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# Plurilinear Modeling and discrete $\mu$ -Synthesis Control of a Hysteretic and Creeped Unimorph Piezoelectric Cantilever

Micky Rakotondrabe, Yassine Haddab and Philippe Lutz  
Laboratoire d'Automatique de Besançon - CNRS UMR6596  
ENSMM - Université de Franche-Comté  
24, rue Alain Savary  
25000 Besançon - France  
{mrakoton,yhaddab,plutz}@ens2m.fr

**Abstract**— First, we present a survey on modeling and control of bending piezoelectric microactuators. Second, a simple model for nonlinear piezoelectric actuators (hysteresis and creep) is presented. It is based on the multilinear approximation. This model requires low computing power and is well adapted for embedded systems. Finally, a  $\mu$ -synthesis controller is implemented. Experiments show that the obtained performances are compatible with the requirements of micromanipulation tasks.

**Index Terms**— Piezoelectric cantilever, hysteresis and creep, plurilinear modeling,  $\mu$ -synthesis control.

## I. INTRODUCTION

Piezoelectric materials have been loyal systems for the actuation of microsystems and microrobots. They offer a good *deformation/force* ratio as well as a good resolution and rapidity. One of their bountiful applications in the microworld is the actuation of microgrippers, as examples: [1][2][3]. A microgripper can be made of two piezoelectric cantilevers. According to the application, the cantilevers may be controlled on position and/or on force. As part of the modular and re-organizable microfactory project in our laboratory [4][5], each actuator inside the microfactory is considered as a module: it can be removed, added or replaced independantly from the other actuators. On the other hand, to allow the re-organizability of the microfactory, the modularity of the control is also needed. For that, we must consider and study independantly the control of the actuators. A microgripper is composed of two modules: the two cantilevers. The force control and the displacement (bending) control of each cantilever must be studied independently and a high level controller (supervisor) manages the functioning of the whole system. As example, when transporting a micropart, one cantilever is controlled on force while the other on displacement. On the other hand, when picking or releasing a micropart, the two cantilevers are controlled on displacement. However, to complete the modularity of the microfactory, each module should have its own local intelligence. We use a microcontroller for each module where the low level control is implemented.

The aim of this paper is the modeling and the control of

the displacement of a piezoelectric cantilever, dedicated to a modular microgripper, subjected to nonlinear phenomena: hysteresis and creeping. Many works have been done for several years about the modeling and control of piezoelectric materials. Thus, the first and second sections are dedicated to a survey on modeling and control of piezoelectric materials. After that, we describe our modeling of the nonlinearity which is based on plurilinear (also called multilinear) approach. Then, the following section presents the design and the results of  $\mu$ -synthesis controller.

## II. SURVEY ON PIEZOELECTRIC MODELING

### A. Linear modeling

The static (or quasi-static) linearity phenomenon of a piezoelectric cantilever submitted to mechanical, electrical or thermal excitation has been brought since long time [6]. However, the real physical modeling was first introduced by *Smits* [7] where, in using strain-energy method, he formulated the slope at the tip ( $\alpha$ ), the deflexion  $\delta$ , the displaced volume  $v$  and the charge  $Q$  versus the mechanical moment  $M$ , the force  $F$ , the uniform load  $p$  and the voltage  $U$  (Fig. 1) for bimorph systems. In [8], he introduced the thermal excitation inside the equations. In the case of a voltage excitation  $U$ , *Rogacheva* [9] and *Chang* [10] shown that the partial slope  $\frac{\partial \delta}{\partial U}$  varies according to the applied frequency. On the other hand, *Weinberg* extended the physical formulation to multilayered piezoelectric beam [11].

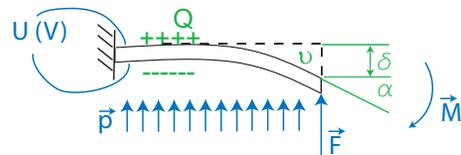


Fig. 1. A piezoelectric cantilever submitted to external excitations.

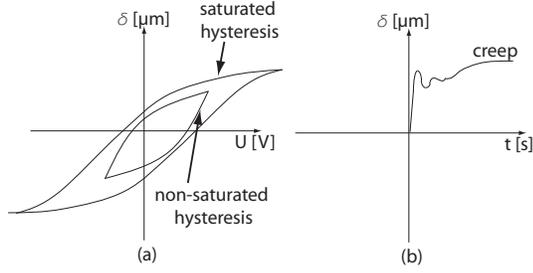


Fig. 2. a: hysteresis phenomenon. b: creeping phenomenon.

### B. Non-linear modeling

When the deflexion  $\delta$  of the beam becomes large, generally higher than 15% of the maximum field strength [45], the linear modeling is not applicable anymore and hysteresis and creeping phenomena arise (Fig. 2). To model the piezoelectric non-linearity, three categories exist: the microscopic, the semi-macroscopic and the macroscopic modelings.

The microscopic modeling is the less used among the three. It is based on energy relation applied at the atomic or molecular level [12]. This approach presents a high degree of detail but its implementation is very difficult due to the large number of parameters. On the other hand, the semi-macroscopic modeling of the piezoelectric materials hysteresis is based both on energy relations for the material and on macroscopic averages to obtain small number of parameters. It combines the Boltzman principles, the domain walls theory and some experiments adjustments (Smith *et al* [13][14]). Finally, the macroscopic modeling, also called phenomenological modeling, is based on mathematical expressions with parameters determined experimentally. To approach piezoelectric nonlinearity and hysteresis, such works have arisen in 90's [15][16][17]. However, the celebrity and arising of mathematical analysis of the Preisach operator have lead the authors to use it since the end of 90's [18][19][20][21][22]. In addition, Lining introduced the notion of tuning voltage to model and to decide about the shape of the hysteresis [23]. The same principle has been done by Low [24]. To complete the phenomenological model of the nonlinear piezoelectric materials, Jung [25] proposed a logarithm based formula to express the time evolution of the creep. In the same aim, Kuhnen [26] introduced the notion of linear creep operator combined with the Preisach operator. The linear creep operator consists on considering the creep as a linear combination of solutions of linear dynamic systems of the first order. In other term, the creep is a generalized Maxwell model. This concept has also been found in [27] coupled with the Preisach operator and in [28] to model the whole hysteresis.

## III. SURVEY ON PIEZOELECTRIC CONTROL

### A. Open-loop control

The piezoelectric materials, notably beam structures, present nonlinear (hysteresis and creep) characteristics. When applying a known voltage at their input, the repeatability and then the predictability of the output is very bad. Thus, the authors try to linearize the nonlinear behaviour by two methods: model compensation and charge compensation. Both are open-loop control type.

1) *Model compensation*: An inversion of the hysteresis operator is used and placed before the plant to control. Thus, the whole becomes linear and the quality of the result depends on the exactitude of the hysteresis model and on the inversion method. As examples, [19][22][29] use the Preisach modeling and its inversion for piezoelectric materials. On the other hand, [25][27][30][31] inverse the Preisach model coupled with the creep model. Another way is to consider that the hysteresis is approximated by a phaser with a negative phase for periodic inputs. Cruz-Hernandez introduce the notion of variable phaser to reduce the nonlinearity (hysteresis and saturation) [32][33]. Finally, Lining [23] uses his model with a computer based inversion calculation to linearize the hysteresis of the piezoelectric actuator.

2) *Q-charge compensation*: While the bending of a piezoelectric beam is nonlinear versus the voltage, it is nearly linear versus the applied charge. Thus, the authors use that principle in converting the voltage input into a charge input using electronic circuits [34][35]. This method takes into account the hysteresis nonlinearity but not the creep. On the other hand, to obtain a charge control, some authors propose the control of time-integration of a constant current [36][37][38]. According to [39], these methods are not efficient for industrial applications because of the impossibility to maintain the constance of the charge endlessly. Agnus [40] proposes then the Q/V control which combines the voltage and the charge controls.

### B. Closed-loop control

When open-loop controlled system is submitted to disturbances, a divergence from the reference point appears and sometimes may generate instability. To solve this problem, closed-loop principle is used. In the case of small displacement where the linearity expression is valid, the PID controller has given good performances. When the displacement is larger, other methods are used to ensure the stability and the performances. Ge [18] and Choi [41] use an inversion principle of the hysteresis model and close the loop with a PID-controller. Other authors [42] [43] propose robust controllers coupled with the model inversion. However, Chen [44] linearizes the hysteresis model of Low [24] before the use of the  $H_\infty$  based controller. While Zhong [45] uses a semi-macroscopic model coupled with optimal control (minimization of input and output energies) to

ensure performances, *Gorbet* employs the Preisach operator and a passivity based controller [46]. Finally, author authors propose adaptive controllers. As the hysteresis phenomenon is generally non-differentiable, most of them use an artificial intelligence to model in real-time the system and often apply a modulable PID-controller. The real-time modeling or the nonlinear adaptive controller are based either on learning methods [47][48][49] or on neural network principle [50][51][52][53].

#### IV. PLURILINEAR MODELING

##### A. Principle of the static modeling

The linear modeling is not appropriate when the bending is large enough. Thus, the literature proposes the use of mathematical hysteresis models which are taken into account when synthesising a controller. Sometimes, open-loop control based on non-linearity linearization or compensation is sufficient but when the system is submitted to disturbances, a closed loop controller must be used. Then, many authors propose the use of closed-loop linear controller to the compensated or linearized model. The most of the compensation is based on the Preisach hysteresis model. In this paper, we propose a simple model. As no inversion is used, it does not consume memory and time for the microcontroller. The Fig. 3-a shows the plurilinear approximation of a hysteresis. A variable straightline ( $\Delta$ ) represents the curve. The offset  $\delta_0$  and the slope  $\alpha$  are dependent on the past and present values of input  $U$ . We note them  $\delta_{U0}(\cdot)$  and  $\alpha(\cdot)$ . Thus, the approximation model of the hysteresis without the creep is the pseudolinear formula:

$$\delta(t) = \alpha(\cdot) \cdot U(t) + \delta_{U0}(\cdot) \quad (1)$$

When the external excitations are force  $F$  and voltage  $U$ , the linear formulation of [7] or [11] gives:

$$\delta(t) = c_p \cdot \left( U + \frac{c_e}{c_p} \cdot F \right) \quad (2)$$

where  $c_p$  is the piezoelectric constant and  $c_e$  the elastic constant.

This expression means that the force behaves like a voltage excitation and vice-versa. Thus, from equ. (1), an external force applied at the tip of the cantilever would also generate a hysteresis and creeping phenomena. Considering the two excitations, we have:

$$\delta(t) = \alpha(\cdot) \cdot U(t) + \beta(\cdot) \cdot F(t) + \delta_{UF0}(\cdot) \quad (3)$$

Here,  $\beta(\cdot)$  is the elastic constant depending on the past and present value of  $F(t)$  and  $\delta_{UF0}(\cdot)$  is the new offset considering the voltage and the force offsets.

To take into account the creeping, we propose to consider it as an error due to a fictive time-variant force  $F_C(t)$ . The

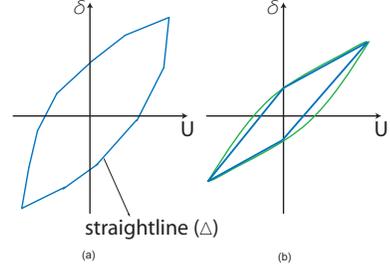


Fig. 3. a: plurilinear approximation of a hysteresis. b: quadrilateral approximation.

final model which includes the hysteretic and creep effects is:

$$\delta(t) = \alpha(\cdot) \cdot U(t) + \beta(\cdot) \cdot F(t) + \beta_C(\cdot) \cdot F_C(t) + \delta_0(\cdot) \quad (4)$$

where  $\beta_C(\cdot)$  is the fictive elastic constant and  $\delta_0(\cdot)$  an equivalent offset.

When the hysteresis does not reach the saturation, a quadrilateral approximation modeling (Fig. 3-b) is sufficient.

##### B. Quadrilateral modeling in our case

In our application, the choice of the cantilever is guided by the bending range of the micromanipulation ( $[-50\mu m, 50\mu m]$ ). To satisfy these constraints, we use a unimorph piezoelectric cantilever based on *PIC151* material and a thin *Cu* plate. The total thickness is  $0.275mm$  ( $0.2$  for the *PIC151* and  $0.075$  for the *Cu*), the width is  $2mm$  and the length is  $16mm$ . These values lead to a non-saturated hysteresis (Fig. 4). The experiments also show the rate-dependency of the hysteresis. The non-saturation and the convenable (not very large) area of the hysteresis lead to use the quadrilateral modeling. Let  $(\Delta_M)$  and  $(\Delta_m)$  represent the two straightlines of the quadrilateral with respectively the maximal and the minimal slopes:

$$\begin{cases} (\Delta_M) : \delta(t) = \alpha_M \cdot U(t) + \delta_M(\cdot) \\ (\Delta_m) : \delta(t) = \alpha_m \cdot U(t) + \delta_m(\cdot) \end{cases} \quad (5)$$

where  $\alpha_M$  ( $\alpha_m$ ) represents the maximal (minimal) slope and  $\delta_M(\cdot)$  ( $\delta_m(\cdot)$ ) represents the general equivalent offset which includes the fictive force, the force applied to the tip of the cantilever and the equivalent offset as seen in (4).

Let  $\alpha_O$  be the middle value of the maximal and the minimal slopes and let  $\alpha_E$  be their radius:

$$\begin{cases} \alpha_O = \frac{\alpha_M + \alpha_m}{2} \\ \alpha_E = \frac{\alpha_M - \alpha_m}{2} \end{cases} \quad (6)$$

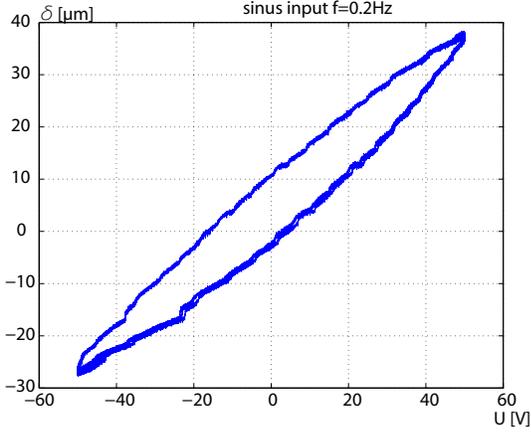


Fig. 4. The unimorph piezoelectric cantilever has non-saturated hysteresis. Here, a sinusoidal voltage with  $f = 0.2\text{Hz}$  and  $U = 50\text{V}$  is applied.

To model the hysteresis, we propose to use one nominal straightline with an nominal slope and a new general offset  $\delta_P(\cdot)$ :

$$\delta(t) = \alpha_O \cdot U(t) + \delta_P(\cdot) \quad (7)$$

Thereby, the real system has the following parameters:

$$\begin{cases} \delta(t) = \alpha_{real} \cdot U(t) + \delta_P(\cdot) \\ \alpha_O - \alpha_E \leq \alpha_{real} \leq \alpha_O - \alpha_E \end{cases} \quad (8)$$

Using experimental results, we obtain:  $\alpha_O = 671 \times 10^{-9}$  and  $\alpha_E = 149 \times 10^{-9}$ .

### C. Dynamic identification

The identification of the dynamic parameters of the cantilever are obtained using the step response (voltage input). We focus on the transient response before the beginning of the creep effect (Fig. 5).

We conclude that a linear 2nd order model is sufficient when the parameters are well adjusted (Fig. 5-dashed plot). Thus, we can complete the model given by the (7) to get the nominal dynamic expression:

$$\delta(t) = \frac{\alpha_O \cdot U(t) + \delta_P(\cdot)}{a \cdot p^2 + b \cdot p + 1} \quad (9)$$

where  $a = 6.746 \times 10^{-8}$  is the inertial coefficient and  $b = 5.195 \times 10^{-6}$  is the viscous coefficient.

Now, the goal is the research of a corrector which stabilizes a nominal model subjected to a disturbance  $\frac{\delta_P(\cdot)}{\alpha_O}$  and where the nominal statical gain  $\alpha_O$  is followed by an uncertainty (Fig. 6) for the real system.

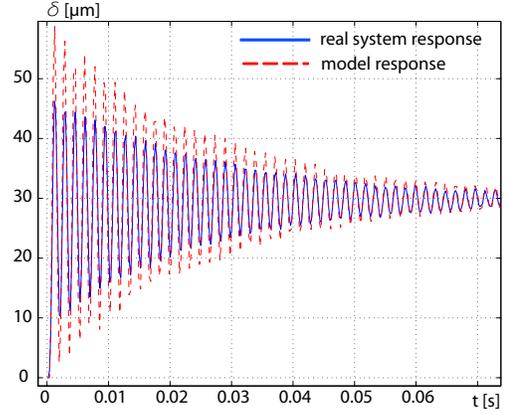


Fig. 5. Response of the piezoelectric cantilever when applying a step voltage signal. Here, we focus on the part before the creep starts in order to identify the dynamic structure and parameters.

## V. DISCRETE $\mu$ -SYNTHESIS CONTROL OF THE PIEZOELECTRIC CANTILEVER

### A. discrete $H_\infty$ and $\mu$ -synthesis

Let  $G(p)$  represent a plant to control where  $p$  is the Laplace variable. When synthesising a controller  $K(p)$ , it is possible to analyze the performances through the bode diagrams characterizing the closed-loop system (Fig. 7-a). If  $S(p) = 1/(1 + KG)$  is the sensitivity function, these diagrams are given by the following transfer functions:

$$\begin{cases} S(j\omega) = \varepsilon/y_c \\ K(j\omega) \cdot S(j\omega) = U/y_c \\ S(j\omega) \cdot G(j\omega) = y/b \\ K(j\omega) \cdot S(j\omega) \cdot G(j\omega) = y/y_c \end{cases} \quad (10)$$

Where,  $\varepsilon$  indicates the error,  $y_c$  indicates the setpoint,  $U$  is the input voltage,  $b$  is the external perturbation and  $y$  is the output (Fig. 8-a). Thus, to impose some performances, frequential gabarits are used (Fig. 8). These gabarits are dependant on three filters  $W_1(p)$ ,  $W_2(p)$  and  $W_3(p)$ . The standard  $H_\infty$  problem consists on finding an optimal controller  $K$  and an optimal value  $\gamma$  so that:

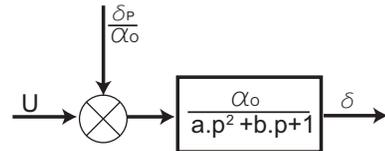


Fig. 6. Scheme of the model to control.

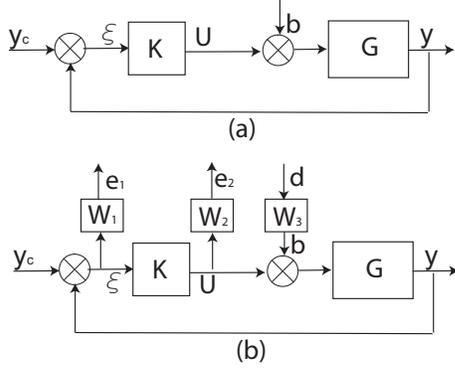


Fig. 7. a: closed-loop scheme. b: augmented scheme.

$$\begin{cases} |S(j\omega)| < \frac{\gamma}{|W_1(j\omega)|} \\ |K(j\omega)S(j\omega)| < \frac{\gamma}{|W_2(j\omega)|} \\ |S(j\omega)G(j\omega)| < \frac{\gamma}{|W_1(j\omega)W_3(j\omega)|} \\ |K(j\omega)S(j\omega)G(j\omega)| < \frac{\gamma}{|W_2(j\omega)W_3(j\omega)|} \end{cases} \quad (11)$$

which is equivalent to:

$$\begin{cases} \|W_1 S\|_\infty < \gamma \\ \|W_2 K S\|_\infty < \gamma \\ \|W_1 S G W_3\|_\infty < \gamma \\ \|W_2 K S G W_3\|_\infty < \gamma \end{cases} \quad (12)$$

Thereby, an augmented closed-loop scheme is obtained (Fig. 7-b). To solve the problem (12), the most used method is the Glover-Doyle algorithm which is based on the Riccati equations [54][55]. The issued controller  $K$  is robust in the fact that it ensures the stability and the performances even if the system  $G$  has uncertainty vis-à-vis of the real plant. The limit of this uncertainty is found à-posteriori with the  $\mu$ -analysis tool.

If the uncertainty can be formulated à-priori, the  $\mu$ -synthesis is the tool which permits finding the optimal controller  $K$  and the optimal value  $\gamma$  and which ensures the target performances in the uncertainty domain. The  $\mu$ -synthesis is based on the  $H_\infty$  and the  $\mu$  (structured singular values) tools. It necessitates to structure the uncertainty inside a diagonal matrix  $\Delta$ .  $\Delta$  is composed of the dynamic uncertainty  $\Delta_i$ , of the parametric uncertainty  $\partial_i$  and the phase and gain uncertainties  $\epsilon_i$ :

$$\Delta = \text{diag}\{\Delta_1(s), \dots, \Delta_q(s), \partial_1 \cdot I_{r1}, \dots, \partial_r I_{rr}, \epsilon_1 I_{c1}, \dots, \epsilon_c I_{cc}\} \quad (13)$$

with:  $\Delta_i(s) \in RH_\infty$   $\partial_i \in R$   $\epsilon_i \in C$

and where the normalization condition must be verified:

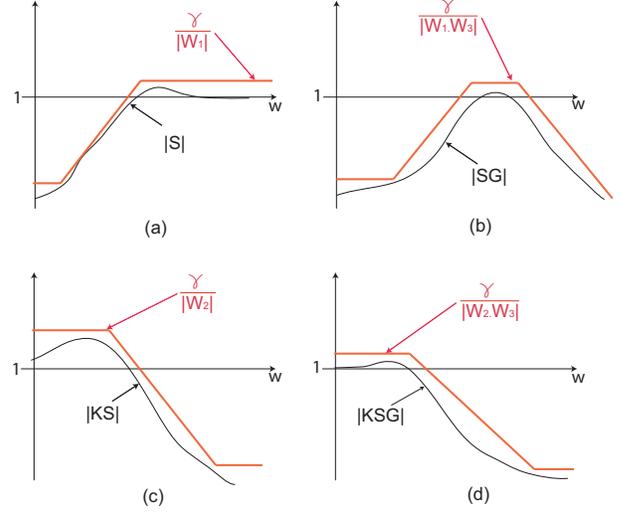


Fig. 8. Appearances of the bode diagrams of the gabarits.

$$\|\Delta\|_\infty < 1 \Leftrightarrow (\|\Delta_i(s)\| < 1; -1 < \partial_i < 1; |\epsilon_i| < 1) \quad (14)$$

Fig. 9-a represents the augmented system  $P$  connected by LFT-lower with the controller  $K$  and by LFT-upper with the uncertainty  $\Delta$ . The goal of the  $\mu$ -synthesis is to find the optimal controller  $K$  so that  $H_\infty$  norm of the transfer between *ext-input* and *ext-output* remains smaller than 1. According to the Small-Gain theorem [56][60], that condition is obtained if and only if the structure in Fig. 9-b remains stable whatever the fictive uncertainty  $\Delta_f$  is, such as,  $\|\Delta_f(p)\|_\infty < 1$ . The latter condition is equivalent to:

$$\forall \omega \quad \mu_{\bar{\Delta}}(F_l(P(j\omega), K(j\omega))) \leq 1 \quad (15)$$

where  $\bar{\Delta}$  takes into account  $\Delta$  and  $\Delta_f$ .  $F_l(P(j\omega), K(j\omega))$  is the connexion of  $P$  on its lower part with  $K$ .

$\mu_{\bar{\Delta}}(M)$  represents the structured singular value of a system  $M$  relatively to  $\bar{\Delta}$ .  $1/\mu_{\bar{\Delta}}(M)$  represents the minimal uncertainty  $\{\Delta, \Delta_f\} \in \bar{\Delta}$  which destabilizes  $M$ .

$\mu_{\bar{\Delta}}(M)$  is defined as follow:

$$\begin{cases} \mu_{\bar{\Delta}}(M) := \frac{1}{\inf_{\{\Delta, \Delta_f\} \in \bar{\Delta}} (\bar{\sigma}(\Delta) : \det(I - \Delta M) = 0)} \\ \mu_{\bar{\Delta}}(M) := 0 \quad \text{if } \forall \{\Delta, \Delta_f\} \in \bar{\Delta} : \det(I - \Delta M) \neq 0 \end{cases} \quad (16)$$

$\bar{\sigma}(\Delta)$  is the maximal singular value of  $\{\Delta, \Delta_f\}$ .

One of the most used methods to solve the problem (15) is the  $D-K$ -iteration [56]. This method will be used for our application.

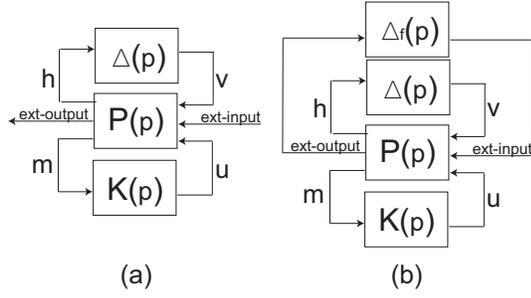


Fig. 9. a: robust synthesis. b: configuration for the  $\mu$ -synthesis.

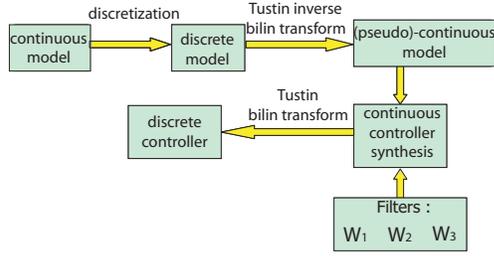


Fig. 10. The stages to synthesize a discrete controller.

As the controller is intended to be implemented in a microcontroller, this paper synthesizes a discrete controller. Nevertheless, the best and simplest way to design a discrete  $H_\infty$  based optimal controller is via bilinear transformation [59][60][61], ie, working in an equivalent continuous domain of the discrete plant. The main reasons are that the continuous analysis is simplest, more standard and possesses more physical sense than the discrete analysis (Fig. 10). The discretization of the continuous model (first step of the transformation in the figure) is necessary in order to take into account the sampling time on the model.

The bilinear transformation is defined as a bijective application  $p = f(z)$ , where  $p$  indicates the Laplace variable and  $z$  the discrete operator:

$$p = \frac{\chi_1 \cdot z + \chi_2}{\chi_3 \cdot z + \chi_4} \quad (17)$$

In Tustin bilinear transformation, we have  $\chi_1 = -\chi_2 = 2$  and  $\chi_3 = \chi_4 = T_e$  where  $T_e$  is the sampling time. From a discrete system, the inverse Tustin bilinear transformation gives a continuous model where the stability analysis may be done.

### B. Experimental application

The experimental setup is shown in Fig. 11-a. An optical sensor with a resolution of  $10nm$  and an accuracy of  $40nm$

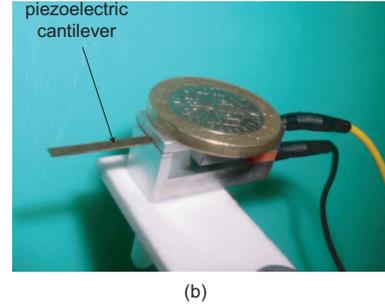
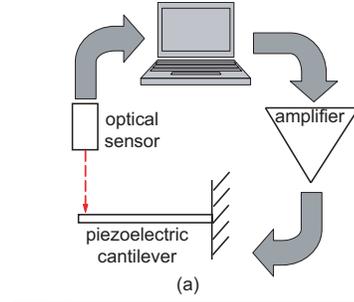


Fig. 11. a : the experimental setup principle. b : photography of the cantilever.

is used. We use a dSpace real-time calculator, a computer and the Matlab-Simulink software to control the process.

The characteristics of the chosen piezoelectric cantilever give the required deflexion with low voltage ( $< 100V$ ). Thereby, the gabarit  $\gamma/W_2$  will not be considered. The initial continuous model (before discretization) is based on the nominal model in (9). From the (9) and (5), the uncertainty  $\Delta = \{r : -1 < r < 1\}$  is built (see Fig. 12).

Among the specifications, the response time must be less than  $10ms$ , the statical error less than  $0.1\%$  and the statical error due to perturbation less than  $\frac{1}{3}\mu m/mN$ . So, we propose the following gabarits:

$$\begin{cases} \frac{1}{W_1} = 10^{-3} \times \frac{(3p+1)}{(0.003p+1)} \\ \frac{1}{W_1 W_3} = 10^{-7} \times \frac{(0.02p+1)}{(0.0002p+1)} \end{cases} \quad (18)$$

The sampling time is  $T_e = 0.2ms$ . The obtained controller  $K$  has a  $9^{th}$  order structure. We reduced the order to 7 by using the equilibrium reduction:

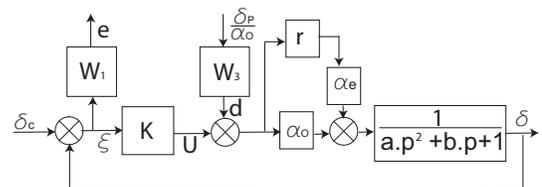


Fig. 12. Bloc-scheme including the uncertainty.

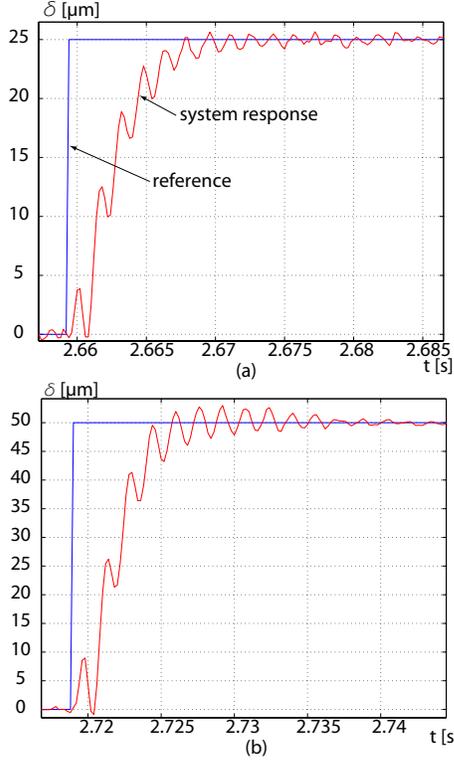


Fig. 13. Results with  $\mu$ -synthesis control. a: application of a serie of steps. b: zoom in a step.

$$K = \frac{\begin{matrix} 7.127 \times 10^5 z^7 - 3.232 \times 10^6 z^6 + 5.551 \times 10^6 z^5 \\ - 3.265 \times 10^6 z^4 - 2.629 \times 10^6 z^3 + 5.619 \times 10^6 z^2 \\ - 3.634 \times 10^6 z + 8.775 \times 10^5 \end{matrix}}{\begin{matrix} z^7 - 3.234z^6 + 3.185z^5 + 0.7548z^4 \\ - 3.786z^3 + 2.571z^2 - 0.3252z - 0.166 \end{matrix}}$$

$$\gamma_{opt} = 1.434 \quad (19)$$

The experiments give good results. The Fig. 13 show the responses obtained with  $25\mu\text{m}$  and  $50\mu\text{m}$  of steps input signal (reference). The response time is around  $8\text{ms}$  and the static error corresponds to the specifications. The control voltages never exceed  $100\text{V}$  (Fig. 14).

## VI. CONCLUSION

In this paper, we first present a survey on the modeling and the control of bending piezoelectric actuators notably in the case where the nonlinearities (hysteresis and creeping) are taken into account. After that, we propose a method to model these nonlinearities using the plurilinear principle. One of the main advantages of this principle is that it requires less real-time resources (memory and calculus) than other methods. In particular if the actuator is used in the non-saturated hysteresis domain, the quadrilateral modeling is a good

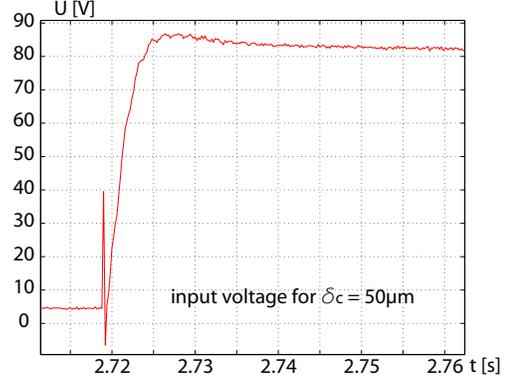


Fig. 14. Input voltage obtained with a setpoint of  $50\mu\text{m}$ .

approximation. In the other hand, the creep phenomenon is taken into account in the model. Knowing the uncertainty, a discrete  $\mu$ -synthesis robust controller is presented in order to maintain performances required in micromanipulation. It has been demonstrated that the resulting controller gives good performances. The next stage will be the study of the force control.

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