



**HAL**  
open science

## Erratum: Precession of a planet with a satellite

G. Boué, Jacques Laskar

► **To cite this version:**

| G. Boué, Jacques Laskar. Erratum: Precession of a planet with a satellite. 2008. hal-00335321v2

**HAL Id: hal-00335321**

**<https://hal.science/hal-00335321v2>**

Preprint submitted on 29 Oct 2008

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Erratum: Precession of a planet with a satellite.

G. Boué and J. Laskar

*Astronomie et Systèmes Dynamiques, IMCCE-CNRS UMR8028,*  
Observatoire de Paris, 77 Av. Denfert-Rochereau, 75014 Paris, France

October 29, 2008

**Abstract.** We correct a mistake in the proof of the proposition 1, given in section 4 of the paper *Icarus*, 185 (2006) 312-330 (Paper I). The proof is slightly modified but the results remain identical.

In section 4 (Global solution),  $\mathcal{W}$  is the matrix  $(\mathbf{w}, \mathbf{w}_1, \mathbf{w}_2)$  and  $V$  the Gram matrix of the basis  $(\mathbf{w}, \mathbf{w}_1, \mathbf{w}_2)$

$$V = \begin{pmatrix} 1 & x & y \\ x & 1 & z \\ y & z & 1 \end{pmatrix}, \quad (1)$$

We have incorrectly stated that the averaged differential system (51) of Paper I

$$\begin{aligned} \dot{\mathbf{w}} &= -\frac{\mathbf{a}x}{\gamma}\mathbf{w}_1 \wedge \mathbf{w} - \frac{\mathbf{b}y}{\gamma}\mathbf{w}_2 \wedge \mathbf{w}, \\ \dot{\mathbf{w}}_1 &= -\frac{\mathbf{c}z}{\beta}\mathbf{w}_2 \wedge \mathbf{w}_1 - \frac{\mathbf{a}x}{\beta}\mathbf{w} \wedge \mathbf{w}_1, \\ \dot{\mathbf{w}}_2 &= -\frac{\mathbf{b}y}{\alpha}\mathbf{w} \wedge \mathbf{w}_2 - \frac{\mathbf{c}z}{\alpha}\mathbf{w}_1 \wedge \mathbf{w}_2, \end{aligned} \quad (2)$$

can be written as

$$\dot{\mathcal{W}} = \mathcal{W}\mathcal{B} \quad \text{where} \quad \mathcal{B} = vV^{-1}\mathcal{A} \quad (3)$$

is a matrix depending only on  $(x, y, z)$ . In fact, the correct expression is (see Appendix B of Paper I)

$$\dot{\mathcal{W}} = vV^{-1}\mathcal{W}\mathcal{A} \quad (4)$$

which cannot be reduced to the previous one. As  $vV^{-1}$  and  $\mathcal{A}$  depend only on  $(x, y, z)$  that are periodic functions of period  $T$ , it is still true that if  $\mathcal{W}(t)$  is a solution of (4), then  $\mathcal{W}(t+T)$  is also a solution, but the remaining part of the proof has to be modified, as Eq. (96) of Paper I is no longer correct. Let us still denote

$$\mathcal{R}_T(t) = \mathcal{W}(t+T)\mathcal{W}(t)^{-1}. \quad (5)$$

We need to prove that  $\mathcal{R}_T(t)$  is constant with  $t$ . As the Gram matrix  $V$  of the vectors  $(\mathbf{w}(t), \mathbf{w}_1(t), \mathbf{w}_2(t))$  is  $T$ -periodic, the norm is conserved by the linear transformation  $\mathcal{R}_T(t)$  that send  $\mathcal{W}(t)$  into  $\mathcal{W}(t+T)$ , and  $\mathcal{R}_T(t)$  is

thus an isometry of  $\mathbb{R}^3$ . Moreover, this isometry is positive, as the volume  $v$  is conserved over a full period  $T$  (see section 3.4 of Paper I). The invariance of the total angular momentum  $\mathbf{W}_0$  (Eq. 52 of Paper I) then implies that  $\mathcal{R}_T(t)$  is a rotation matrix of axis  $\mathbf{W}_0$ .

As  $\mathcal{R}_T(t)$  is a rotation in  $\mathbb{R}^3$ , we have for all  $\mathbf{w}_i, \mathbf{w}_j \in \{\mathbf{w}, \mathbf{w}_1, \mathbf{w}_2\}$ ,

$$\begin{aligned} \mathbf{w}_i(t+T) \wedge \mathbf{w}_j(t+T) &= (\mathcal{R}_T(t)\mathbf{w}_i(t)) \wedge (\mathcal{R}_T(t)\mathbf{w}_j(t)) \\ &= \mathcal{R}_T(t)(\mathbf{w}_i(t) \wedge \mathbf{w}_j(t)). \end{aligned} \quad (6)$$

From Eq. (2), we can thus derive

$$\dot{\mathcal{W}}(t+T) = \mathcal{R}_T(t)\dot{\mathcal{W}}(t). \quad (7)$$

On the other hand, as  $\mathcal{W}(t+T) = \mathcal{R}_T(t)\mathcal{W}(t)$  (Eq. 5), we deduce that for all  $t$ ,

$$\dot{\mathcal{R}}_T(t)\mathcal{W}(t) = 0. \quad (8)$$

$\mathcal{R}_T(t)$  is thus a constant matrix  $\mathcal{R}_T$ . The last part of the proof remains identical. Let us denote  $\mathcal{R}(t)$  the rotation of axis  $\mathbf{W}_0$  and angle  $t\theta_T/T$  (i.e.  $\mathcal{R}(T) = \mathcal{R}_T$ ). We have

**Proposition 1.** The complete solution  $\mathcal{W}(t)$  can be expressed in the form

$$\mathcal{W}(t) = \mathcal{R}(t)\tilde{\mathcal{W}}(t), \quad (9)$$

where  $\tilde{\mathcal{W}}(t)$  is periodic with period  $T$ , and  $\mathcal{R}(t)$  a uniform rotation of axis  $\mathbf{W}_0$  and angle  $t\theta_T/T$ . The motion has two periods: the (usually) short period  $T$  and the precession period

$$T' = \frac{2\pi}{\theta_T}T. \quad (10)$$

### Miscellaneous misprints

We take the opportunity to also report here some misprints that remain in the published version of Paper I :

Eq. (3) of Paper I should read

$$\begin{pmatrix} \tilde{\mathbf{r}}_0 \\ \tilde{\mathbf{r}}_1 \\ \tilde{\mathbf{r}}_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -\delta & 1 - \delta \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{u}}_0 \\ \tilde{\mathbf{u}}_1 \\ \tilde{\mathbf{u}}_2 \end{pmatrix} .$$

### Appendix A. Averaged quantities

The averaged value of  $\mathbf{r}/r^5$  is

$$\left\langle \frac{\mathbf{r}}{r^5} \right\rangle = + \frac{e}{a^4(1 - e^2)^{5/2}} \mathbf{i} .$$

### Reference :

Boué, G., Laskar, J.: 2006, Precession of a planet with a satellite, *Icarus*, **185**, 312–330