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### Medium-modified average multiplicity and multiplicity fluctuations in jets

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Abstract: The energy evolution of average multiplicities and multiplicity fluctuations in jets produced in heavy-ion collisions is investigated from a toy QCD-inspired model. In this model, we use modified splitting functions accounting for medium-enhanced radiation of gluons by a fast parton which propagates through the quark gluon plasma. The leading contribution of the standard production of soft hadrons is enhanced by a factor  $\sqrt{N_s}$  while next-to-leading order (NLO) corrections are suppressed by  $1/\sqrt{N_s}$ , where the parameter  $N_s > 1$  accounts for the induced-soft gluons in the medium. Our results for such global observables are cross-checked and compared with their limits in the vacuum.

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Recent experiments at the Relativistic Heavy Ion Collider (RHIC) have established a phenomenon of strong high-transverse momentum hadron suppression [1], which supports the picture that hard partons going through dense matter suffer a significant energy loss prior to hadronization in the vacuum (for recent review see [2]).

Predictions concerning multi-particle production in nucleus-nucleus collisions can be carried out by using a toy QCD-inspired model introduced by Borghini and Wiedemann in [3]; it allows for analytical computations and may capture some important features of a more complete QCD description. In this model, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) splitting functions  $q \rightarrow g\bar{q}$  and  $g \rightarrow gg$  [4] of the QCD evolution equations were distorted so that the role of soft emissions was enhanced by multiplying the infra-red singular terms by the medium factor  $N_s$ . The model [3] was further discussed and used on the description of final states hadrons produced in heavy-ion collisions [5].

Within the model, we make predictions for the medium-modified average multiplicity  $N_A$  in quark and gluon jets (A = q, g) produced in such reactions, for the ratio  $r = N_g/N_q$  and finally for the second multiplicity correlators  $\langle N_A(N_A - 1) \rangle / N_A^2$ , which determines the width of the multiplicity distribution. The starting point of our analysis is the NLO or Modified-Leading-Logarithmic-Approximation (MLLA) master evolution equation for the generating functional [4] which determine the jet properties at all energies together with the initial conditions at threshold at small x, where x is the fraction of the outgoing jet energy carried away by a single gluon. Their solutions with medium-modified splitting functions can be resummed in powers of  $\sqrt{\alpha_s/N_s}$  and the leading contribution can be represented as an exponential of the medium-modified anomalous dimension which takes into account the  $N_s$ -dependence:

$$N_A \simeq \exp\left\{\int^Y \gamma_{\text{med}}\left(\alpha_s(Y)\right) dY\right\},\tag{1}$$

where  $\gamma_{\rm med}(\alpha_s)$  can be expressed as a power series of  $\sqrt{\alpha_s/N_s}$  in the symbolic form:

$$\gamma_{\text{med}}\left(\alpha_{s}\right)\simeq\sqrt{N_{s}}\times\sqrt{\alpha_{s}}\left(1+\sqrt{\frac{\alpha_{s}}{N_{s}}}+\mathcal{O}\left(\frac{\alpha_{s}}{N_{s}}\right)\right).$$

Within this logic, the leading double logarithmic approximation (DLA,  $\mathcal{O}(\sqrt{N_s \alpha_s})$ ), which resums both soft and collinear gluons, and NLO (MLLA,  $\mathcal{O}(\alpha_s)$ ), which resums hard collinear partons and accounts for the running of the coupling constant  $\alpha_s$ , are complete. The choice  $dY = d\Theta/\Theta$ , where  $\Theta \ll 1$  is the angle between outgoing couples of partons in independent partonic emissions, follows from Angular Ordering (AO) in intra-jet cascades [4]. In order to obtain the hadronic spectra, we advocate for the Local Parton Hadron Duality (LPHD) hypothesis [6]: global and differential partonic observables can be normalized to the corresponding hadronic observables via a certain constant  $\mathcal{K}$  that can be fitted to the data, i.e.  $N_{q,q}^h = \mathcal{K} \times N_{q,q}$ .

The evolution of a jet of energy E and half-opening angle  $\Theta$  involves the DLA anomalous dimension  $\gamma_0$  related to the coupling constant  $\alpha_s$  through  $\gamma_0^2 = 2N_c\alpha_s/\pi$ , with  $\alpha_s = 2\pi/4N_c\beta_0(Y+\lambda)$ , where  $Y = \ln(Q/Q_0)$  ( $Q = E\Theta$  is the hardness or maximum transverse momentum of the jet),  $\lambda = \ln(Q_0/\Lambda_{\rm QCD})$  is a parameter associated with hadronization ( $Q_0$  is the collinear cut-off parameter,  $k_T > Q_0$ , and  $\Lambda_{\rm QCD}$  is the intrinsic QCD scale) and  $\beta_0 = \frac{1}{4N_c} \left(\frac{11}{3}N_c - \frac{4}{3}T_R\right)$ , where  $T_R = n_f/2$ ,  $n_f$  being the number of active flavors. At MLLA, as a consequence of angular ordering in parton cascading, the average multiplicity inside a gluon and a quark jet,  $N_{g,q}$ , obey the system of two-coupled evolution equations [7]

(the subscript Y denotes d/dY)

$$N_{gY} = \int_{0}^{1} dx \,\gamma_{0}^{2} \left[ \Phi_{g}^{g} \left( N_{g}(x) + N_{g}(1-x) - N_{g} \right) + n_{f} \Phi_{g}^{q} \left( N_{q}(x) + N_{q}(1-x) - N_{g} \right) \right], \quad (2)$$

$$N_{qY} = \int_0^1 dx \,\gamma_0^2 \left[ \Phi_q^g \left( N_g(x) + N_q(1-x) - N_g \right) \right],\tag{3}$$

which follow from the MLLA master evolution equation for the generating functional;  $N_{g,q} \equiv N_{g,q}(Y)$ ,  $N_{g,q}(x) \equiv N_{g,q}(Y + \ln x)$ ,  $N_{g,q}(1 - x) \equiv N_{g,q}(Y + \ln(1 - x))$ ,  $\Phi_A^B$  denotes the medium-modified DGLAP splitting functions:

$$\Phi_{g}^{g}(x) = \frac{N_{s}}{x} - (1-x)[2-x(1-x)],$$

$$\Phi_{g}^{q}(x) = \frac{1}{4N_{c}}[x^{2} + (1-x)^{2}],$$

$$\Phi_{q}^{g}(x) = \frac{C_{F}}{N_{c}}\left(\frac{N_{s}}{x} - 1 + \frac{x}{2}\right),$$
(4)

which accounts for parton energy loss in the medium by enhancing the singular terms like  $\Phi \approx N_s/x$  as  $x \ll 1$  as proposed in the Borghini-Wiedemann model [3]. Thus, when  $N_s$  increases the DLA becomes dominant and energy-momentum conservation plays a less important role.

For  $Y \gg \ln x \sim \ln(1-x)$ ,  $N_{g,q}(x)$   $(N_{g,q}(1-x))$  can be replaced by  $N_{g,q}$  in the hard partonic splitting region  $x \sim 1 - x \sim 1$  (non-singular or regular parts of the splitting functions), while the dependence at small  $x \ll 1$  is kept in the singular term  $\Phi(x) \approx N_s/x$  as done in the vacuum. Furthermore, the integration over x can be replaced by the integration over  $Y(x) = \ln\left(\frac{xE\Theta}{Q_0}\right)$ . Thus, one is left with the approximate system of two-coupled equations,

$$\frac{d^2}{dY^2}N_g(Y) = \gamma_0^2 \left(N_s - a_1 \frac{d}{dY}\right) N_g(Y), \tag{5}$$

$$\frac{d^2}{dY^2}N_q(Y) = \frac{C_F}{N_c}\gamma_0^2 \left(N_s - \tilde{a}_1\frac{d}{dY}\right)N_g(Y),\tag{6}$$

with the initial conditions at threshold  $N_A(0) = 1$  and  $N'_A(0) = 0$  and the hard constants:

$$a_1 = \frac{1}{4N_c} \left[ \frac{11}{3} N_c + \frac{4}{3} T_R \left( 1 - 2 \frac{C_F}{N_c} \right) \right], \quad \tilde{a}_1 = 3/4.$$

The quantum corrections  $\propto a_1$ ,  $\tilde{a}_1$  in (5,6) arise from the integration over the regular part of the splitting functions, they are  $\mathcal{O}(\sqrt{\alpha_s/N_s})$  suppressed and *partially* account for energy conservation as happens in the vacuum.

These equations can be solved by applying the inverse Mellin transform to the self-contained gluonic equation (5), which leads to

$$N_g^h(Y) \simeq \mathcal{K} \times \int_C \frac{d\omega}{2\pi i} \omega^{\frac{a_1}{\beta_0} - 2} \exp\left[\omega(Y + \lambda) + \frac{N_s}{\beta_0 \omega}\right],\tag{7}$$

where the contour C lies to the right of all singularities of  $N_g(\omega)$  in the complex plane. Since we are concerned with the asymptotic solution of the equation as  $Y \gg 1$  ( $E\Theta \gg Q_0$ ), that is the high-energy limit, the inverse Mellin transform (7) can be estimated by the steepest descent method. Indeed, the large

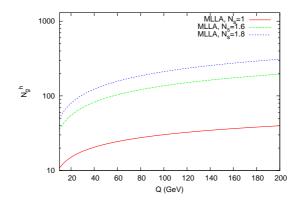


Figure 1: MLLA (8) medium-modified average multiplicity as a function of  $Q = E\Theta$  in the vacuum  $(N_s = 1)$  and in the medium  $(N_s = 1.6 \text{ and } N_s = 1.8)$  for  $n_f = 3$ .

parameter is Y and the function in the exponent presents a saddle point at  $\omega_0 = \sqrt{N_s/\beta_0(Y+\lambda)}$ , such that the asymptotic solution reads

$$N_g^h(Y) \simeq \mathcal{K} \times (Y+\lambda)^{-\frac{\sigma_1}{\beta_0}} \exp\sqrt{\frac{4N_s}{\beta_0}(Y+\lambda)},\tag{8}$$

where  $\sigma_1 = \frac{a_1}{2} - \frac{\beta_0}{4}$ . The constant  $\sigma_1$  is  $N_s$ -independent because it resums vacuum corrections. Therefore, the production of soft gluons in the medium becomes  $\exp\left[2(\sqrt{N_s}-1)\sqrt{(Y+\lambda)/\beta_0}\right]$  higher than the standard production of soft gluons in the vacuum [4]. From (1) and (8) one obtains the mediummodified MLLA anomalous dimension  $\gamma_{\text{med}} = \frac{1}{N_g} \frac{dN_g}{dY} = \sqrt{N_s}\gamma_0 - \sigma_1\gamma_0^2$ , which is nothing but the MLLA rate of multi-particle production with respect to the *evolution-time* variable Y in the dense medium. In Fig. 1, we display the medium-modified average multiplicity (8) with predictions in the vacuum ( $N_s = 1$ ) in the range  $10 \leq Q(\text{GeV}) \leq 200$ ; we set  $Q_0 = \Lambda_{\text{QCD}} = 0.23$  GeV in the limiting spectrum approximation [7], and take  $\mathcal{K} = 0.2$  from [7]. The values  $N_s = 1.6$  and  $N_s = 1.8$ in the medium may be realistic for RHIC and LHC phenomenology [3, 5]; the jet energy subrange  $10 \leq Q(\text{GeV}) \leq 50$  displayed in Fig. 1 has been recently considered by the STAR collaboration, which reported the first measurements of charged hadrons and particle-identified fragmentation functions from p+p collisions [8] at  $\sqrt{s_{\text{NN}}} = 200$  GeV. Finally, the whole jet energy range in the same figure, in particular for those values at  $Q \geq 50$  GeV, will be reached at the LHC, i.e Q = 100 GeV is an accessible value in this experiment (see [3] and references therein).

We find, as expected, that the production of soft hadrons increases as  $N_s > 1$ : the available phase space for the production of harder collinear hadrons is restricted as the model itself states. The mediummodified MLLA gluon to quark average multiplicity ratio,  $r = N_g/N_q = N_g^h/N_q^h$ , following from (8) and (3) reads

$$r = r_0 \left[ 1 - r_1 \frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right) \right], \ r_0 = \frac{N_c}{C_F},\tag{9}$$

where we introduced the coefficient  $r_1 = a_1 - \tilde{a}_1$  in the term suppressed by  $\gamma_0/\sqrt{N_s}$  as  $N_s > 1$ . Therefore, if compared with its behavior at  $N_s = 1$ , we check, as expected from the model [3], that r becomes closer to its asymptotic DLA limit  $r_0 = N_c/C_F = 9/4$ , as depicted in Fig. 2. Setting  $N_s = 1$  in (9), one recovers the appropriate limits in the vacuum [4, 9, 10]. Finally, the gluon jets are still more

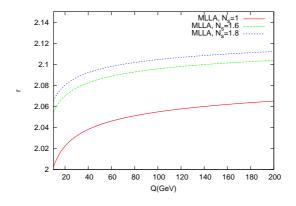


Figure 2: MLLA ratio r (9) as a function of  $Q = E\Theta$  in the vacuum ( $N_s = 1$ ) and in the medium ( $N_s = 1.6$  and  $N_s = 1.8$ ) for  $n_f = 3$ .

active than the quark jets in producing secondary particles and the shape of the curves are roughly the same.

The normalized second multiplicity correlator  $A_2 = \langle N_A(N_A - 1) \rangle / N_A^2$  defines the width of the multiplicity distribution and is related to its dispersion by the formula  $D_A^2 = (A_2 - 1)N_A^2 + N_A$  [9]. These moments, which are less inclusive than the average multiplicity, prove to be  $\mathcal{K}$ -independent and therefore provide a pure test of multiparticle production. The medium-modified system of two-coupled evolution equations for this observable follows from the MLLA master equation for the azimuthally averaged generating functional [4] and can be written in the convenient form

$$\frac{d}{dY}(N_g^{(2)} - N_g^2) = \int_0^1 dx \gamma_0^2 \Phi_g^g \left[ N_g^{(2)}(Y + \ln x) + \left( N_g^{(2)}(Y + \ln(1 - x)) - N_g^{(2)}(Y) \right) \right. \\ \left. + \left( N_g(Y + \ln x) - N_g(Y) \right) \left( N_g(Y + \ln(1 - x)) - N_g(Y) \right) \right] \\ \left. + n_f \int_0^1 dx \gamma_0^2 \Phi_g^q \left[ 2 \left( N_q^{(2)}(Y + \ln x) - N_q^2(Y + \ln x) \right) - \left( N_g^{(2)}(Y) - N_g^2(Y) \right) \right. \\ \left. + \left( 2N_q(Y + \ln x) - N_g(Y) \right) \left( 2N_q(Y + \ln(1 - x)) - N_g(Y) \right) \right], \tag{10}$$

$$\frac{d}{dY}(N_q^{(2)} - N_q^2) = \int_0^1 dx \gamma_0^2 \Phi_q^g \left[ N_g^{(2)}(Y + \ln x) + \left( N_q^{(2)}(Y + \ln(1 - x)) - N_q^{(2)}(Y) \right) + 2 \left( N_g(Y + \ln x) - N_q(Y) \right) \left( N_q(Y + \ln(1 - x)) - N_q(Y) \right) \right], \quad (11)$$

which proves to be more suitable for obtaining analytical solutions in the following. We use a new method to compute solutions at MLLA by replacing  $N_A^{(2)} = A_2 N_A^2$  on both sides of the expanded equations at  $x \sim 1 - x \sim 1$ . The notations in (10,11) follow the same logic as those in (5,6). Applying the analysis that led to the system (5,6), we obtain from (10,11)

$$\frac{d^2}{dY^2} \left( N_g^{(2)} - N_g^2 \right) = \gamma_0^2 \left( N_s - a_1 \frac{d}{dY} \right) N_g^{(2)} + (a_1 - b_1) \gamma_0^2 \frac{d}{dY} N_g^2, \tag{12}$$

$$\frac{d^2}{dY^2} \left( N_q^{(2)} - N_q^2 \right) = \frac{C_F}{N_c} \gamma_0^2 \left( N_s - \tilde{a}_1 \frac{d}{dY} \right) N_g^{(2)},\tag{13}$$

where

$$b_1 = \frac{1}{4N_c} \left[ \frac{11}{3} N_c - 4 \frac{T_R}{N_c} \left( 1 - 2 \frac{C_F}{N_c} \right)^2 \right].$$

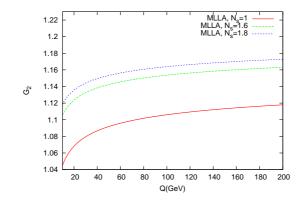


Figure 3: MLLA second multiplicity correlator inside a gluon jet (15) as a function of  $Q = E\Theta$  in the vacuum ( $N_s = 1$ ) and in the medium ( $N_s = 1.6$  and  $N_s = 1.8$  for  $n_f = 3$ .

The constant  $N_s$  only affects the leading double logarithmic term of the equations. The terms proportional to  $a_1$ ,  $(a_1 - b_1)$  and  $\tilde{a}_1$  are hard vacuum corrections, which *partially* account for energy conservation, indeed  $\gamma_0^2 \frac{dN}{dY} \approx \sqrt{N_s} \gamma_0^3$  and the relative correction to DLA is  $\mathcal{O}(\sqrt{\alpha_s/N_s})$ .

Setting  $N_g^{(2)} = G_2 N_g^2$  in (12) and making use of (8), the system can be solved iteratively by taking terms up to  $\mathcal{O}(\alpha_s)$  into consideration. The analytical solution reads,

$$G_{2} - 1 = \frac{1 - \left(\frac{2}{3}a_{1} + 2b_{1}\right)\frac{\gamma_{0}}{\sqrt{N_{s}}} + \mathcal{O}\left(\frac{\gamma_{0}^{2}}{N_{s}}\right)}{3 - (4a_{1} - \beta_{0})\frac{\gamma_{0}}{\sqrt{N_{s}}} + \mathcal{O}\left(\frac{\gamma_{0}^{2}}{N_{s}}\right)},$$
(14)

while its expansion in the form  $1 + \gamma_0 / \sqrt{N_s}$  leads to

$$G_2 - 1 \approx \frac{1}{3} - c_1 \frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right),\tag{15}$$

where the linear combination of color factors can be written in the form

$$c_1 = \frac{1}{4N_c} \left( \frac{55}{9} - 4\frac{T_R}{N_c} + \frac{112}{9} \frac{T_R}{N_c} \frac{C_F}{N_c} - \frac{32}{3} \frac{T_R}{N_c} \frac{C_F^2}{N_c^2} \right).$$
(16)

We use (15) and (9) and substitute  $N_q^{(2)} = Q_2 N_q^2$  into (13) such that the solution reads

$$Q_2 - 1 \approx \frac{N_c}{C_F} \left( \frac{1}{3} - \tilde{c}_1 \frac{\gamma_0}{\sqrt{N_s}} \right) + \mathcal{O}\left( \frac{\gamma_0^2}{N_s} \right), \tag{17}$$

where we obtain the combination of color factors

$$\tilde{c}_1 = \frac{1}{4N_c} \left( \frac{55}{9} + \frac{4}{9} \frac{T_R}{N_c} \frac{C_F}{N_c} - \frac{8}{3} \frac{T_R}{N_c} \frac{C_F^2}{N_c^2} \right).$$
(18)

Setting  $N_s = 1$  in (15) and (17) we get a perfect agreement with the vacuum results [9]. In Fig. 3 and Fig. 4, we compare our results for the medium-modified second multiplicity correlators (15) and (17) with predictions in the vacuum ( $N_s = 1$ ) [9] in the limiting spectrum approximation inside the typical range  $10 \le Q(\text{GeV}) \le 200$  for RHIC and LHC phenomenology. Similarly to the MLLA ratio  $r(N_s)$ , Eq. (9), the hard corrections  $\mathcal{O}(\gamma_0)$  are suppressed by a factor  $1/\sqrt{N_s}$ . As expected from the model,

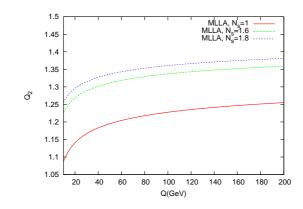


Figure 4: MLLA second multiplicity correlator inside a quark jet (17).

we check that these results approach their DLA limits when  $N_s$  increases; moreover, the multiplicity fluctuations of individual events must be larger for quark jets as compared to gluon jets just like in the vacuum [9]. Another interesting feature of these observables concerns the shape of the curves. They are roughly identical and prove not to depend on the medium parameter  $N_s$ . Moreover, there exists evidence for a flattening of the slopes as the hardness of the jet  $Q = E\Theta$  increases for  $N_s \ge 1$  (vacuum and medium). This kind of scaling behavior is known as the Koba-Nielsen-Olsen (KNO) scaling [11]: it was discovered by Polyakov in quantum field theory [12] and experimentally confirmed by  $e^+e^$ measurements [13] for the second and higher order multiplicity correlators.

In this paper we have dealt with the medium-modified average multiplicity and the medium-modified second multiplicity correlator in quark and gluon jets at RHIC and LHC energy scales. The starting point of our calculations is based on the Borghini-Wiedemann work [3], which models parton energy loss in a nuclear medium. The average multiplicity is found to be enhanced by the factor  $\sqrt{N_s}$  acting on the exponential leading contribution (8); this leads in particular to the rescaling of the anomalous dimension  $\gamma_{\text{med}} (\gamma \rightarrow \gamma_{\text{med}} \approx \sqrt{N_s}\gamma_0)$  or equivalently, to the enhancement of the in medium coupling constant. Since hard corrections are suppressed by the extra factor  $1/\sqrt{N_s}$ , it is straightforward to check that r,  $G_2$  and  $Q_2$  approach the asymptotic DLA limits  $r_0 = N_c/C_F$ ,  $G_2 = 4/3$  and  $Q_2 = 1 + N_c/3C_F$  [4] when  $N_s$  increases. The previously mentioned KNO-scaling experienced by  $G_2$  and  $Q_2$  proves no special sensibility to the model and should normally hold like in the vacuum.

Finally, since these results are model-dependent, they may still be improved in the future, specially after the  $N_s$ -dependence of the non-singular parts of the splitting functions (4) has been exactly computed.

Perspective: Many experimental characterizations of the medium-modified intrajet structure in heavyion collisions at RHIC and at the LHC require a soft momentum cut-off  $p_T^{\text{cut}}$ , with  $Q > p_T^{\text{cut}}$  to remove the effects of the high multiplicity background. In [3], the soft background was subtracted by integrating the single inclusive differential distribution  $\frac{dN}{d\ln p_T}$  ("hump-backed plateau") over the range  $Q \ge p_T \ge p_T^{\text{cut}}$ , with  $p_T^{\text{cut}} > \Lambda_{QCD}$ . Accordingly, the equivalent computation should be performed for the second multiplicity correlator by integrating the double differential inclusive distribution (two-particle correlation)  $\frac{d^2N}{d\ln p_{1,T}d\ln p_{2,T}}$  over  $p_{i,T}$ , with the lower bounds of integration  $p_{i,T}^{\text{cut}} > \Lambda_{QCD}$  (i = 1, 2). Imposing such a cut-off in our calculations will affect the normalization rather than the behavior and the shape of these observables as a function of  $N_s$  and the jet energy scale of the process Q [14].

#### References

- K. Adcox et al. (PHENIX Collab.), Phys. Rev. Lett. 88 (2002) 022301;
   S.S. Adler et al. (PHENIX Collab.), Phys. Rev. Lett. 91 (2003) 072301;
   C. Adler et al. (STAR Collab.), Phys. Rev. Lett 89 (2002) 202301.
- [2] F. Arleo, hep-ph/0810.1193 and references therein;S. Peigné & A.V. Smilga, hep-ph/0810.5702.
- [3] N. Borghini & U.A. Wiedemann, hep-ph/0506218.
- [4] Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller & S.I. Troyan, Basics of Perturbative QCD, Editions Frontières, Paris (1991).
- [5] S.Sapeta & U.A Wiedemann, Eur. Phys. J. C55 (2008) 293.
- [6] Ya.I. Azimov, Yu.L. Dokshitzer, V.A. Khoze & S.I. Troian, Z. Phys. C 27 (1985) 65; Yu.L. Dokshitzer, V.A. Khoze & S.I. Troian, J. Phys. G 17 (1991) 1585.
- [7] I.M. Dremin, V.A. Nechitailo, Mod. Phys. Lett. A 9 (1994) 1471; JETP Lett. 58 (1993) 945.
- [8] M. Heinz for the STAR Collaboration, nucl-exp/0809.3769.
- [9] E.D. Malaza & B.R. Webber, Phys. Lett. B 149 (1984) 501; E.D. Malaza & B.R. Webber, Nucl. Phys. B 267 (1986) 702.
- [10] V.A. Khoze & W. Ochs, Int. J. Mod. Phys. A 12 (1997) 2949.
- [11] Yu.L. Dokshitzer, Phys. Lett. B 305 (1993) 295.
- [12] A.M. Polyakov, Sov. Phys. JETP 32 (1971) 296.
- [13] HRS Coll., Phys. Rev. D 34 (1986) 3304; AMY Coll., Phys. Rev. D 42 (1990) 737; DELPHI Coll.,
   Z. Phys. C-Particles and Fields 50 (1991) 185.
- [14] N. Borghini & R. Perez-Ramos, in preparation.