



**HAL**  
open science

## Addendum to "Maximal regularity and Hardy spaces"

Pascal Auscher, Frédéric Bernicot, Jiman Zhao

► **To cite this version:**

Pascal Auscher, Frédéric Bernicot, Jiman Zhao. Addendum to "Maximal regularity and Hardy spaces". *Collectanea Mathematica*, 2009, 60 (2), pp.239-240. hal-00335114

**HAL Id: hal-00335114**

**<https://hal.science/hal-00335114>**

Submitted on 28 Oct 2008

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Addendum to Maximal regularity and Hardy spaces

Pascal Auscher, Frédéric Bernicot and Jiman Zhao

Université de Paris-Sud, Orsay et CNRS 8628, 91405 Orsay Cedex, France

*E-mail address:* pascal.auscher@math.u-psud.fr

Université de Paris-Sud, Orsay et CNRS 8628, 91405 Orsay Cedex, France

*E-mail address:* Frederic.Bernicot@math.u-psud.fr

School of Mathematical Sciences, Beijing Normal University

Beijing 100875, P.R. China

*E-mail address:* jzhao@bnu.edu.cn

October 16, 2008

## Abstract

We correct an inaccuracy in [1].

**Key words:** maximal regularity, Laplace-Beltrami operator, heat kernel, Hardy spaces, atomic decomposition.

**AMS2000 Classification:** 34G10, 35K90, 42B30, 42B20, 47D06

Proposition 3.14 in [1] states as follows: Let  $H_1, H_2$  be any Hardy spaces obtained via an atomic decomposition. Any linear operator mapping the atoms of  $H_1$  into a bounded set in  $H_2$  has a bounded extension from  $H_1$  into  $H_2$ .

M. Bownik [2], based on an example of Y. Meyer in Meyer, Taibleson and Weiss [6], showed that the Hardy space norm and the finite Hardy space norm may not be equivalent on the finite Hardy space (defined by restricting atomic decompositions to be finite sums and the atoms are  $L^\infty$ -atoms in the sense of Coifman-Weiss). Hence, Proposition 3.14 is not correct as stated.

The paper by Meda, Sjögren and Vallarino [4] establishes, among other things, that if one replaces  $L^\infty$  atoms by  $L^2$  atoms, the equivalence holds. Hence, for an operator to have a bounded extension it suffices it is bounded on  $L^2$ -atoms. Moreover, the extension coincides with the original operator on  $H^1 \cap L^2$ . So Proposition 3.14 is correct if  $H_1$  is the original Coifman-Weiss Hardy space and atoms are  $L^2$ -atoms. As the atoms in [1] are  $L^2$ -atoms on a space of homogeneous type, this applies directly to the spaces  $H_z^1(X)$  and (with little extra work)  $H_r^1(X)$  defined in [1]: the maximal regularity operator and its adjoint have the boundedness property announced in Theorem 2.1 there.

Another possibility to reach our conclusion is to apply the weak-type (1,1) result of Coulhon and Duong [3] under our hypotheses. This implies that the maximal regularity operator and its adjoint are defined on  $L^1$ , thus on the Hardy space. Taking approximations of these operators by truncating integrals in their definitions, one can adapt the proof of Lemma 1 in Chapter VI of Meyer's book [5].

## References

- [1] Auscher, Pascal; Bernicot, Frédéric; Zhao, Jiman. Maximal regularity and Hardy spaces. *Collect. Math.* 59 (2008), no. 1, 103–127.
- [2] Bownik, Marcin. Boundedness of operators on Hardy spaces via atomic decompositions. *Proc. Amer. Math. Soc.* 133 (2005), no. 12, 3535–3542.
- [3] Coulhon, Thierry; Duong, Xuan Thinh. Maximal regularity and kernel bounds: observations on a theorem by Hieber and Prüss. *Adv. Differential Equations* 5 (2000), no. 1-3, 343–368.
- [4] Meda, Stefano; Sjögren, Peter; Vallarino, Maria. On the  $H^1$ - $L^1$  boundedness of operators. *Proc. Amer. Math. Soc.* 136 (2008), no. 8, 2921–2931.
- [5] Meyer, Yves. *Ondelettes et opérateurs. II.* Hermann, Paris, 1990.
- [6] Meyer, Yves; Taibleson, Mitchell H.; Weiss, Guido. Some functional analytic properties of the spaces  $B_q$  generated by blocks. *Indiana Univ. Math. J.* 34 (1985), no. 3, 493–515.
- [7] Yang, Dachun; Zhou, Yuan. A boundedness criterion via atoms for linear operators in Hardy spaces, to appear in *Constr. Approx.* DOI 10.1007/s00365-008-9015-1