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OBSERVER-BASED H_∞ CONTROL FOR TIME-DELAY SYSTEMS : A NEW LMI SOLUTION

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Abstract: We propose here a new LMI solution to H_∞ observer-controller design that ensures a disturbance attenuation level for the controlled output and for the state estimation error, which is an open problem. This will be compared with a well-known solution. An application to a wind tunnel model is provided.
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1. INTRODUCTION

For sake of simplicity, we consider single delayed-state systems as:

$$\begin{cases} \dot{x} &= A_0x + A_1x_h + Bu + Ew \\ y &= Cx + Fw \\ z &= Dx \\ x(\theta) &= \phi(\theta); \quad \theta \in [-h, 0] \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $x_h = x(t-h)$, $u \in \mathbb{R}^r$ is the control input, $y \in \mathbb{R}^p$ is the output measurement vector, $z \in \mathbb{R}^m$ is the controlled output, $w \in R^q$ is the square-integrable disturbance vector, $\phi(\theta) \in \mathcal{C}[-h, 0]$ is the functional initial condition and $h \in \mathbb{R}^+$ is the delay.

An H_∞ robust observer-controller is here proposed in a delay independent framework, the design of which being much simpler than existing ones. To emphasize the interest of our method, the provided results will be compared (on an illustrative example) with existing strategies (e.g (Choi and Chung, 1996)). On the other hand, when the observer delay is different from the system one (due to uncertainties), we develop here a robustness analysis, and provide the allowable maximal delay uncertainty that preserves the stability of the (extended) closed-loop system.

The contributions of the paper are the following:

- We provide an LMI solution to the H_∞ observer-controller design, much simpler and powerful than previous results.
- The given solution allows to get an H_∞ disturbance attenuation property for the controlled output and for the state estimation errors as well, while, in previous contributions, only the controlled output is considered which makes the observer useless.
- In practice, the observer delay may be different from the system one (which may be unknown or difficult to measure). In this case, we provide here a robustness analysis (with respect to delay uncertainties) and give an evaluation of the maximum allowable uncertainty.

The outline of this paper is as follows. In section 2, some necessary background is developed. In section 3 a new simple method of H_∞ observer-controllers design is proposed in an LMI framework. The robustness of observer-controller scheme is studied in section 4 to get the maximal delay uncertainty that keeps stability. The illustrative example, i.e., the wind tunnel model, is presented in section 5. Finally, some concluding remarks end the paper.

2. BACKGROUND

This part is devoted to the results of (Choi and Chung, 1996) for observer-based controllers of the form (2). Note that it includes a specific term $EG\hat{x}(t)$ that represents the coupling within the observer and the control. In an H_∞ framework it represents an

estimation of the worst possible disturbance (see (Trentelman *et al.*, 2001) for the linear non delay case).

$$\begin{cases} \dot{\hat{x}} = (A_0 + EG)\hat{x} + A_1\hat{x}_h + Bu(t) - L(C\hat{x} - y) \\ u = K\hat{x} \end{cases} \tag{2}$$

The observer and the control are then obtained through two coupled Riccati equations including 7 parameters (2 matrices and 5 constants) which is very involved to use. Nevertheless the estimation error stability is guaranteed for closed-loop systems only and no performance (in terms of H_∞ gain) is ensured for the observer. On the other hand, such a performance is obtained for the closed-loop system, as stated below.

Proposition 1. Consider the time-delay system (1) (with $F = I_p$) and the observer-based controller (2), and suppose that the control parameters are given by:

$$K = -\frac{1}{\epsilon_c}B^T P_c, \quad G = \frac{1}{\gamma^2\epsilon_c}E^T P_c, \quad L = \frac{1}{\epsilon_o}P_o C^T$$

where ϵ_c and ϵ_o are some positive constants, and P_c and P_o are positive definite solution matrices to the following Riccati-like equations for some for some positive constants δ_c and δ_o and some positive-definite weighting matrices Q_c and Q_o:

$$\begin{aligned} & A_0^T P_c + P_c A_0 - \frac{1}{\epsilon_c}P_c(BB^T - \frac{1}{\delta_c}A_1 A_1^T - \frac{1}{\gamma^2}EE^T)P_c \\ & +\epsilon_c(\delta_c I_n + D^T D + Q_c) = 0 \\ & (A_0 + EG)P_o - \frac{1}{\epsilon_o}P_o(C^T C - \frac{1}{\gamma^2}K^T K - \frac{\delta_o}{\gamma^2}I_n)P_o \\ & +P_o(A_0 + EG)^T + \epsilon_o(\frac{\gamma^2}{\delta_o}A_1 A_1^T + EE^T + Q_o) = 0 \end{aligned} \tag{3}$$

Then, for all h, the closed-loop system is asymptotically stable and such that : $\|T_{zw}\|_\infty \leq \gamma$.

3. A NEW H_∞ OBSERVER-CONTROLLER DESIGN METHOD

This section contains the main contribution of this paper, i.e. the design of an H_∞ observer-based controller in an LMI framework. The provided method ensures, not only the stability of the extended system, but also an a priori H_∞ attenuation property of the disturbance, for the controlled output and the estimation error, which is not the case in the previous mixed design methods. This guarantees that the observer can be used, in this control structure, to get a good estimation of the state variables. For instance, this can be useful also for diagnosis purpose where the residuals, generated as the difference between measured and estimated variables, are used for fault detection. First, an H_∞ stability criterion is recalled for a time-delay system of the form (1), as an extension of results in (Gu *et al.*, 2003), or in a similar form as in (Kokame *et al.*, 1998).

Lemma 2. Consider system (1). Given a positive scalar γ, if there exist some positive definite matrices $P = P^T$ and S such that

$$\mathcal{L} = \begin{bmatrix} A_0'P + PA_0 + D'D + S & PA_1 & PE \\ A_1'P & -S & 0 \\ E'P & 0 & -\gamma^2 I_q \end{bmatrix} < 0,$$

then

- the trivial solution of (1) with $w \equiv 0$, $u \equiv 0$, is asymptotically stable for any delay, and
- $\|T_{zw}(s)\|_\infty \leq \gamma$, for zero initial condition and some positive scalar γ

Note that this result can directly be extended to the case of time-varying delay, as precised in (Gu *et al.*, 2003).
Finally the optimal value of the attenuation bound γ can be found by solving

$$\begin{aligned} \gamma*^2_{min} = & \min_{P,S} \gamma^2 \\ & \text{s.t. } \mathcal{L} < 0, \quad P > 0, \quad S > 0, \end{aligned} \tag{4}$$

which is a convex optimization problem.
Now, the following observer-controller scheme is considered:

$$\begin{cases} \dot{\hat{x}} = A_0\hat{x} + A_1\hat{x}_h + Bu - L(C\hat{x} - y) + G\hat{x} \\ u = K\hat{x} \end{cases} \tag{5}$$

Noting $e := x - \hat{x}$, and the extended state $x_e = \begin{bmatrix} x & e \end{bmatrix}^T$, the extended closed-loop system with observer and control (5) is:

$$\begin{aligned} \dot{x}_e &= \begin{bmatrix} A_0 + BK & -BK \\ -G & A_0 - LC + G \end{bmatrix} x_e + \\ & \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} (x_e)_h + \begin{bmatrix} E \\ E - LF \end{bmatrix} w \\ &= \mathcal{A}_0 x_e + \mathcal{A}_1 (x_e)_h + \mathcal{E} w \end{aligned} \tag{6}$$

The aim is here to provide results that ensure H_∞ stability for the controlled output z and the estimation error e. Two different cases are investigated in this part:

(1) obtain an H_∞ stabilization of a new controlled output, combining z and e (with a unique attenuation bound γ)
(2) get the H_∞ stabilization of the controlled output $z = Dx$ and of the estimation error e (with two different attenuation bounds, γ_c and γ_o resp.).

Definition 3. The system (5) is said to be an H_∞ observer-controller for system (1) if

- the trivial solution of (6) with $w(t) \equiv 0$, is asymptotically stable, and
- (1) $\|T_{z_e w}(s)\|_\infty \leq \gamma$ for zero initial condition and some positive scalar γ, with $z_e = \begin{bmatrix} D & 0 \\ 0 & I_n \end{bmatrix} x_e = \mathcal{D}x_e$, or
  (2) $\|T_{z_c w}(s)\|_\infty \leq \gamma_c$ and $\|T_{z_o w}(s)\|_\infty \leq \gamma_o$, $z_c = \begin{bmatrix} D & 0 \end{bmatrix} x_e$ = and $z_o = \begin{bmatrix} 0 & I_n \end{bmatrix} x_e$, for zero initial condition and some positive scalars γ_c and γ_o.

3.1 Case 1

This first case consists in applying the result of lemma 2 to the extended system (6). This leads to the following solution.

Theorem 4. Consider the time-delay system (1) and the observer-controller (5). Given a positive scalar γ, if there exist positive definite matrices $P_c = P_c^T$, $P_o = P_o^T$, S_c and S_o, and some matrices $X \in \mathbb{R}^{m\times n}$, $Y \in \mathbb{R}^{n\times p}$ satisfying the following matrix inequality:

$$\mathcal{L}_{oc} = \begin{bmatrix} M_c & 0 & A_1P_c & 0 & E & P_cD^T \\ * & M_o & 0 & P_oA_1 & P_oE - LF & 0 \\ * & * & -S_c & 0 & 0 & 0 \\ * & * & * & -S_o & 0 & 0 \\ * & * & * & * & -\gamma^2 I_q & 0 \\ * & * & * & * & * & -I_n \end{bmatrix} < 0, \quad (7)$$

where $*$ means the symmetric element, and

$$\begin{aligned} M_c &= A_0P_c + P_cA_0^T + BX + X^TB^T + S_c, \\ M_o &= A^TP_o + P_oA - P_c^{-1}(XB + B^TX^T)P_c^{-1} \\ &\quad + C^TY^Y + YC + I_n + S_o \end{aligned}$$

then the system (5) is an H_∞ observer-controller according to Definition 3, with the disturbance attenuation level γ and the observer-controller gains:

$$L = -P_o^{-1}Y, \; K = XP_c^{-1}, G = -K^TB^TP_c^{-1}P_o^{-1}$$

Proof: The aim of this proof is to apply Lemma 2 to the extended system, which leads to the LMI:

$$\begin{bmatrix} \mathcal{A}_0'\mathcal{P} + \mathcal{P}\mathcal{A}_0 + \mathcal{D}'\mathcal{D} + \mathcal{S} & \mathcal{P}\mathcal{A}_1 & \mathcal{P}\mathcal{E} \\ \mathcal{A}_1'\mathcal{P} & -\mathcal{S} & 0 \\ \mathcal{E}'\mathcal{P} & 0 & -\gamma^2 I_q \end{bmatrix} < 0 \quad (8)$$

Assuming that $\mathcal{P}$ and $\mathcal{S}$ are of the form:

$$\mathcal{P} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \quad \mathcal{S} = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \quad (9)$$

we obtain:

$$\begin{bmatrix} L_{11} & -P_1BK - G^TP_2 & P_1A_1 & 0 & P_1E \\ * & L_{22} & 0 & P_2A_1 & P_2(E - LF) \\ * & * & -S_1 & 0 & 0 \\ * & * & * & -S_2 & 0 \\ * & * & * & * & -\gamma^2 I_q \end{bmatrix} < 0$$

with

$$\begin{aligned} L_{11} &= (A^T + K^TB^T)P_1 + P_1(A + BK) + D^TD + S_1 \\ L_{22} &= (A^T - C^TL^T + G^T)P_2 + P_2(A - LC + G) + I_n + S_2 \end{aligned}$$

Now let us choose G such as $G^TP_2 = -P_1BK$ and multiplying both sides of the previous LMI by $diag([P_1^{-1},\ I_n,\ P_1^{-1},\ I_n,\ I_q])$ it leads to:

$$\mathcal{L}2 < 0, \text{ with}$$

$$\mathcal{L}2 = \begin{bmatrix} L2_{11} & 0 & A_1P_1^{-1} & 0 & E \\ * & L2_{22} & 0 & P_2A_1 & P_2(E - LF) \\ * & * & -P_1^{-1}S_1P_1^{-1} & 0 & 0 \\ * & * & * & -S_2 & 0 \\ * & * & * & * & -\gamma^2 I_q \end{bmatrix}$$

and

$$\begin{aligned} L2_{11} &= P_1^{-1}(A^T + K^TB^T) + (A + BK)P_1^{-1} \\ &\quad + P_1^{-1}D^TDP_1^{-1} + P_1^{-1}S_1P_1^{-1} \\ L2_{22} &= (A^T - C^TL^T + G^T)P_2 + P_2(A - LC + G) + I_n + S_2 \end{aligned}$$

Noting $P_c = P_1^{-1}$, $P_o = P_2$, $S_c = P_1^{-1}S_1P_1^{-1}$, $S_o = S_2$, $X = KP_1^{-1}$, $Y = -P_2L$, we get:

$$\mathcal{L}2 = \begin{bmatrix} L2_{11} & 0 & A_1P_c & 0 & E \\ * & L2_{22} & 0 & P_oA_1 & P_oE + YF \\ * & * & -S_c & 0 & 0 \\ * & * & * & -S_o & 0 \\ * & * & * & * & -\gamma^2 I_q \end{bmatrix} < 0 \quad (10)$$

$$\begin{aligned} L2_{11} &= P_cA^T + AP_c + X^TB^T + BX + P_cD^TDP_c + S_c \\ L2_{22} &= A^TP_o + P_oA - P_c^{-1}(XB + B^TX^T)P_c^{-1} \\ &\quad + C^TY^Y + YC + I_n + S_o \end{aligned}$$

Using the Schur complement, it leads to the inequality (7). □

If the minimal attenuation bound is to be searched, then the following optimization problem has to be solved:

$$\gamma_{min}^2 = \min_{P_c,P_o,X,Y,S_c,S_o} \gamma^2 \quad (11)$$

$$\text{s.t. } \mathcal{L}_{oc} < 0, \; P_c > 0, \; P_o > 0, \quad (12)$$

$$S_c > 0, \quad S_o > 0,$$

Of course the problem to be solved (11) is not convex due to the term $P_c^{-1}(XB + B^TX^T)P_c^{-1}$ in M_o. Note that this can be rewritten as:

$$\begin{aligned} M_o &= A^TP_o + P_oA + C^TY^Y + YC \\ &\quad + I_n + S_o - Z^T - Z \quad (13) \\ &\quad (\text{ with } Z = -G^TP_o = P_c^{-1}BK) \quad (14) \end{aligned}$$

A first attempt to solve this non convex problem is given below.

Proposition 5. A solution to the observer-controller design is to follow the iterative procedure:

Step 1: Initialisation: solve the LMI problem (11) with M_o of the form (13), Z being unknown and get Z, $\gamma^1 = \gamma_{min}^1$. Set $test = 1$, $tol = 1e-3$, $i = 1$ and $Niter = 50$.

Step 2: (1) while (test==1) and ($i < Niter$),
- (2) Solve the LMI problem (11) with M_o of the form (13) (Z being the one obtained at step 1) and get P_c, P_o, X, Y, S_c, S_o, γ_{min}
- (3) Calculate $Z = P_c^{-1}BX$ and set $\gamma^i = \gamma_{min}^i$
- (4) test=($\frac{\gamma^i - \gamma^{i-1}}{\gamma^i} > tol$)
- (5) $i = i + 1$, $\gamma^{i-1} = \gamma^i$
- (6) end

Step 3: If test=0, then γ_{min}=γ_i. calculate $L = -P_o^{-1}Y$, $K = XP_c^{-1}$ and $G = -K^TB^TP_c^{-1}P_o^{-1}$

Step 4: Check the H_∞ closed-loop stability by solving the optimisation problem (4) (following Lemma 2) on the extended system (6) with L, K and G obtained at step 3. This allows to get the minimal attenuation bound ensured with the H_∞ stabilizing observer-controller.

Note that, in order to reduce the convergence time when the attenuation bound approaches zero, the optimal problem (11) is solved with the constraints $\gamma > 0.01$. This will avoid to do many iterations when γ is approaching 0.

3.2 Case 2

In that case the result of lemma 2 is applied to the extended system (6) in both configurations:

(1) $z_c = \begin{bmatrix} D & 0 \end{bmatrix} x_e = \mathcal{D}_c x_e$ (requiring an attenuation bound γ_c)

(2) $z_o = \begin{bmatrix} 0 & I_n \end{bmatrix} x_e = \mathcal{D}_o x_e$ (requiring an attenuation bound γ_o)

This leads to the following result:

Theorem 6. Consider the time-delay system (1) and the observer-controller (5). Given some positive scalars γ_c and γ_o, if there exist positive definite matrices $P_c = P_c^T$, $P_o = P_o^T$, S_c and S_o, and some matrices $X \in \mathbb{R}^{m\times n}$, $Y \in \mathbb{R}^{n\times p}$ satisfying the following matrix inequalities:

$$\mathcal{L}_c = \begin{bmatrix} M_c & 0 & A_1P_c & 0 & E & P_cD^T \\ * & M_o^c & 0 & P_oA_1 & P_oE+YF & 0 \\ * & * & -S_c & 0 & 0 & 0 \\ * & * & * & -S_o & 0 & 0 \\ * & * & * & * & -\gamma_c^2 I_q & 0 \\ * & * & * & * & * & -I_n \end{bmatrix} < 0, \quad (15)$$

$$\mathcal{L}_o = \begin{bmatrix} M_c & 0 & A_1P_c & 0 & E \\ * & M_o^o & 0 & P_oA_1 & P_oE+YF \\ * & * & -S_c & 0 & 0 \\ * & * & * & -S_o & 0 \\ * & * & * & * & -\gamma_o^2 I_q \end{bmatrix} < 0, \quad (16)$$

where $*$ means the symmetric element, and

$$\begin{aligned} M_c &= A_0P_c + P_cA_0^T + BX + X^TB^T + S_c, \\ M_o^c &= A^TP_o + P_oA - P_c^{-1}(XB + B^TX^T)P_c^{-1} \\ &\quad + C^TY^Y + YC + S_o, \\ M_o^o &= A^TP_o + P_oA - P_c^{-1}(XB + B^TX^T)P_c^{-1} \\ &\quad + C^TY^Y + YC + I_n + S_o \end{aligned}$$

then the system (5) is an H_∞ observer-controller according to Definition 3, with the disturbance attenuation levels γ_c, γ_o (respectively for the controlled output $z = Dx$ and for the state estimation error e) and the observer-controller gains:

$$L = -P_o^{-1}Y, \; K = XP_c^{-1}, \; G = -K^TB^TP_c^{-1}P_o^{-1} \quad (17)$$

If a minimal attenuation bound is to be searched, a solution is to solve the following optimization problem:

$$\gamma_{min} = \min_{P_c,P_o,X,Y,S_c,S_o} \frac{1}{2}(\gamma_c + \gamma_o) \quad (18)$$

$$\text{s.t. } \mathcal{L}_c < 0, \mathcal{L}_o < 0, \; P_c > 0, \; P_o > 0, \quad (19)$$
$$S_c > 0, \quad S_o > 0,$$

Proof: The proof directly follows the methodology of the previous proof of Theorem 4. Applying Lemma 2 for both configurations (z_c and z_o), leads to:

with z_c : We obtain $\mathcal{L}2$ with

$$\begin{aligned} L2_{11} &= P_cA^T + AP_c + X^TB^T + BX + P_cD^TDP_c + S_c \\ L2_{22} &= A^TP_o + P_oA - P_c^{-1}(XB + B^TX^T)P_c^{-1} \\ &\quad + C^TY^Y + YC + S_o \end{aligned}$$

with z_o : We obtain $\mathcal{L}2$ with

$$\begin{aligned} L2_{11} &= P_cA^T + AP_c + X^TB^T + S_c \\ L2_{22} &= A^TP_o + P_oA - P_c^{-1}(XB + B^TX^T)P_c^{-1} \\ &\quad + C^TY^Y + YC + S_o + I_n \end{aligned}$$

which leads to the above theorem. □

Finally the intuitive procedure given above to overcome the nonlinearity in the matrix inequality can directly be applied to this case, with the new optimisation problem (18).

4. ROBUSTNESS ANALYSIS W.R.T DELAY UNCERTAINTY

In this part we assume that the delay is uncertain, i.e the delay of the real system is $h = d + \theta$ and may be different from the one used in the observer (the nominal delay d. Note that the method here used has been developed in (Verriest *et al.*, 2002). It can be directly extended to other types of uncertainties.

In this case, the observer-controller is then of the form

$$\begin{cases} \dot{\hat{x}} = A_0\hat{x} + A_1\hat{x}(t-d) + Bu - L(C\hat{x} - y) + G\hat{x} \\ u = K\hat{x} \end{cases} \quad (20)$$

and the extended closed-loop system is:

$$\begin{aligned} \dot{x}_e &= \begin{bmatrix} A_0 + BK & -BK \\ -G & A_0 - LC + G \end{bmatrix} x_e + \begin{bmatrix} A_1 & 0 \\ A_1 & 0 \end{bmatrix} (x_e)_h \\ &\quad + \begin{bmatrix} 0 & 0 \\ -A_1 & A_1 \end{bmatrix} (x_e)_d + \begin{bmatrix} E \\ E - LF \end{bmatrix} w \\ &= \mathcal{A}_0 x_e + \mathcal{A}_h (x_e)_h + \mathcal{A}_d (x_e)_d + \mathcal{E} w \end{aligned} \quad (21)$$

Now, assuming $h = d + \theta$, we can write:

$$e^{-sh} = e^{-sd} + (e^{-s(d+\theta)} - e^{-sd}) = e^{-sd}(1 - \Delta(s))$$

with $\Delta(s) = 1 - e^{-s\theta}$. Therefore the characteristic equation of the above system can be written as:

$$\Psi(s) = \det[\Psi_0(s)] \det[I_n + \Psi_0^{-1}(s)\mathcal{A}_h e^{-sd}\Delta(s)]$$

where $\Psi_0(s) = sI_{2n} - \mathcal{A}_0 - (\mathcal{A}_h + \mathcal{A}_d)e^{-sd}$.

Now, the previous design ensures that the nominal extended system is stable, i.e. $\det[\Psi_0(s)]$ is stable. Then the perturbed closed loop system remains stable if $\det[I_n + \Psi_0^{-1}(s)\mathcal{A}_h e^{-sd}\Delta(s)]$ does not change sign when s sweeps the imaginary axis. Invoking Rouché's theorem, it follows that the condition for stability is

$$\|Q_d(s)\Delta(s)\|_\infty < 1. \quad (22)$$

where $Q_d(s) = \Psi_0^{-1}(s)\mathcal{A}_h e^{-sd}$. As shown in (Verriest *et al.*, 2002), this means that the maximal uncertainty bound that preserves stability may be determined as:

$$\theta_{max} = 1/\|se^{-sd}\Psi_0^{-1}(s)\mathcal{A}_h\|_\infty \quad (23)$$

Then for all $\theta \in (-\theta_{max}, \theta_{max})$, the determinant has a fixed sign, implying the absence of zero crossings,

and henceforth the stability of the perturbed system (provided the nominal one is stable).
The result here given can be summarized by the following proposition.

Proposition 7. Let the real system (1) be defined with an uncertain delay $h = d+\theta$, where d is known and θ is the uncertainty (unknown and bounded). Assume that an observer-controller (5) has been designed in the nominal case (i.e. $\theta = 0$). Then the applied observer-controller in the real case (20) preserves the closed-loop stability for all uncertainty up to

$$\theta_{max} = 1/\|se^{-sd}\Psi_0^{-1}(s)\mathcal{A}_h\|_\infty \tag{24}$$

where $\Psi_0(s) = sI_{2n} - \mathcal{A}_0 - (\mathcal{A}_h + \mathcal{A}_d)e^{-sd}$.

5. APPLICATION TO A WIND TUNNEL MODEL

This example in (Manitius, 1984) is a simplified mathematical model of the Mach number dynamic response to guide vane changes. The delay in one state variable represents the transportation time between the guide vanes of the fan and the test section of the tunnel. It has been tackled also in (Germani *et al.*, 2000), where an approximation approach is used to design a LQG control, i.e. in the presence of Gaussian noise, and assuming the exact knowledge of the delay. In steady-state operating conditions (fan speed, liquid nitrogen injection rate and gaseous-nitrogen vent rate) the dynamic response of the Mach number is given by the following system (Germani *et al.*, 2000):

$$\dot{x} = \begin{bmatrix} -0.5091 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -36 & -9.6 \end{bmatrix} x + \begin{bmatrix} 0 & -0.005956 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x_h + \begin{bmatrix} 0 \\ 0 \\ 36 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} w$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x + w$$

and $h = 0.33sec.$, x_1 is the Mach number, x_2 is the guide vane angle and $x_3 = \dot{x}_2$. The disturbance is a resistant torque on the input motor.
Note that we have decided to control the Mach number but also the vane angle, i.e.

$$z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x \tag{25}$$

For simulation purpose an initial value at time $t = 0sec.$ is used to generate a functional initial condition on $t \in [0, 0.5]$. The observer acts at $t = 0.5sec.$ A unit step disturbance is applied at $t = 10sec.$

5.1 Choi-Chung method

Using the method of (Choi and Chung, 1996) we can obtain the following result (note that due to the 7 parameters to be set, this procedure is quite involved and not systematic). A solution can be obtained for $\gamma = 1.4$ (for the closed-loop system). Results are shown in figure 1. The estimated error response e

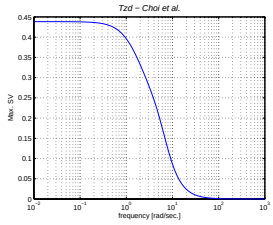

Fig. 1. $\sigma_{max}(T_{zw}(jw))$

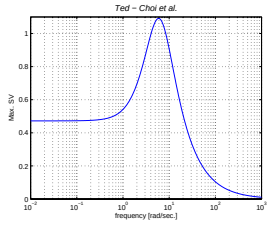

Fig. 2. $\sigma_{max}(T_{ew}(jw))$

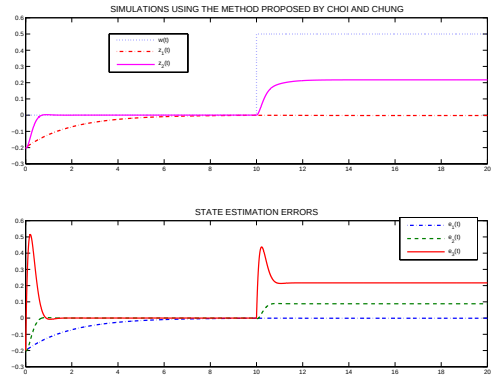

Fig. 3. Controlled output and estimated errors

and the controlled output z are shown in figure 3 for $h = 0.33sec.$
We can note in vector e of figure 3 that, as no robust property is guaranteed for the observer in this case, the disturbance attenuation for the state estimation error may not be good. This can be appreciated in the maximum singular value frequency plot of $T_{ew}(j\omega)$ (Fig. 2).
Note also, that, even if different values of the parameters can be chosen to get a smaller γ, the obtained attenuation property may be larger. Indeed, this depends on the coupling between the observer and controller.
Applying the robustness analysis w.r.t delay uncertainties, in this framework, leads to $\theta_{max} = 118.7$ *sec.*
To conclude from this example only, this does not emphasize the interest of this mixed design. The methodology is too complex (too much parameters to set a priori) and the results are not so far different from what could be obtained using a separated design.

5.2 Proposed method- case 1

Following the methodology given in proposition 5, a solution is obtained in 4 iterations and is such that $\gamma_{min} = 0.01$ and

$$G \simeq 0_{3\times 3}, \quad K = \begin{bmatrix} 0 \\ -4.482310^6 \\ -29315 \end{bmatrix}^T, \quad L = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

Now, when applying the step 4 of proposition 5, we obtain

$$\gamma_{min}^{step4} = 1.55\ 10^{-6},$$

which, as we can see in figures 4 and 5 is a good estimation of the real obtained disturbance attenuation level.

Of course we can note that some gains are very large. This is due to the fact that the minimal attenuation bound is required which, in this case, is near 0. If one aims to solve a sub optimal problem only (i.e. with γ given a priori) the gain will be much less large.

With this design the frequency and temporal behaviors of the observer-controller are presented in figures 4, 5 and 6. As we can see on these figures, the designed

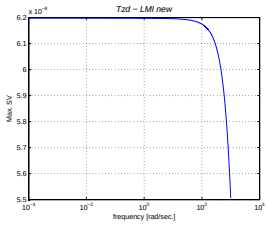

Fig. 4. $\sigma_{max}(T_{zw}(jw))$

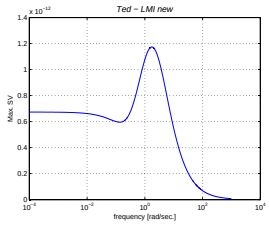

Fig. 5. $\sigma_{max}(T_{ew}(jw))$

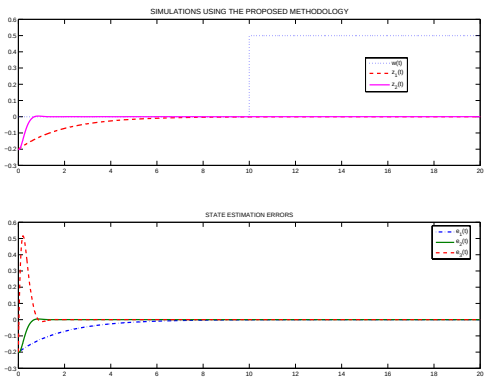

Fig. 6. Controlled output and estimated errors

observer controller scheme is not affected by the disturbance input (neither the controlled output nor the state estimation errors), which is a great advantage compared to both previous designs.

Note that solving case 2 would lead here to very similar results.

Finally, applying proposition 7, the maximal delay uncertainty that preserves stability is $\theta_{max} = 118.7$ *sec*. Hence, simulations with an observer delay d much different from the system one h could show the robust property. As the transient behaviors are very few affected by such a difference, results are not shown here.

To conclude on our method, it clearly points out the efficiency of the LMI formulation, which allows to get a solution even with a great disturbance attenuation property, and with no parameter to be set a priori. Also the H_∞ disturbance attenuation can be ensured for both the controlled output and the state estimation errors which greatly improves the existing results.

6. CONCLUDING REMARKS

A new observer-controller design is proposed and solved in a procedure including LMIs. This new design allows to get a closed-loop system and an observer which both satisfy an H_∞ attenuation property. The proposed method is simple to be solved as it does not contain any parameter to be chosen a priori, which differs from the current solution in the literature. As the solution is based on an optimisation procedure, it is worth noting that one can design (if possible) the best observer-controller w.r.t a disturbance attenuation property but could also design an observer-controller scheme with attenuation levels specified a priori for the estimation errors and for the controlled outputs, allowing to tackle the usual trade-off performance (w.r.t disturbance attenuation) /robustness (w.r.t uncertainties). This emphasizes the great flexibility of the methodology.

The given results could be directly extended to the case of multiple time-delay and also for time-varying delays. As a further extension, it could be possible to take into account some constraints on the observer and controller gain.

Further study may also concern the derivation of delay-dependent H_∞ observer-controller; however, to reduce conservatism, complex Lyapunov-Krasovskii functionals are generally used (Gu *et al.*, 2003) which may lead in our case to non convex matrix inequalities more difficult to be relaxed to get LMIs.

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