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Addendum to Maximal regularity and Hardy spaces

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Abstract

We correct an inaccuracy in [1].

Key words: maximal regularity, Laplace-Beltrami operator, heat kernel, Hardy spaces, atomic decomposition.

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Proposition 3.14 in [1] states as follows: Let H_1, H_2 be any Hardy spaces obtained via an atomic decomposition. Any linear operator mapping the atoms of H_1 into a bounded set in H_2 has a bounded extension from H_1 into H_2.

M. Bownik [2], based on an example of Y.Meyer in Meyer, Taibleson and Weiss [6], showed that the Hardy space norm and the finite Hardy space norm may not be equivalent on the finite Hardy space (defined by restricting atomic decompositions to be finite sums and the atoms are L^∞-atoms in the sense of Coifman-Weiss). Hence, Proposition 3.14 is not correct as stated.

The paper by Meda, Sjögren and Vallarino [4] establishes, among other things, that if one replaces L^∞ atoms by L^2 atoms, the equivalence holds. Hence, for an operator to have a bounded extension it suffices it is bounded on L^2-atoms. Moreover, the extension coincides with the original operator on $H^1 \cap L^2$. So Proposition 3.14 is correct if H_1 is the original Coifman-Weiss Hardy space and atoms are L^2-atoms. As the atoms in [1] are L^2-atoms on a space of homogeneous type, this applies directly to the spaces $H^1_z(X)$ and (with little extra work) $H^1_r(X)$ defined in [1]: the maximal regularity operator and its adjoint have the boundedness property announced in Theorem 2.1 there.

Another possibility to reach our conclusion is to apply the weak-type (1,1) result of Coulhon and Duong [3] under our hypotheses. This implies that the maximal regularity operator and its adjoint are defined on L^1, thus on the Hardy space. Taking approximations of the these operators by truncating integrals in their definitions, one can adapt the proof of Lemma 1 in Chapter VI of Meyer's book [5].

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