HAL open science

Efimov states with strong three-body losses

Felix Werner

To cite this version:

Felix Werner. Efimov states with strong three-body losses. EPL - Europhysics Letters, 2009, 86, pp.66006. hal-00340101v2

HAL Id: hal-00340101

https://hal.science/hal-00340101v2

Submitted on 10 Jun 2009

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Efimov states with strong three-body losses

Félix Werner [(a)]

Laboratoire Kastler Brossel, École Normale Supérieure,
Université Pierre et Marie Curie-Paris 6, CNRS,
24 rue Lhomond, 75231 Paris Cedex 05, France

PACS 21.45.-v – Few-body systems
PACS 67.85.-d – Ultracold gases

Abstract. - We determine analytically how Efimov trimer states are modified by three-body losses within the model of Braaten and Hammer. We find a regime where the energies approach the positive real axis and the decay rates vanish.

Introduction. – Motivated by nuclear physics, Efimov discovered that three particles with short-ranged two-body interactions of scattering length $|a| = \infty$ can support an infinite series of weakly bound trimer states [1]. In contrast to systems of nucleons or of ^{4}He atoms, ultracold gases of alkali metal atoms offer the possibility to tune a by using a Feshbach resonance, and thus to go more deeply into the limit $|a| \to \infty$ where the Efimov effect sets in [2–9]. However the situation for alkali metal atoms is complicated by the existence of deeply bound dimer states: a weakly bound trimer necessarily decays into a deeply bound dimer and a free atom [6, 10]. This three-body loss process has prevented so far to produce weakly bound trimers experimentally. The first evidence for the existence of a weakly bound trimer state is the recent observation of a peak in the three-body loss rate from an ultracold atomic Bose gas at large negative a [2]. Such peaks were predicted to occur for the values of a where the energy of an Efimov trimer reaches zero [5, 11]. More recently, a similar effect was studied experimentally and theoretically in a three-component Fermi gas [12–15]. The simplest description of three-body losses is the model of Braaten and Hammer, where three incoming low-energy unbound atoms are reflected back as three unbound atoms with a probability $e^{-4\eta_*}$ and recombine to a deeply bound dimer and an atom with a probability $1 - e^{-4\eta_*}$, η_* being the so-called inelasticity parameter, which depends on the short-range details of the interaction potential and of its deeply bound states [3, 4, 11]. This model was used to obtain the decay rate of Efimov trimers in the regime of small inelasticity parameter $\eta_* \ll 1$ [4, 10], as well as 3- and 4-body scattering properties [3, 4, 16] and the decay rate of efimovian 3-body states in a harmonic trap [17, 18]. This model is expected to become exact in the zero-range limit where $|a|$ and the inverse relative momenta between atoms in the initial state are much larger than the range and effective range of the interaction potential [3, 4, 10, 11, 19].

In this Letter, we determine how Efimov trimers are modified for an arbitrary inelasticity parameter η_* by solving analytically the model of Braaten and Hammer. We find that the Efimov spectrum rotates counterclockwise in the complex plane by an angle proportional

[(a)] *Present address:* Department of Physics, University of Massachusetts, Amherst, MA 01003, USA

to η_*. When η_* reaches the critical value where this angle equals π, the discrete states disappear into the continuum. When η_* approaches this critical value from below, the energies approach the positive real axis, so that the decay rates tend to zero. Thus a large inelasticity parameter $\eta_* \sim 1$ can paradoxically give rise to long-lived three-body states.

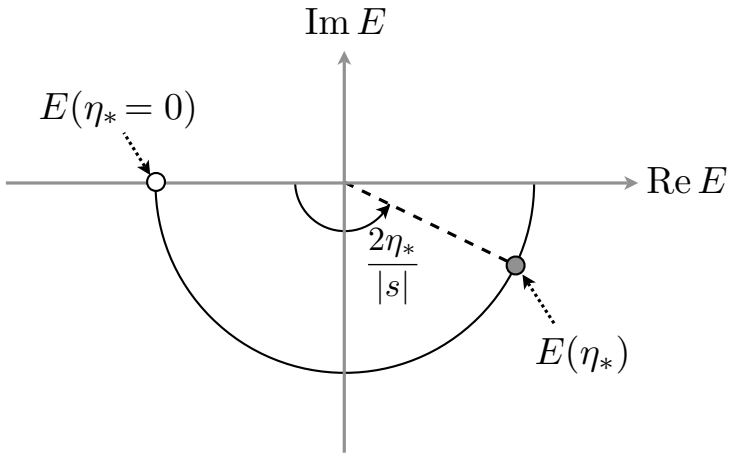

Fig. 1: Without three-body losses the inelasticity parameter $\eta_* = 0$ and the energy of an Efimov state lies on the negative real axis. The effect of three-body losses is to rotate this energy in the complex plane by an angle $2\eta_*/|s|$, where $|s| = 1.00624\ldots$. When this angle approaches π, the decay rate $\Gamma = -2\,\mathrm{Im}\,E/\hbar$ vanishes.

Efimov states without three-body losses. – We start by reviewing Efimov's solution of the three-body problem in the absence of three-body losses. We restrict for simplicity to the case of three identical bosonic particles, where the wavefunction $\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ is completely symmetric.[1] In the limit where the range of the interaction potential is negligible compared to $|a|$ and to the typical de Broglie wavelength, the interaction can be replaced exactly by the zero-range model, see e.g. [1,3,7,9,16,17,21–24]. An eigenstate of the zero-range model solves (i) the Schrödinger equation

$$-\frac{\hbar^2}{2m}\sum_{i=1}^{3}\Delta_{\mathbf{r_i}}\psi = E\,\psi \tag{1}$$

when all interparticle distances r_{ij} are different from zero, and (ii) the Bethe-Peierls boundary condition, imposing that there exists a function A such that

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \underset{r_{ij}\to 0}{=} \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) A(\mathbf{R}_{ij}, \mathbf{r}_k) + O(r_{ij}) \tag{2}$$

in the limit $r_{ij} \to 0$ where particles i and j approach each other while the position of their center of mass $\mathbf{R}_{ij}$ and of the third particle $\mathbf{r}_k$ are fixed. In what follows we assume that

[1]The case of different masses and statistics can be included without difficulty: only the value of s is modified in the hyperradial problem [20].

$|a|$ is much larger than the typical distance between particles, so that we can set $|a| = \infty$. Equations (1,2) are then solved by Efimov's Ansatz [1]

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = F(R) \left(1 + \hat{P}_{13} + \hat{P}_{23}\right) \frac{1}{r\rho} \sin\left[s \arctan\left(\frac{\rho}{r}\right)\right] \tag{3}$$

where $\hat{P}_{ij}$ exchanges particles i and j, $s \simeq i \cdot 1.00624$ is the only solution $s \in i \cdot (0; +\infty)$ of the equation $s \cos(s\pi/2) - 8/\sqrt{3} \sin(s\pi/6) = 0$, the Jacobi coordinates are $r = \|\mathbf{r}_2 - \mathbf{r}_1\|$ and $\rho = \|2\mathbf{r}_3 - \mathbf{r}_1 - \mathbf{r}_2\|/\sqrt{3}$, the hyperradius is $R = \sqrt{(r^2 + \rho^2)/2}$, and the hyperradial wavefunction $F(R)$ solves the hyperradial Schrödinger equation

$$\left[-\left(\frac{d^2}{dR^2} + \frac{1}{R}\frac{d}{dR}\right) + \frac{s^2}{R^2}\right] F(R) = \frac{2m}{\hbar^2} E\, F(R). \tag{4}$$

In the limit $R \to 0$ where all three particles approach each other, the attractive effective potential s^2/R^2 diverges strongly, and it is necessary to impose a boundary condition on the hyperradial wavefunction $F(R)$, as first realized by Danilov [25]. This boundary condition is conveniently expressed as

$$F(R) \underset{R\to 0}{\propto} \left(\frac{R}{R_t}\right)^{-s} - \left(\frac{R}{R_t}\right)^{s} \tag{5}$$

where the three-body parameter R_t depends on short-range physics and is a parameter of the zero-range model. The solution of Eqs. (4,5) is given by the famous geometric series of Efimov trimers

$$E_n^0 = -\frac{2\hbar^2}{m{R_t}^2} \exp\left[\frac{2}{|s|} \arg\Gamma(1+s)\right] \exp\left(n\frac{2\pi}{|s|}\right), \quad n \in \mathbb{Z}, \tag{6}$$

with a (here unnormalized) wavefunction

$$F(R) = K_s(\kappa R) \tag{7}$$

where K is a Bessel function and κ is defined by

$$E = -\frac{\hbar^2\kappa^2}{2m} \tag{8}$$

with the determination $\kappa > 0$ ensuring that $F(R)$ decays exponentially for $R \to \infty$. Efimov's spectrum (6) is unbounded from below, in agreement with the Thomas effect [26] and with the fact that the spectrum for an interaction of finite range b coincides with Efimov's spectrum only in the zero-range limit, i.e. for weakly bound trimers satisfying $|E| \ll \hbar^2/(mb^2)$ [1,3,4,21,27–29].

Effect of three-body losses. – We now determine how Efimov's result is modified by three-body losses within the model of Braaten and Hammer. The only difference between the Braaten-Hammer model and the zero-range model is that the boundary condition (5) is replaced by [3,4]

$$F(R) \underset{R\to 0}{\propto} \left(\frac{R}{R_t}\right)^{-s} - e^{-2\eta_*} \left(\frac{R}{R_t}\right)^{s} \tag{9}$$

where the inelasticity parameter η_* and the three-body parameter R_t are parameters of the Braaten-Hammer model, whose values depend on the details of the finite-range interactions which one wishes to model.[2] The physical meaning of (9) is that the outgoing wave $\propto R^s$

[2] E.g. for ^{133}Cs atoms near the -11 G Feshbach resonance, the fit of the theoretical result of [11] to the experimental data of [2] gives $\eta_* \simeq 0.06$ [2] and $R_t \simeq 30$ nm [2,17].

has an amplitude which is smaller than the amplitude of the ingoing wave $\propto R^{-s}$ by a factor $e^{-2\eta_*}$, i.e. three ingoing atoms are reflected with a probability $e^{-4\eta_*}$ and are lost by three-body recombination with a probability $1-e^{-4\eta_*}$.

The Braaten-Hammer model is expected to become exact in the zero-range limit [3, 4, 10, 11]. This is supported by numerical calculations of the three-body loss rate from an atomic gas with finite-range interaction potentials, which are in good agreement with the prediction of the Braaten-Hammer model in the zero-range regime [19]. Moreover this can be explained using the adiabatic hyperspherical description [3, 6, 30, 31]: in addition to the 'atomic' adiabatic channel responsible for the Efimov effect, there is one 'molecular' channel associated with each deep two-body bound state; one can thus look for decaying Gamov states with a complex energy E by imposing outgoing boundary conditions in the molecular channels; the coupling between the atomic channel and the molecular channels is only effective at distances on the order of the potential range, where the wavefunction is insensitive to E in the zero-range limit; and matching this short-distance wavefunction to the atomic-channel wavefunction at hyperradii much larger than the range and much smaller than the typical atomic-channel de Broglie wavelength yields the boundary condition (9).

In the absence of losses ($\eta_*=0$), the boundary condition (9) reduces to Efimov's boundary condition (5), and one recovers Efimov's spectrum (6):

$$E_n(\eta_*=0)=E_n^0. \tag{10}$$

In presence of losses ($\eta_*>0$), we solve the hyperradial problem (4,9) with the additional boundary condition that $F(R)$ must decay quickly enough at infinity.[3] Using the known properties of Bessel functions [32], we obtain the energies

$$E_n(\eta_*)=\exp\left(i\,\frac{2\eta_*}{|s|}\right)\,E_n(\eta_*=0), \tag{11}$$

i. e. the spectrum is rotated in the complex plane counterclockwise around the origin by an angle $2\eta_*/|s|$. The result (11) only holds if the angle $2\eta_*/|s|<\pi$, i. e. for $\eta_*<\eta_{*c}$ with

$$\eta_{*c}=\frac{\pi|s|}{2}=1.5806\ldots, \tag{12}$$

while for $\eta_*>\eta_{*c}$ there is no normalisable solution.

The wavefunction is still given by Eqs. (7,8), now with the determination

$$\mathrm{Re}\,\kappa>0 \tag{13}$$

of the sign of κ, which ensures that the wavefunction is normalisable since

$$F(R)\underset{R\to\infty}{\propto}\frac{e^{-\kappa R}}{\sqrt{R}}. \tag{14}$$

The decay rate

$$\Gamma\equiv-\frac{2}{\hbar}\,\mathrm{Im}\,E \tag{15}$$

is given by

$$\hbar\Gamma=2\sin\left(\frac{2\eta_*}{|s|}\right)\,|E(\eta_*=0)|. \tag{16}$$

In the limit of small losses $\eta_*\ll 1$ we recover the known result [10]

$$\hbar\Gamma\simeq\frac{4\eta_*}{|s|}\,|E(\eta_*=0)|. \tag{17}$$

[3] More precisely the normalisation condition is $\int_0^\infty dR\,R\,|F(R)|^2<\infty$, since the quantity $\int d\mathbf{r_1}\,d\mathbf{r_2}\,|\psi(\mathbf{r_1},\mathbf{r_2},\mathbf{r_3})|^2$, which does not depend on $\mathbf{r_3}$, has to be finite.

The proportionality between Γ and the energy was first observed in numerical calculations with finite-range interactions and explained using the adiabatic hyperspherical description in [6].

An interesting effect occurs when η_* approaches the critical value η_{*c} from below: the energies approach the opposite of the energies of the Efimov states without losses

$$E(\eta_*) \underset{\eta_* \to \eta_{*c}}{\longrightarrow} |E(\eta_* = 0)|, \tag{18}$$

so that the decay rates tend to zero. Moreover, the sizes of the states diverge: since the imaginary part of κ tends to a positive value and its real part tends to 0^+, the behavior (14) of the wavefunction at large R is an ingoing wave with a *slowly* decaying envelope. Physically, it is not surprising that the states become increasingly delocalised before disappearing into the continuum. Since the wavefunction is normalized, this divergence of the size implies that the probability for the three particles to be close to each other vanishes, so that Γ vanishes.

This effect occurs within the effective low-energy model of Braaten and Hammer. We thus expect that it also occurs for finite-range interactions supporting deeply bound dimers, provided the two-body interaction potential is tuned in such a way that η_* is slightly below η_{*c}; this could be checked numerically using the methods of [5, 6, 8, 19, 31].

Experimentally, it is rather unlikely to find a Feshbach resonance close to which η_* is slightly below η_{*c}. However this regime may become accessible if interatomic interactions are tuned using an additional control parameter, e. g. an electric field [33].

* * *

I thank E. Braaten, Y. Castin, B. Esry, H.-W. Hammer, S. Jochim, C. Mora and A. Wenz for discussions. LKB is a *Unité Mixte de Recherche* of ENS, Université Paris 6 and CNRS. The ENS cold atoms group is a member of *Institut Francilien de Recherche sur les Atomes Froids*.

REFERENCES

[1] Efimov V. N., *Yad. Fiz.* , **12** (1970) 1080 [Sov. J. Nucl. Phys. **12**, 589 (1971)].
[2] Kraemer T., Mark M., Waldburger P., Danzl J. G., Chin C., Engeser B., Lange A. D., Pilch K., Jaakkola A., Naegerl H.-C. and Grimm R., *Nature* , **440** (2006) 315.
[3] Braaten E. and Hammer H.-W., *Physics Reports* , **428** (2006) 259.
[4] Braaten E. and Hammer H.-W., *Ann. Phys.* , **322** (2007) 120.
[5] Esry B. D., Greene C. H. and Burke J. P., *Phys. Rev. Lett.* , **83** (1999) 1751.
[6] Nielsen E., Suno H. and Esry B. D., *Phys. Rev. A* , **66** (2002) 012705.
[7] Petrov D. S., *Phys. Rev. Lett.* , **93** (2004) 143201.
[8] Lee M. D., Köhler T. and Julienne P. S., *Phys. Rev. A* , **76** (2007) 012720.
[9] Gogolin A. O., Mora C. and Egger R., *Phys. Rev. Lett.* , **100** (2008) 140404.
[10] Braaten E., Hammer H.-W. and Kusunoki M., *Phys. Rev. A* , **67** (2003) 022505.
[11] Braaten E. and Hammer H.-W., *Phys. Rev. A* , **70** (2004) 042706.
[12] Ottenstein T. B., Lompe T., Kohnen M., Wenz A. N. and Jochim S., *Phys. Rev. Lett.* , **101** (2008) 203202.
[13] Huckans J. H., Williams J. R., Hazlett E. L., Stites R. W. and O'Hara K. M., *Phys. Rev. Lett.* , **102** (2009) 165302.
[14] Braaten E., Hammer H.-W., Kang D. and Platter L., *arxiv:0811.3578v1.*
[15] Naidon P. and Ueda M., *arxiv:0811.4086v1.*
[16] Marcelis B., Kokkelmans S. J. J. M. F., Shlyapnikov G. V. and Petrov D. S., *Phys. Rev. A* , **77** (2008) 032707.
[17] Werner F., *PhD Thesis* (Université Paris VI) 2008 chap. 3. `http://tel.archives-ouvertes.fr/tel-00285587`
[18] Werner F. and Castin Y., in preparation.
[19] D'Incao J. P., Greene C. H. and Esry B. D., *J. Phys. B* , **42** (2009) 044016.

[20] Efimov V., *Nucl. Phys. A*, **210** (1973) 157.
[21] Albeverio S., Høegh-Krohn R. and Wu T. T., *Phys. Lett. A* , **83** (1981) 105.
[22] Albeverio S., Gesztesy F., Hoegh-Krohn R. and Holden H., *Solvable models in quantum mechanics* (Springer-Verlag, Berlin) 1988.
[23] Jonsell S., Heiselberg H. and Pethick C. J., *Phys. Rev. Lett.* , **89** (2002) 250401.
[24] Werner F. and Castin Y., *Phys. Rev. Lett.* , **97** (2006) 150401.
[25] Danilov G. S., *JETP* , **40** (1961) 498 [Sov. Phys. JETP **13**, 349 (1961)].
[26] Thomas L. H., *Phys. Rev.* , **47** (1935) 903.
[27] Amado R. D. and Noble J. V., *Phys. Lett* , **35B** (1971) 25.
[28] Amado R. D. and Noble J. V., *Phys. Rev. D* , **5** (1972) 1992.
[29] Albeverio S., Lakaev S. N. and Muminov Z. I., *Ann. Henri Poincaré* , **5** (2004) 743.
[30] Nielsen E., Fedorov D. V., Jensen A. S. and Garrido E., *Phys. Rep.* , **347** (2001) 373.
[31] D'Incao J. P. and Esry B. D., *Phys. Rev. Lett.* , **94** (2005) 213201.
[32] Lebedev N. N., *Special functions and their applications* (Prentice-Hall) 1965.
[33] Marcelis B., Verhaar B. and Kokkelmans S., *Phys. Rev. Lett.* , **100** (2008) 153201.