + \beta_i)$$

where $h(t)$ is the time-varying number of hops, F contains the propagation and transmission delays, $i = 1 \dots h(t)$ indicates the transmission node considered, α_i is the time to wait before sending a data packet (typically a random variable) and β_i is the time induced by an Automatic Repeat reQuest (ARQ) mechanism. For simulation purposes, we consider the experimental data presented in [8], scaled to represent a tunnel of length L (initial measurements performed in a 10 m corridor, approximately).

B. Concentration profiles

The pollutants (CO_x or NO_x) volume concentration profiles $c_j(z,t)$, where $z \in [0; h_{room}]$ is the height in the extraction room, h_{room} is the room height and j indicates the pollutant considered, is approximated with the sigmoid distribution

$$c_j(z,t) = \frac{\alpha_j(t)}{1 + e^{-\beta_j(t)(z-\gamma_j(t))}}$$

where $\alpha_j(t)$ is the amplitude, $\beta_j(t)$ is the dilatation and $\gamma_j(t)$ is the inflection point of the distribution. Note that any function can be approximated with the desired precision level by a sum of such sigmoid functions: we suppose here that one curve is sufficiently accurate for control purposes and considering the system uncertainties. This simplified modeling approach was proposed for distributed systems involving smooth energy transport phenomena in [9], where a grey-box identification method allowing for the distinction between transient and steady-state behavior, the use of a switched model and the conservation of global physical

properties is proposed. This method was successfully applied to the modeling of temperature profiles in Tokamak plasmas and validated with experimental results.

The shape parameters α_j, β_j, γ_j can be related to the global parameters (room temperature and pressure, number of trucks and engines power, etc.) with an appropriate identification method. The pollutant mass $m_j(t)$ in the room is obtained from the concentration distribution thanks to the relationship

$$\begin{aligned} m_j(t) &= S_{room} \int_0^{h_{room}} c_j(z,t)dz \\ &= S_{room} \left[\int_0^{h_{door}} c_j(z,t)dz + \alpha_j(t)\Delta h \right] \end{aligned}$$

where S_{room} is the room surface, h_{door} is the door height and $\Delta h = h_{room} - h_{door}$. The last equality is established by supposing that the breathing and engine levels are below the room entrance level h_{door}. This hypothesis is reasonable in mining ventilation applications as, typically, the ceiling is blasted and the engines/humans enter the room only to work on the ore at the ground level. The pollutant dynamics is set thanks to the mass conservation law

$$\dot{m}_j(t) = \dot{m}_{j,in}(t) - \dot{m}_{j,out}(t) - \dot{m}_{j,chem}(t) \quad (1)$$

where $\dot{m}_{j,in}$ is the incoming pollutant mass rate due to the engines (we neglect human contribution) given by appropriate specifications and $\dot{m}_{j,chem}$ is the mass variation due to chemical reactions between components j and k at a rate $\eta_{jk}(z,T) = 1 - \eta_{kj}(z,T)$. The mass conservation equation (1) sets the shape parameters dynamics with

$$\begin{aligned} \dot{m}_j(t) &= S_{room} \left[\int_0^{h_{door}} \dot{c}_j(z,t)dz + \dot{\alpha}_j(t)\Delta h \right] \\ \dot{m}_{j,out}(t) &= \frac{1}{h_{door}} \int_0^{h_{door}} c_j(z,t)dz \times Q_{out} \\ \dot{m}_{j,chem}(t) &= S_{room} \left[\int_0^{h_{door}} \eta_{jk} c_j(z,t) c_k(z,t)dz \right. \\ &\quad \left. + \eta_{jk}\alpha_j(t)\alpha_k(t)\Delta h \right] \end{aligned}$$

where $Q_{out} = S_{tarp}\, \nu u_{fan}(t - \tau_{tarp})$ is the volume rate of flow leaving the room, S_{tarp} is the cross section of the tarpaulin tube and u_{fan} is the airflow speed provided by the fan, scaled by a constant ν (flow momentum losses due to the tube geometry). We supposed here that there is no pollutant in the tarpaulin tube and that the flow in the room is incompressible.

The variable-step integration method described in [9] is now introduced to obtain a differential equation representation of the distributed model

$$E_j \begin{bmatrix} \dot{\alpha}_j(t) \\ \dot{\beta}_j(t) \\ \dot{\gamma}_j(t) \end{bmatrix} = \dot{m}_{j,in}(t) - B_j\, u_{fan}(t - \tau_{tarp}) - D_{jk}$$

with

$$\begin{aligned} E_j &\doteq S_{room} \left(V_{int} \begin{bmatrix} \vdots & \vdots & \vdots \\ \frac{\partial C_{j,i}}{\partial \alpha_j} & \frac{\partial C_{j,i}}{\partial \beta_j} & \frac{\partial C_{j,i}}{\partial \gamma_j} \\ \vdots & \vdots & \vdots \end{bmatrix} + \begin{bmatrix} \Delta h \\ 0 \\ 0 \end{bmatrix}^T \right) \\ B_j &\doteq \frac{1}{h_{door}} V_{int} \begin{bmatrix} \vdots \\ C_{j,i} \\ \vdots \end{bmatrix} \times S_{tarp}\, \nu \\ D_{jk} &= S_{room} \left[V_{int} \begin{bmatrix} \vdots \\ \eta_{jk,i}\, C_{j,i}\, C_{k,i} \\ \vdots \end{bmatrix} + \eta_{jk}\alpha_j\alpha_k\Delta h \right] \end{aligned}$$

where $i = 1 \ldots N$, $V_{int} = M_{int}(N, 1:N)$, N is the number of discretization points and $C_{j,i} = c_j(z_i, t)$. The system dynamics is finally represented as a set of delay differential equations of the form:

$$\begin{aligned} \begin{bmatrix} \dot{\alpha}_j(t) \\ \dot{\beta}_j(t) \\ \dot{\gamma}_j(t) \end{bmatrix} &= E^{\dagger} \left[\dot{m}_{j,in}(t) - B_j\, u_{fan}(t - \tau_{tarp}) - D_{jk} \right] \\ y_j(t) &= \frac{\alpha_j(t)}{1 + e^{-\beta_j(t)(z_r - \gamma_j(t))}} \end{aligned}$$

where $\dagger$ denotes the Moore-Penrose inverse and $y_j(t)$ is the concentration of gas j at height z_r.

III. Control scheme

The control *objective* is to minimize the fan energy consumption while ensuring an acceptable air quality in the extraction room. Due to the height-dependent model proposed in the previous section, the air quality objective can be rephrased as guaranteeing a maximum pollutant concentration at a given height z_r

$$\max_{\forall j} y_j(t) \leq \bar{y}_j$$

where $\bar{y}_j$ is the threshold value on pollutant j (CO_x, CH_4, SO_2 and N_xO_x are classically associated with the trucks engines). Communication constraints, such as delays, timeout, packet losses and bandwidth limitations also should be taken into account in the optimization algorithm.

A. Receding Horizon Control

Predictive control is a model-based design technique, described in Figure 2. It is based on the on-line solution of successive optimization problems. The idea is to solve a scheduling optimization problem at time kT, (T: sampling time) based on future desired outputs $y_{j,des}$ and prediction of future disturbances $\hat{\dot{m}}_{j,in}$ (pollutant sources from trucks in this case), to apply the first control values to the system, to update the prediction model, and to repeat the whole procedure at time $(k+1)T$. The scheduling algorithm can be stated as

$$\min_{u_i, i \in \{1,\ldots,N_u\}} \int_{kT}^{kT+N} \Big(\sum_j (\hat{y}_j(\tau) - y_{j,des}(\tau))^2 + \lambda u_{fan}^2(\tau) \Big) d\tau,$$

where $u_{fan}(\tau) = u_i$ if $\tau \in [kT + (i-1)N/N_u, kT + iN/N_u]$, and $\hat{y}_j$ is the prediction of future outputs, computed from the simulation of a prediction model. For the prediction model, predicted values of disturbances $\hat{\dot{m}}_{j,in}$ are of course considered. The tuning parameters of the control law are N,

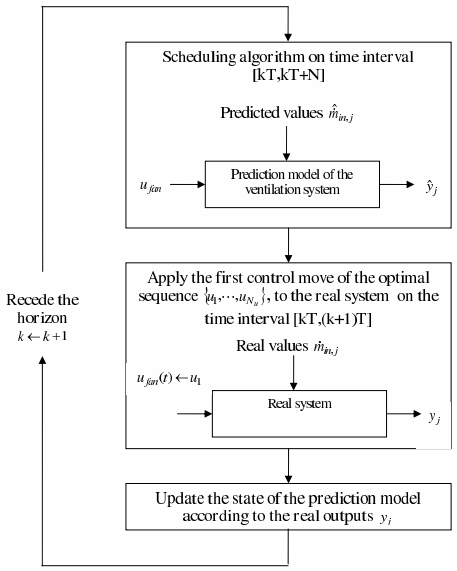

Fig. 2. Receding horizon control synopsis.

N_u and λ. Here, N represents the prediction horizon length. It should be chosen large enough to provide information about the transient behavior. Next, N_u represents the number of degrees of freedom in the control action during the prediction horizon. A trade-off has to be found between the increased precision (large number of degrees of freedom) and the consequent augmented complexity of the optimal control problem to be solved at each sampling period. Finally, λ is the weighting factor between the control effort and the disturbance rejection performances.

B. Implementation issues

In this paper the "real system" will be the reference model which has been defined in section II. Being a real-time optimization procedure, the main implementation problems are related to the speed of convergence of the scheduling algorithm. This implies that the prediction model has to be simpler than the reference model. The time varying delay $\tau_{tarp}(t)$ is chosen as a constant equal to the maximal time delay which may occur in the real system.

The second important issue is the feasibility of the successive optimization procedure, as the scheduling algorithm has to be solved with the constraints defined in section III.A. If the problem appears to be unfeasible, than one has to define alternative strategies. This point is discussed in section IV.C.

IV. Simulation results

A. Unconstrained case - tuning of the control law

We consider the ventilation problem for two pollutants, NO_x and CO_x. The control law is tested with Matlab 7.5, its *Optimization Toolbox 3.1.2* and Simulink 7.0 on a Pentium IV, 2.0 GHz. First, the unconstrained problem is simulated (relaxing the bound constraints of section III.A). A sampling time of $T = 5\ s$ is chosen, as the time response of the open loop is about $50\ s$. Desired outputs are set to $y_{j,des} = 0$, and a null pollutant source prediction error is supposed ($\hat{\dot{m}}_{j,in} = \dot{m}_{j,in}$). This configuration highlights the influence of the control law parameters. As discussed in section III.B, N has to be higher than the time response of the system and is consequently choosen as $N = 50s$. We first study the influence of λ for a fixed $N_u = 2$. The optimization problem is solved in about 20 seconds. Figure 3 presents the simulation of the closed-loop system with a predictive law with a small weight $\lambda = 10^{-7}$. This small weight leads to high control values, as they are not penalized, while the rejection of pollutant gas is quite satisfactory with small values for concentrations. Notice that the prediction capabilities enable changes in the control action according to the future evolution of the pollutant dejections. Decreasing λ

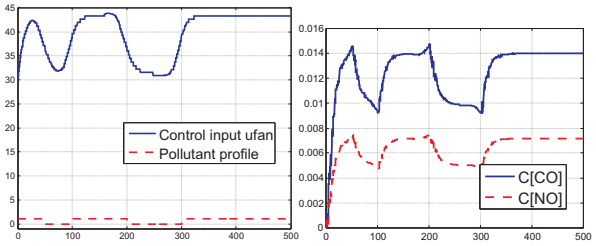

Fig. 3. Simulation with $N = 50; N_u = 2; \lambda = 1e-7$.

(Fig. 4) is associated with a decrease of the magnitude of the control action by one order of magnitude. The increase in the pollutant concentration is not following the same proportion even if an increase can be noticed. This fact illustrates the non linearity of the system, and the fact that the choice of a linear controller such as a PI is not well suited for this problem (see section IV.D). For $\lambda = 10^{-3}$ the preservation

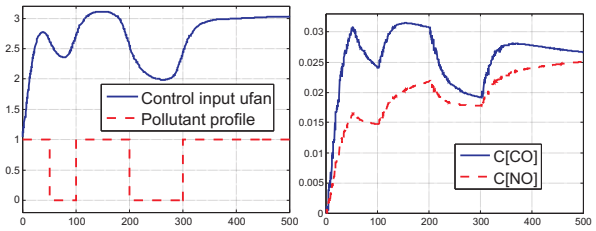

Fig. 4. Simulation with $N = 50; N_u = 2; \lambda = 1e-5$.

of the control energy becomes the main objective and by consequence a poor rejection is observed (Fig. 5). Finally, for smaller λ, the rejection is better, but the application of the control law is more expensive. The influence of N_u can be seen in Fig. 6, which presents the performances of the control law with $N_u = 5$ changes in the control action during the prediction horizon. The comparison between $N_u = 2$ and $N_u = 5$ is analyzed in Fig. 7: for small N_u values,

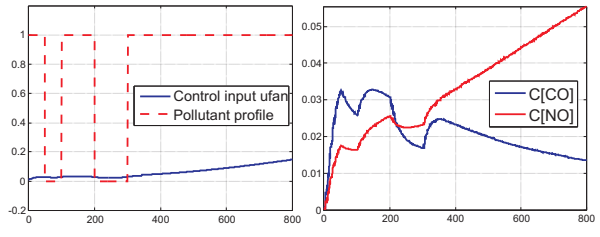

Fig. 5. Simulation with $N = 50; N_u = 2; \lambda = 1e-3$.

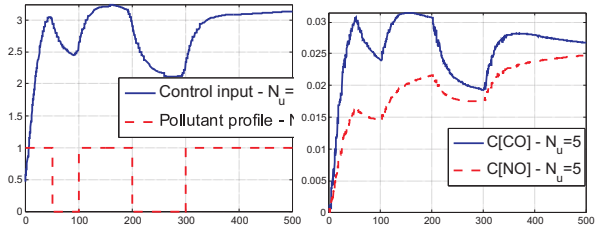

Fig. 6. Simulation with $N = 50; N_u = 5; \lambda = 10^{-5}$.

the control effort is concentrated in the first control actions and the curve corresponding to $N_u = 2$ "reacts" well in "advance" to the change in pollutant profile. Furthermore, note that the number of optimization variables is N_u, and an increase in N_u leads to an increase of the computational effort which may become intractable for on-line solutions. In the sequel, we consider $N = 50\ s$ and $N_u = 2$.

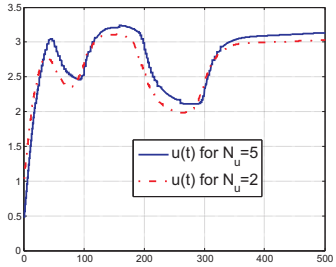

Fig. 7. Comparison of control laws for $N_u = 2$ and $N_u = 5$.

B. Constrained case

We focus now on the constrained case. The goal is to regulate the system, and we set $y_{j,des} = 0.025$, with a bound constraint of $y_j(t) \leqslant 0.028$. Figures 8 and 9 show simulation results for $\lambda = 10^{-3}, \lambda = 10^{-5}$. For these results, no pollutant prediction error has been considered. As can be seen from the figures, the constraints are satisfied. Once again, choosing $\lambda = 10^{-3}$ leads to poor regulation performances, while $\lambda = 10^{-5}$ leads to better results, but with larger control values. Note that the regulation problem is multiobjective, as one needs to regulate two outputs (NO_x and CO_x concentrations), by using only one control input.

C. Robustness against pollutant prediction errors

In the receding horizon strategy, one has to consider a prediction model of the system. In our case, the prediction

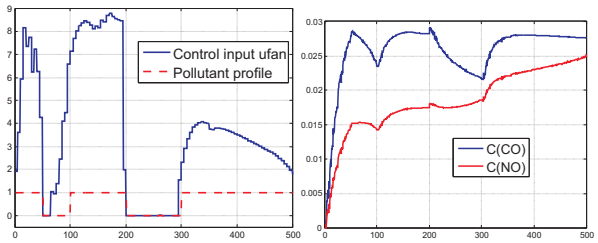

Fig. 8. Constrained case - Simulation with $\lambda = 10^{-3}$.

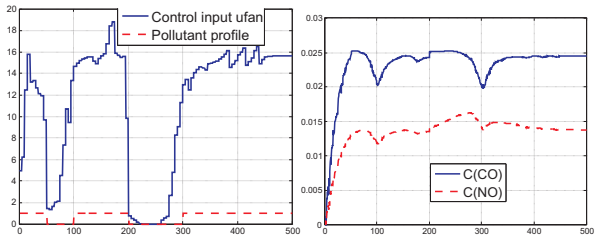

Fig. 9. Constrained case - Simulation with $\lambda = 10^{-5}$.

model was made of the reference model, with the maximum time delay, and prediction of pollutant sources. Feasibility of successive optimization problems appears to be a crucial point in the receding horizon strategies. If the sequence of control inputs leads to an unfeasible optimization problem, an alternative strategy has to be used. This idea of feasibility of successive optimization problem is strongly linked to the classical Automatic Control concept of robustness against parametric and modeling uncertainties.

Choosing maximum time-delay values may not lead necessarily to infeasibility of optimization problems since the predictive control law reacts in advance, and a decrease in pollutant concentrations is observed. Consequently, an error in the time delay has just an economic influence (higher energy consumption), but the technical behavior of the system remains more than satisfactory. Obviously, this is not the case for pollutant sources prediction errors, and the control law has to be robust against these errors. In this study, we are proposing an alternative strategy that can be resumed as follows: If infeasibility occurs, we decide to temporally forget the constraints and to use the unconstrained case with a small $\lambda = 10^{-7}$. As seen in section IV.A, this value leads to large values of the controller parameters but also a decrease in pollutant concentrations. As a result, the alternative strategy allows reaching feasibility again a few sampling times after. We consider now one of the worst case situations, where the pollutant source is always underestimated: the real pollutant sources is 50% higher than the prediction. Figure 10 gives the corresponding results. On the bottom graph, one can see the chosen strategy: 1 (2) meaning constrained (unconstrained) case. The constraints are sometimes not satisfied due to important prediction errors, but the alternative strategy proposed here allows reaching feasibility again.

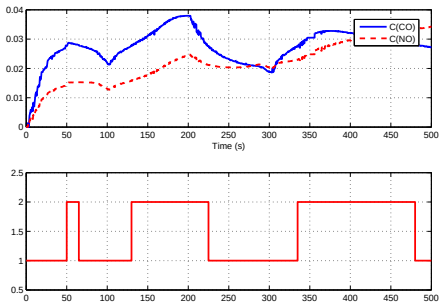

Fig. 10. Results for underestimated prediction and used strategies

D. Comparisons with a linear controller

In this section, a Proportional Integral (PI) controller is defined for comparison with the proposed strategy. As mentioned previously, the problem is multiobjective, as two concentrations have to be controlled from one input. However, those concentrations are linked together (see section II), and for the PI controller, only one concentration will be regulated at 0.028. A PI controller has been tuned, $C(s) = 500\,(1 + 1/200s)$, leading to the time response of figure 11, for a pollutant step at time 200. The PI seems

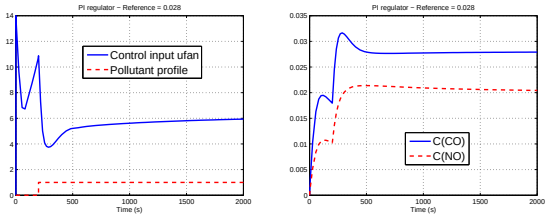

Fig. 11. Simulation with PI for reference 0.028

to have a satisfactory behavior in this case. However, when the reference has been set to 0.035 (simulation in figure 12) the results show that a slight increase in the reference can lead to instability, proving the lack of robustness of the linear controller. Furthermore, note that the economic aspect (energy consumption) can not be explicitly taken into account by using PI controllers.

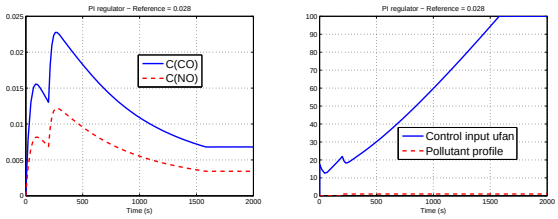

Fig. 12. Simulation with PI for reference 0.035

E. Discussion

Finally, the updating of the prediction model (see figure 2) supposes that a non linear observer is available, so as to estimate the states of the system from the measures given by the WSN. In this paper, the WSN delay is not taken into account for the design of the predictive law, so that it is possible to measure the system states. This point will have to be improved in future works.

V. Conclusions

In this paper, a receding horizon control law has been defined for the climate control in metal mine extraction rooms which appears to be a challenging automation problem. The first point of the study is to define a suitable model of the system, capturing all relevant phenomena which have to be dealt with. From this system modeling, a prediction model is established which can be used to define a receding horizon control law for the system. The idea of the proposed approach is to consider pollutant sources prediction in order to react in advance and to reject these disturbances in the regulation procedure. The proposed method is based on the "on-line" solutions of optimization problems. The procedure allows taking into account economic aspects as a penalization term on energy consumption is added to the optimized criterion. Furthermore, the approach is quite versatile as many technical constraints and objectives can be explicitly taken into account. Results show that the proposed strategy lead to very satisfactory results with the possibility of weighting the two main "contradictory" objectives: performing an efficient regulation, but with low energy consumption. The robustness of the proposed method against pollutant prediction errors has also been studied with an alternative strategy to reach feasibility again. Finally, the proposed method leads to a controller more robust than a classical PI controller.

Forthcoming works deal with the influence of the WSN in the control strategy, as one has to estimate the systems states from the measurements of the sensors to update the prediction model.

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