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Nonadditive Quantum Codes from \mathbb{Z}_4 -Codes

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Abstract—Non additive binary quantum codes are constructed by using classical \mathbb{Z}_4 -linear binary codes.

I. INTRODUCTION

The first quantum codes, the so called stabilizer codes were constructed by using linear quaternary codes [3]. Non additive quantum codes were constructed later in [2], [10] by using union of linear codes or permutation group action [9]. In the present work, we construct binary non additive quantum codes from binary \mathbb{Z}_4 -linear codes. The argument is based on a description of quantum codes in terms of orthogonal arrays combined with Delsarte celebrated theorem on the equivalence of unrestricted (viz not necessarily linear) codes with given dual distance and orthogonal arrays of given strength [5].

II. CONSTRUCTION

The following characterization of quantum codes was given in [7]:

Theorem II.1. *There exists a quantum $((n, K, d))_q$ -code with $K \geq 2$ if and only if there exist K nonzero mappings $\phi_i : \mathbb{F}_q^n \rightarrow \mathbb{C}$ ($1 \leq i \leq K$) satisfying the following condition:*

for each partition $\{1, 2, \dots, n\} = A \cup B$ with $|A| = d - 1$ and $|B| = n - d + 1$, and any $\mathbf{c}_A, \mathbf{c}'_A \in \mathbb{F}_q^{d-1}$, $1 \leq i, j \leq n$,

$$\sum_{\mathbf{c}_B \in \mathbb{F}_q^{n-d+1}} \overline{\phi_i(\mathbf{c}_A, \mathbf{c}_B)} \phi_j(\mathbf{c}'_A, \mathbf{c}_B) = \delta_{i,j} f$$

where f is independent of i and depends only on \mathbf{c}_A and \mathbf{c}'_A and δ is Kronecker's symbol.

Instead of [7, Lemma 2.3, Proposition 2.4 and Corollary 2.5], in order to use \mathbb{Z}_4 -linear codes for our construction, we use the following analogous results:

Lemma II.2. *Let C be a linear \mathbb{Z}_4 -code of length n and type $4^{k_1}2^{k_2}$, such that the minimum Lee distance*

$d(C^\perp)$ of the dual code is at least d . Then for each partition $\{1, 2, \dots, 2n\} = A \cup B$ with $|A| = d - 1$ and $|B| = 2n - d + 1$, and any $\mathbf{c}_A \in \mathbb{F}_2^{d-1}$ and $\mathbf{v} \in \mathbb{F}_2^{2n}$, one has

$$\#\{\mathbf{c}_B \in \mathbb{F}_2^{2n-d+1} : (\mathbf{c}_A, \mathbf{c}_B) \in \mathbf{v} + C\} = 2^{2k_1+k_2-d+1}.$$

Proof. The Gray image of C is a binary code (not necessarily linear) of length $2n$ and size $2^{2k_1+k_2}$, whose formal dual distance is $d(C^\perp)$. By the equivalence between codes and orthogonal arrays [5], any translate of this Gray image is an orthogonal array of level 2 and strength $d - 1$. \square

Proposition II.3. *Let C be a linear \mathbb{Z}_4 -code of length n and $V = \{\mathbf{v}_i\}_{i=1}^K$ be a set of K distinct vectors in \mathbb{Z}_4^n . Put*

$$d_v := \min\{w_L(\mathbf{v}_i - \mathbf{v}_j + \mathbf{c}) : 1 \leq i \neq j \leq K \text{ and } \mathbf{c} \in C\}$$

and $d = \min\{d_v, d(C^\perp)\}$, where w_L denotes the Lee weight. If $d > 0$, then the Gray image of $\bigcup_{i=1}^K (\mathbf{v}_i + C)$ is a binary $((n, K, d))$ -quantum code.

Proof. For each $1 \leq i \leq K$, define a mapping $\phi_i : \mathbb{F}_2^{2n} \rightarrow \mathbb{C}$ given by

$$\mathbf{u} \mapsto \begin{cases} 1 & \text{if } \mathbf{u} \in \phi(\mathbf{v}_i + C) \\ 0 & \text{if } \mathbf{u} \notin \phi(\mathbf{v}_i + C). \end{cases}$$

It is necessary to verify that the condition in Theorem II.1 is satisfied. For each partition $\{1, 2, \dots, 2n\} = A \cup B$ with $|A| = d - 1$ and $|B| = 2n - d + 1$, and any $\mathbf{c}_A, \mathbf{c}'_A \in \mathbb{F}_2^{d-1}$,

$$\overline{\phi_i(\mathbf{c}_A, \mathbf{c}_B)} \phi_i(\mathbf{c}'_A, \mathbf{c}_B) \neq 0$$

if and only if

$$\phi_i(\mathbf{c}_A, \mathbf{c}_B) = \phi_i(\mathbf{c}'_A, \mathbf{c}_B) = 1,$$

i.e., $(\mathbf{c}_A, \mathbf{c}_B), (\mathbf{c}'_A, \mathbf{c}_B) \in \phi(\mathbf{v}_i + C)$. This is equivalent to

$$\phi^{-1}(\mathbf{c}_A, \mathbf{c}_B), \phi^{-1}(\mathbf{c}'_A, \mathbf{c}_B) \in \mathbf{v}_i + C, \quad (\text{II.1})$$

