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Redamy Perez Ramos. Medium-modified average multiplicity and multiplicity fluctuations in jets. 2008. hal-00338947v1

HAL Id: hal-00338947

<https://hal.science/hal-00338947v1>

Preprint submitted on 14 Nov 2008 (v1), last revised 19 May 2009 (v2)

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Medium-modified average multiplicity and multiplicity fluctuations in jets

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The energy evolution of medium-modified average multiplicities and multiplicity fluctuations in quark and gluon jets produced in heavy-ion collisions is discussed at next-to-leading order (NLO) in perturbative QCD. We use modified splitting functions, accounting for induced soft gluon radiation from a hard parton, which propagates through quark gluon plasma. The leading contribution of the standard production of soft hadrons is found to be enhanced by the factor $\sqrt{N_s}$ while NLO corrections are suppressed by $1/\sqrt{N_s}$, where the parameter $N_s > 1$ accounts for the induced-soft gluons in the hot medium. Our results for such global observables are cross-checked and compared with their limits in the vacuum.

Recent experiments at the Relativistic Heavy Ion Collider (RHIC) have established a phenomenon of strong high-transverse momentum hadron suppression [1], which supports the picture that hard partons going through dense matter suffer a significant energy loss prior to hadronization in the vacuum (for recent review see [2]).

Predictions concerning multi-particle production in such reactions can be carried out by using the QCD interpretation of the medium-induced modification of the one-particle single inclusive distribution inside high energy jets [3], the so-called “distorted hump-backed plateau”, which was considered by Borghini and Wiedemann in [4]. The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) splitting functions [3] of the QCD evolution equations were distorted so that the role of soft emissions was enhanced by introducing the medium factor N_s in the infra-red singular terms.

In this letter we are concerned with multi-particle production in quark and gluon jets, $A = q, g$, produced in nucleus-nucleus collisions at very high energy. We make predictions at NLO for the medium-modified average multiplicity N_A , for the ratio $r = N_G/N_Q$ and finally for the second multiplicity correlators $\langle N_A(N_A - 1) \rangle / N_A^2$ which defines the width of the multiplicity distribution.

The starting point of our analysis is the NLO or Modified-Leading-Logarithmic-Approximation (MLLA) master evolution equation for the generating functional [3] which determine the jet properties at all energies together with the initial conditions at threshold at small x , where x is the fraction of the outgoing jet energy carried away by a single gluon. Their solutions with medium-modified splitting functions can be resummed in powers of $\sqrt{\alpha_s/N_s}$ and the leading contribution can be represented as an exponential of the medium-modified anomalous dimension which takes into account the N_s -dependence:

$$N_A \simeq \exp \left\{ \int^Y \gamma_{\text{med}}(\alpha_s(Y)) dY \right\}, \quad (1)$$

where $\gamma_{\text{med}}(\alpha_s)$ can be expressed as a power series of

$\sqrt{\alpha_s/N_s}$ in the symbolic form:

$$\gamma_{\text{med}}(\alpha_s) \simeq \sqrt{N_s} \times \sqrt{\alpha_s} \left(1 + \sqrt{\frac{\alpha_s}{N_s}} + \mathcal{O}\left(\frac{\alpha_s}{N_s}\right) \right).$$

Within this logic, the leading double logarithmic approximation (DLA, $\mathcal{O}(\sqrt{N_s\alpha_s})$), which resums both soft and collinear gluons, and NLO (MLLA, $\mathcal{O}(\alpha_s)$), which resums hard collinear partons and accounts for the running of the coupling constant α_s , are complete. The choice $dY = d\Theta/\Theta$, where $\Theta \ll 1$ is the angle between outgoing couples of partons in independent partonic emissions, follows from Angular Ordering (AO) in intra-jet cascades [3]. In order to obtain the hadronic spectra, we advocate for the Local Parton Hadron Duality (LPHD) hypothesis [5]: global and differential partonic observables can be normalized to the corresponding hadronic observables via a certain constant \mathcal{K} that can be fitted to the data, i.e. $N_{g,q}^h = \mathcal{K} \times N_{g,q}$.

The evolution of a jet of energy E and half-opening angle Θ involves the DLA anomalous dimension γ_0 related to the coupling constant α_s through $\gamma_0^2 = 2N_c\alpha_s/\pi$, with $\alpha_s = 2\pi/4N_c\beta_0(Y + \lambda)$, where $Y = \ln(Q/Q_0)$ ($Q = E\Theta$ is the hardness or maximum transverse momentum of the jet) and $\lambda = \ln(Q_0/\Lambda_{\text{QCD}})$ is a parameter associated with hadronization (Q_0 is the collinear cut-off parameter, $k_T > Q_0$, and Λ_{QCD} is the intrinsic QCD scale). At MLLA, as a consequence of angular ordering in parton cascading, the average multiplicity inside a gluon and a quark jet, $N_{g,q}$, obey the system of two-coupled evolution equations [6] (the subscript Y denotes d/dY)

$$N_{gY} = \int_0^1 dx \gamma_0^2 \left[\Phi_g^g(N_g(x) + N_g(1-x) - N_g) + n_f \Phi_g^q(N_q(x) + N_q(1-x) - N_g) \right], \quad (2)$$

$$N_{qY} = \int_0^1 dx \gamma_0^2 \left[\Phi_q^g(N_g(x) + N_q(1-x) - N_g) \right], \quad (3)$$

which follow from the MLLA master evolution equation for the generating functional; n_f is the number of light quark flavors, $N_{g,q} \equiv N_{g,q}(Y)$, $N_{g,q}(x) \equiv N_{g,q}(Y + \ln x)$, $N_{g,q}(1-x) \equiv N_{g,q}(Y + \ln(1-x))$, Φ_A^B denotes the

medium-modified DGLAP splitting functions:

$$\Phi_g^q(x) = \frac{N_s}{x} - (1-x)[2-x(1-x)], \quad (4)$$

$$\Phi_g^q(x) = \frac{1}{4N_c} [x^2 + (1-x)^2], \quad (5)$$

$$\Phi_q^g(x) = \frac{C_F}{N_c} \left(\frac{N_s}{x} - 1 + \frac{x}{2} \right), \quad (6)$$

which accounts for parton energy loss in the medium by enhancing the singular terms like $\Phi \approx N_s/x$ as $x \ll 1$ as proposed in the Borghini-Wiedemann model [4].

For $Y \gg \ln x \sim \ln(1-x)$, $N_{g,q}(x)$ ($N_{g,q}(1-x)$) can be replaced by $N_{g,q}$ in the hard partonic splitting region $x \sim 1-x \sim 1$ (non-singular or regular parts of the splitting functions), while the dependence at small $x \ll 1$ is kept in the singular term $\Phi(x) \approx N_s/x$ as done in the vacuum. Furthermore, the integration over x can be replaced by the integration over $Y(x) = \ln\left(\frac{x E \Theta}{Q_0}\right)$. Thus, one is left with the approximate system of two-coupled equations,

$$\frac{d^2}{dY^2} N_g(Y) = \gamma_0^2 \left(N_s - a_1 \frac{d}{dY} \right) N_g(Y), \quad (7)$$

$$\frac{d^2}{dY^2} N_q(Y) = \frac{C_F}{N_c} \gamma_0^2 \left(N_s - \tilde{a}_1 \frac{d}{dY} \right) N_q(Y), \quad (8)$$

with the initial conditions at threshold $N_A(0) = 1$ and $N'_A(0) = 0$ and the hard constants:

$$a_1 = \frac{1}{4N_c} \left[\frac{11}{3} N_c + \frac{4}{3} T_R \left(1 - 2 \frac{C_F}{N_c} \right) \right], \quad a_1 = 3/4,$$

where we have introduced $T_R = n_f/2$. The quantum corrections $\propto a_1, \tilde{a}_1$ in (7,8) arise from the integration over the regular part of the splitting functions, they are $\mathcal{O}(\sqrt{\alpha_s/N_s})$ suppressed and *partially* account for energy conservation as happens in the vacuum.

These equations can be solved by applying the inverse Mellin transform to the self-contained gluonic equation (7), which leads to

$$N_g(Y) \simeq \int_C \frac{d\omega}{2\pi i} \omega^{\frac{a_1}{\beta_0} - 2} \exp \left[\omega(Y + \lambda) + \frac{N_s}{\beta_0 \omega} \right], \quad (9)$$

where the contour C lies to the right of all singularities of $N_g(\omega)$ in the complex plane. Since we are concerned with the asymptotic solution of the equation as $Y \gg 1$ ($E\Theta \gg Q_0$), that is the high-energy limit, the inverse Mellin transform (9) can be estimated by the steepest descent method. Indeed, the large parameter is Y and the function in the exponent presents a saddle point at $\omega_0 = \sqrt{N_s/\beta_0(Y + \lambda)}$, such that the asymptotic solution reads

$$N_g^h(Y) \simeq \mathcal{K} \times (Y + \lambda)^{-\frac{\sigma_1}{\beta_0}} \exp \sqrt{\frac{4N_s}{\beta_0} (Y + \lambda)}, \quad (10)$$

where $\sigma_1 = \frac{a_1}{2} - \frac{\beta_0}{4}$ with $\beta_0 = \frac{1}{4N_c} \left(\frac{11}{3} N_c - \frac{4}{3} T_R \right)$. The constant σ_1 is N_s -independent because it resums vacuum

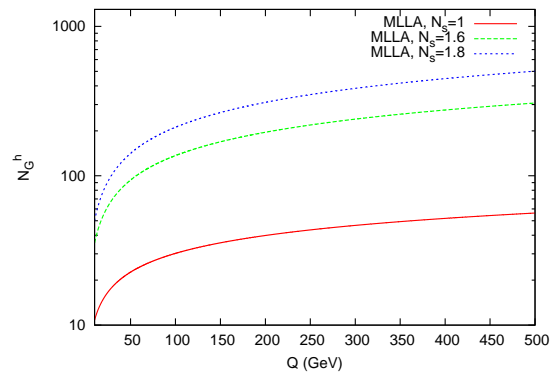


FIG. 1: MLLA (10) medium-modified average multiplicity as a function of $Q = E\Theta$ in the vacuum ($N_s = 1$) and in the medium ($N_s = 1.6$ and $N_s = 1.8$) for $n_f = 3$.

corrections. Therefore, the production of soft gluons in a dense medium becomes $\exp \left[2(\sqrt{N_s} - 1) \sqrt{(Y + \lambda)/\beta_0} \right]$ higher than the standard production of soft gluons in the vacuum [3]. From (1) and (10) one obtains the medium-modified MLLA anomalous dimension $\gamma_{\text{med}} = \frac{1}{N_g} \frac{dN_g}{dY} = \sqrt{N_s} \gamma_0 - \sigma_1 \gamma_0^2$, which is nothing but the MLLA rate of multi-particle production with respect to the *evolution-time* variable Y in the dense medium. Fixing $\gamma_{\text{med}} = \text{const}$, we are able to give the rough dependence of the coupling constant as a function of N_s , that is $\alpha_s(N_s) \simeq 1/N_s$. This behavior follows the trends first observed in [7, 8] with fixed-coupling numerical solution. In Fig. 1, we display the medium-modified average multiplicity (10) with predictions in the vacuum ($N_s = 1$) in the range $10 \leq Q(\text{GeV}) \leq 500$; we set $Q_0 = \Lambda_{\text{QCD}} = 0.23$ GeV in the limiting spectrum approximation [6], and take $\mathcal{K} = 0.2$ from [6]. The values $N_s = 1.6$ and $N_s = 1.8$ in the medium may be realistic for RHIC and LHC phenomenology [4]. We find, as expected, that the production of soft hadrons increases as $N_s > 1$, which implies that the available phase space for the production of harder collinear hadrons is restricted.

The medium-modified MLLA gluon to quark average multiplicity ratio, $r = N_g/N_q = N_g^h/N_q^h$, following from (10) and (3) reads

$$r = r_0 \left[1 - r_1 \frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O} \left(\frac{\gamma_0^2}{N_s} \right) \right], \quad r_0 = \frac{N_c}{C_F}, \quad (11)$$

where we introduced the coefficient $r_1 = a_1 - \tilde{a}_1$ in the term suppressed by $\gamma_0/\sqrt{N_s}$ as $N_s > 1$. Therefore, r approaches its asymptotic DLA limit $r_0 = N_c/C_F = 9/4$ when the coherent radiation of soft gluons is enhanced by the medium, as depicted in Fig. 2. Moreover, r_0 is not affected by N_s and is seen to be universal. Setting $N_s = 1$ in (11), one recovers the appropriate limits in the vacuum [3, 9, 10]. This observable is interesting because it is \mathcal{K} -independent, and it is therefore expected to provide a pure test of multi-particle dynamics [5]. Finally, the

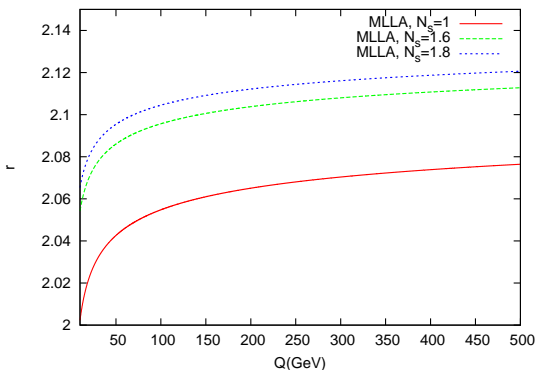


FIG. 2: MLLA ratio r (11) as a function of $Q = E\Theta$ in the vacuum ($N_s = 1$) and in the medium ($N_s = 1.6$ and $N_s = 1.8$) for $n_f = 3$.

gluon jets are still more active than the quark jets in producing secondary particles and the shape of the curves are roughly the same.

The second multiplicity correlator in quark and gluon jets was first considered in [9] at MLLA in the vacuum. It is defined in the form $N_A^{(2)} = \langle N_A(N_A - 1) \rangle$ in gluon ($A = g$) and quark ($A = q$) jets. The normalized second multiplicity correlator defines the width of the multiplicity distribution and is related to its dispersion $D_A^2 = \langle N_A(N_A - 1) \rangle - N_A^2$ by the formula $D_A^2 = (A_2 - 1)N_A^2 + N_A$. The second multiplicity correlators normalized to their own squared average multiplicity are $G_2 = \langle N_g(N_g - 1) \rangle / N_g^2$ and $Q_2 = \langle N_q(N_q - 1) \rangle / N_q^2$ inside a gluon and a quark jet respectively. These moments, which are less inclusive than the average multiplicity, prove to be \mathcal{K} -independent. The medium-modified system of two-coupled evolution equations for this observable follows from the MLLA master equation for the azimuthally averaged generating functional [3] and can be written in the convenient form

$$(N_g^{(2)} - N_g^2)_Y = \int_0^1 dx \gamma_0^2 \Phi_g^g \left[N_g^{(2)}(x) + (N_g^{(2)}(1-x) - N_g^{(2)}) + (N_g(x) - N_g)(N_g(1-x) - N_g) \right] \\ + n_f \int_0^1 dx \gamma_0^2 \Phi_g^q \left[2(N_q^{(2)}(x) - N_q^2(x)) - (N_g^{(2)} - N_g^2) + (2N_q(x) - N_g)(2N_q(1-x) - N_g) \right], \quad (12)$$

$$(N_q^{(2)} - N_q^2)_Y = \int_0^1 dx \gamma_0^2 \Phi_q^g \left[N_g^{(2)}(x) + (N_q^{(2)}(1-x) - N_q^{(2)}) + 2(N_g(x) - N_q)(N_q(1-x) - N_q) \right], \quad (13)$$

which proves to be more suitable for obtaining analytical solutions in the following. We use a new method to compute solutions at MLLA by replacing $N_A^{(2)} = A_2 N_A^2$ on both sides of the expanded equations at $x \sim 1 - x \sim 1$. The notations in (12,13) follow the same logic as those in (7,8).

Applying the analysis that led to the system (7,8), we obtain from (12,13)

$$\frac{d^2}{dY^2} (N_g^{(2)} - N_g^2) = \gamma_0^2 \left(N_s - a_1 \frac{d}{dY} \right) N_g^{(2)} + (a_1 - b_1) \gamma_0^2 \frac{d}{dY} N_g^2, \quad (14)$$

$$\frac{d^2}{dY^2} (N_q^{(2)} - N_q^2) = \frac{C_F}{N_c} \gamma_0^2 \left(N_s - \tilde{a}_1 \frac{d}{dY} \right) N_q^{(2)}, \quad (15)$$

where

$$b_1 = \frac{1}{4N_c} \left[\frac{11}{3} N_c - 4 \frac{T_R}{N_c} \left(1 - 2 \frac{C_F}{N_c} \right)^2 \right].$$

The constant N_s only affects the leading double logarithmic term of the equations. The terms proportional to a_1 , $(a_1 - b_1)$ and \tilde{a}_1 are hard vacuum corrections, which *partially* account for energy conservation, indeed $\gamma_0^2 \frac{dN}{dY} \approx \sqrt{N_s} \gamma_0^3$ and the relative correction to DLA is $\mathcal{O}(\sqrt{\alpha_s/N_s})$.

Setting $N_g^{(2)} = G_2 N_g^2$ in (14) and making use of (10), the system can be solved iteratively by taking terms up to $\mathcal{O}(\alpha_s)$ into consideration. The exact analytical solution reads,

$$G_2 - 1 = \frac{1 - \left(\frac{2}{3} a_1 + 2b_1 \right) \frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right)}{3 - (4a_1 - \beta_0) \frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right)}, \quad (16)$$

while its expansion in the form $1 + \gamma_0/\sqrt{N_s}$ leads to

$$G_2 - 1 \approx \frac{1}{3} - c_1 \frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right), \quad (17)$$

where the linear combination of color factors can be written in the form

$$c_1 = \frac{1}{4N_c} \left(\frac{55}{9} - 4 \frac{T_R}{N_c} + \frac{112 T_R C_F}{9 N_c N_c} - \frac{32 T_R C_F^2}{3 N_c N_c^2} \right). \quad (18)$$

We use (17) and (11) and substitute $N_q^{(2)} = Q_2 N_q^2$ into (15) such that the solution reads

$$Q_2 - 1 \approx \frac{N_c}{C_F} \left(\frac{1}{3} - \tilde{c}_1 \frac{\gamma_0}{\sqrt{N_s}} \right) + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right), \quad (19)$$

where we obtain the combination of color factors

$$\tilde{c}_1 = \frac{1}{4N_c} \left(\frac{55}{9} + \frac{4 T_R C_F}{9 N_c N_c} - \frac{8 T_R C_F^2}{3 N_c N_c^2} \right). \quad (20)$$

Setting $N_s = 1$ in (17) and (19) we get a perfect agreement with the vacuum results [9]. Fixing γ_0 , it is straightforward to obtain the rough behavior of A_2 as a function of N_s : $A_2 = \text{const} - 1/\sqrt{N_s}$, that is increasing with N_s as $N_s > 1$. For $A = Q$, $\text{const} = 1 + N_c/3C_F$ and for $A = G$, $\text{const} = 4/3$. In Fig.3 and Fig.4, we compare

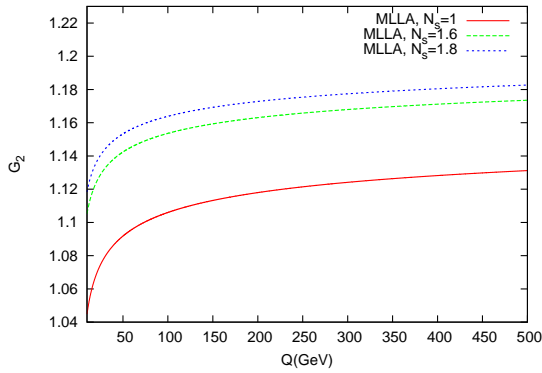


FIG. 3: MLLA second multiplicity correlator inside a gluon jet (17) as a function of $Q = E\Theta$ in the vacuum ($N_s = 1$) and in the medium ($N_s = 1.6$ and $N_s = 1.8$ for $n_f = 3$).

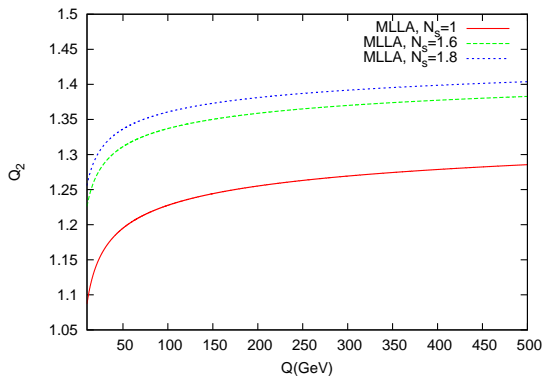


FIG. 4: MLLA second multiplicity correlator inside a quark jet (19).

our results for the medium-modified second multiplicity correlators (17) and (19) with the predictions in the vacuum ($N_s = 1$) in the limiting spectrum approximation. Similarly to the MLLA ratio $r(N_s)$, Eq. (11), the hard corrections $\mathcal{O}(\gamma_0)$ are suppressed by a factor $1/\sqrt{N_s}$. As before, these results provide evidence for the softening of jets in the nuclear medium; moreover, the multiplicity fluctuations of individual events must be larger for quark jets as compared to gluon jets just like in the vacuum [9]. Another interesting feature of these observables concerns the shape of the curves. They are roughly identical and prove not to depend on the medium parameter N_s . Moreover, there exists evidence for a flattening of the slopes as the hardness of the jet $Q = E\Theta$ increases for $N_s \geq 1$ (vacuum and medium). This kind of scaling behavior is known as the Koba-Nielsen-Olsen (KNO) scaling [11]: it was discovered by Polyakov in quantum field theory [12] and experimentally confirmed by e^+e^- measurements [13] for the second and higher order multiplicity correlators. Therefore, the KNO-scaling should

hold in heavy-ion collisions.

In this Letter we have dealt with the medium-modified average multiplicity and the medium-modified second multiplicity correlator in quark and gluon jets. The starting point of our calculations is based on the Borghini-Wiedemann model [4], which models parton energy loss in a dense nuclear medium. The average multiplicity is found, because of multiple rescattering of the hard parton in the medium, to be enhanced by the factor $\sqrt{N_s}$ on the exponential leading contribution (10). The former leads, in particular, to the rescaling of α_s ($\alpha_s \rightarrow \alpha_s/N_s$) and to the medium-modified anomalous dimension γ_{med} ($\gamma \rightarrow \gamma_{\text{med}} \approx \sqrt{N_s}\gamma_0$). Furthermore, the energy increase of the medium-modified average multiplicity at precise values of N_s can serve as a *thermometer* on the nuclear medium of deconfined partons. In general, hard corrections are suppressed by the extra factor $1/\sqrt{N_s}$, which leads to the restriction on the production of hard partons in quark and gluon jets. Thus, r , G_2 and Q_2 approach the asymptotic DLA $r_0 = N_c/C_F$, $G_2 = 4/3$ and $Q_2 = 1 + N_c/3C_F$ [3]; consequently, these vacuum values do not take N_s -dependence and prove to be universal. The previously mentioned KNO-scaling experienced by G_2 and Q_2 also proves its universality.

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