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Fabrice Ardhuin, Louis Marié, Nicolas Raschle, Philippe Forget, Aaron Roland. Observation and estimation of Lagrangian, Stokes and Eulerian currents induced by wind and waves at the sea surface. 2008. hal-00331675v1

**HAL Id: hal-00331675**

**<https://hal.science/hal-00331675v1>**

Preprint submitted on 20 Oct 2008 (v1), last revised 9 Apr 2009 (v4)

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## Observation and estimation of Lagrangian, Stokes and Eulerian currents induced by wind and waves at the sea surface

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(Manuscript received October 2008, in final form )

### ABSTRACT

The surface current response to winds is analyzed in a two-year time series of a 12 MHz (HF) Wellen Radar (WERA) off the West coast of France. Consistent with previous observations, the measured currents, after filtering tides, are of the order of 1.0 to 2.0% of the wind speed, in a direction 10 to 40 degrees to the right of the wind, with systematic trends as a function of wind speed. This Lagrangian current can be decomposed as the vector sum of a quasi-Eulerian current  $U_E$ , representative of the top 1 to 2 m of the water column, and part of the wave-induced Stokes drift  $U_{ss}$  at the sea surface. Here  $U_{ss}$  is estimated from the third moment of the wave frequency spectrum with a directional correction. Using observed wave spectra,  $U_{ss}$  is found to be very well approximated by a simple function of the wind speed and significant wave height, generally increasing quadratically with the wind speed. Results from a new numerical wave model are presented, that reproduce well this variability of  $U_{ss}$ . Focusing on a site located 100 km from the mainland, the estimated contribution of  $U_{ss}$  to the radar measurement has a magnitude of 0.6 to 1.3% of the wind speed, in the wind direction, a fraction that increases with wind speed. The difference  $U_E$  of Lagrangian and Stokes contributions is found to be of the order of 0.4 to 0.8% of the wind speed, and 45 to 70 degrees to the right of the wind. This relatively weak quasi-Eulerian current with a large deflection angle is interpreted as evidence of strong near-surface mixing, likely related to breaking waves. Summer stratification tends to increase the  $U_E$  response by up to a factor 2, and further increases the deflection angle of  $U_E$  by 5 to 10 degrees. At locations closer to coast,  $U_{ss}$  is smaller, and  $U_E$  is larger with a smaller deflection angle. This decomposition into Stokes drift and quasi-Eulerian current is most important for the estimation of energy fluxes to the Ekman layer.

### 1. Introduction

Surface drift constitutes one of the most important applications of the emerging operational oceanography systems, and plays an important role in larvae recruitment and ocean biogeochemistry. A quantitative understanding of the relative contribution of the wave-induced Stokes drift to the near surface velocities is also paramount for the proper estimation of air-sea energy fluxes. The quantitative variation of surface drift as a function of the forcing parameters is still relatively poorly known. In areas of strong currents due to tides or quasi-geostrophic dynamics, the surface drift current is highly correlated to the sub-surface current. Otherwise, winds play a major role in defining the surface velocities.

Recent theoretical and numerical works (Ardhuin et al. 2004; Rascle et al. 2006; Ardhuin et al. 2008c) have sought to reconcile historical measurements of Eulerian and Lagrangian (i.e. drift) velocities with recent knowledge on wave-induced mixing (Agrawal et al. 1992) and wave-induced drift (Rascle et al. 2008). These suggest that the surface Stokes drift  $U_{ss}$  induced by waves typically accounts for 2/3 of the surface wind-induced drift, in the open ocean, and that the wind-related Lagrangian velocity  $U_L$  is the sum of the strongly sheared  $U_{ss}$  and a relatively uniform quasi-Eulerian current  $U_E$ , defined by Jenkins (1987) and generalized by Ardhuin et al. (2008c).

Because the depth-integrated current is constrained by the balance between the Coriolis force and the wind stress, the magnitude of the quasi-Eulerian current is expected to be modulated by the surface stratification, with larger velocities in shallow mixed layers, and directions of  $U_E$  more strongly deflected to the right of the wind

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(in the Northern Hemisphere) than previously expected (Rascle 2007).

These predictions were verified by Rascle (2007) with mooring data at depths greater than 5 m. When extrapolated to the surface using a simple numerical model, they give directions of  $U_E$  between  $45^\circ$  and  $90^\circ$ , more than the  $45^\circ$  given by the constant eddy viscosity model of Ekman (1905), as extended by Gonella (1971), and the  $10^\circ$  given by the linear eddy viscosity model of Madsen (1977). This surface angle is also critical for the estimation of the flux of wind energy to the Ekman layer (e.g. Wang and Huang 2004), or the analysis of near-surface drifter data, which often does not take into account the wave-induced motion (e.g. Rio and Hernandez 2003; Elipot and Lumpkin 2008).

High Frequency (HF) radars can provide measurements much closer to the sea surface, depending on their operating frequency. Using a 30 MHz radar, Mao and Heron (2008) made observations that are also consistent with the idea that the drift current, found to be 2.1% of the wind speed on average, is the sum of  $U_E$  which, according to their theory, depends quadratically on the wind speed, and  $U_{ss}$  which they estimate to depend linearly on the wind speed, with a variation according to the fetch. Unfortunately, their analysis relied on empirical wave estimates that give large relative errors (of the order of 100%, see e.g. Kahma and Calhoun 1992; Ardhuin et al. 2007), and a limited range of wind speeds. Other HF-radar observations give a surface current of the order of 1.5 to 2.5% of  $U_{10}$  (Essen 1993) with 25 to 30 MHz radars. Dobson et al. (1989) also report a ratio of 2.0% using a 22 MHz radar, and Shay et al. (2007) report a ratio of 2 to 3% using a 16 MHz radar in water depths of 20 to 50 m. These analyses are very difficult to interpret due to the filters applied on time series to remove motions (tides, geostrophic currents ...) that are not related to the wind, and also because of the importance of inertial oscillations that make the wind- and wave-driven current a function of the full wind history, and not just a function of the wind vector at the same time and location.

In the present paper we extend the previous analyses of HF radar data by independently estimating the Stokes drift, using an accurate wave model. We find that at our deep water<sup>1</sup> North-East Atlantic site the quasi-Eulerian current  $U_E$  is of the order of 0.6% of the wind speed with a direction that is, on average,  $60^\circ$  to the right of the wind. This result is expected to be representative of the open ocean. Therefore the estimates of the flux of wind energy to the Ekman layer by Wang and Huang (2004), who used an angle of  $45^\circ$ , are likely overestimated by 20%, and a proper analysis of the effects of waves is needed. Data and processing are reviewed in section 2, and the analysis of the stratification effect is presented in section 3 with conclusions in section 4.

<sup>1</sup>This means deeper than both the Stokes depth and the expected Ekman depth, see Polton et al. (2005).

## 2. Lagrangian and quasi-Eulerian current from HF radars

### a. Radar measurements and processing

High frequency radars measure, among other things (e.g. Ivonin et al. 2004), the phase velocity  $C$  of Bragg waves that have a wavelength equal to one half of the radar electromagnetic wavelength and that propagate in directions away from and toward the radar. This phase velocity is a combination of the quasi-Eulerian current  $U_E$  (Stewart and Joy 1974; Kirby and Chen 1989), the phase speed of linear waves  $C_{lin}$ , and a nonlinear wave correction (Weber and Barrick 1977) that can be interpreted as a filtered surface Stokes drift  $U_{Sf}$ . For monostatic systems, the usual radial current velocity in the direction  $\theta_B$  towards one radar can be expressed as

$$\begin{aligned} U_R(\theta_B) &= C(\theta_B) - \mathbf{C}_{lin} \cdot \mathbf{e}_{\theta_B} \\ &= U_{Sf}(\theta_B) + \mathbf{U}_E \cdot \mathbf{e}_{\theta_B}, \end{aligned} \quad (1)$$

where  $\mathbf{e}_{\theta_B}$  is the unit vector in direction  $\theta_B$ . This velocity can be loosely interpreted as the projection in direction  $\theta_B$  of a current vector  $\mathbf{U}_R$ . The reason why this is not exactly true is that  $U_{Sf}(\theta_B)$  for all directions cannot be exactly given by the projection of a vector  $\mathbf{U}_{Sf}$ . In other words,  $U_{Sf}(\theta_B)$  is not exactly proportional to  $\cos(\theta_B)$ , although it is a reasonable approximation (Broche et al. 1983).

In order to express  $U_{Sf}$ , we first define the Stokes drift vector for waves with frequencies up to  $f_c$  from the directional wave spectrum  $E(f, \theta)$ ,

$$\mathbf{U}_{ss}(f_c) = 4\pi \int_0^{f_c} \int_0^{2\pi} f \mathbf{k}(f, \theta) E(f, \theta) df, \quad (2)$$

where  $k(f)$  is the wave number, equal to  $(2\pi f)^2/g$  for linear waves in deep water, and  $g$  is the acceleration of gravity. Starting from the full expression given by Weber and Barrick (1977), Broche et al. (1983) showed that the filtered Stokes drift component that affects the radial current measured by one radar station is well approximated by

$$\begin{aligned} U_{Sf}(k_B, \theta_B) &\simeq \mathbf{U}_{ss}(f_B) \cdot \mathbf{e}_{\theta_B} \\ &+ 4\pi k_B \int_{f_B}^{\infty} \int_0^{2\pi} f \cos(\theta - \theta_B) E(f, \theta) d\theta df \end{aligned} \quad (3)$$

where  $f_B$  is the frequency of the Bragg waves, and  $\mathbf{k}_B$  is the corresponding wavenumber vector, with a direction  $\theta_B$  and magnitude  $k_B$ . The full expression, correcting typographic errors in Broche et al. (1983) is given in Appendix A. In order to simplify the notations, the variable  $k_b$  in  $U_{Sf}$  will now be omitted, but the filtered Stokes drift is always a function of the Bragg wavenumber, thus being different for different radar frequencies.

The depth-varying quasi-Eulerian current  $\hat{u}(z)$  is defined as the difference of the Lagrangian velocity and Stokes drift (Jenkins 1987), and can generally be estimated from the full velocity field using a Generalized Lagrangian Mean (Ardhuin et al. 2008c). The value  $U_E$  estimated from the radar is, according to linear wave theory, a convolution of  $\hat{u}(z)$  and the Bragg wave Stokes drift profile (Stewart and Joy 1974; Kirby and Chen 1989).

Here we use data from a 12.4 MHz WERA HF-radar system (Gurgel et al. 1999), manufactured by Helzel GmbH, for which the Bragg wavelength is 12.1 m, corresponding to a wave frequency of 0.36 Hz in deep water. Thus the value of  $U_E$  corresponds, for 50%, to water depths less than 0.6 m from the moving sea surface, compared to 0.28 m with the 30 MHz radar of Mao and Heron (2008). The relative contributions from deeper layers to  $U_E$  decrease exponentially with depth as  $\exp(2k_B z)$ .

The radar system has been deployed and operated by Actimar SAS, since July 2006 on the west coast of France (figure 1), measuring surface currents and sea states every 20 minutes. The area is characterized by intense tidal currents, in particular between the largest islands where it exceeds  $3 \text{ m s}^{-1}$  during mean spring tides. Also important, the offshore stratification is largely suppressed by mixing due to the currents in the areas shallower than 90 m, resulting in complex temperature fronts that are related to the bottom topography (e.g. Mariette and Le Cann 1985).

Each radar station transmits a chirped continuous wave with a repetition frequency of 4 Hz and a 100 kHz bandwidth which gives a radial resolution of 1.5 km. The raw data is processed to remove most of the interference signals (Gurgel and Barbin 2008). Ensemble-averaging over 4 consecutive segments of 512 pulses yields a velocity resolution  $d_u = 0.09 \text{ m/s}$  for each individual radial current measurement. The receiving antennas are 16-element linear arrays with a spacing of 10 m, giving a typical angular resolution of 15 degrees.

We will consider only the temporal evolution of the wave field at one point of the radars' field of view that is representative of the offshore conditions, with a water depth of 120 m. In practice, interferences and ships cause some data to be rejected in the radar processing, or yield bad measurements, and heavy seas or calm seas also reduce the working radar range. In order to obtain a nearly continuous time series, we compiled and filtered data from a  $0.2^\circ$  in latitude by  $0.3^\circ$  in longitude box around that point (A in figure 1, the arrow spacing indicate the resolution of the radar grid). This compilation was done in two steps. First, based on a visual inspection of the data, at each radar grid point 0.05% of the total number of data points in the radial velocities time-series are considered spurious and removed. These points are selected as the points where the raw radial current time-series differs most from the result of a 3-points median filter. The 0.05% value was selected as a convenient rule-of-thumb, which permits the removal of most of the visibly spurious points, but does not introduce too many unnecessary

gaps in the time-series. Second, the time-series of all the grid points in the box around A were converted to  $u$  and  $v$  components and averaged.

The Cartesian components of  $U_R$  and  $U_E$  with respect to west-east (component  $u$ ) and south-north ( $v$ ) directions are calculated from the radial components  $u_r$  measured by the radars, before and after the subtraction of  $U_{Sf}(\theta_B)$ . These Cartesian components suffer from a geometrical dilution of precision (GDOP), varying with position (Chapman et al. 1997; Shay et al. 2007). The radar beams intersect at point A with an angle  $r = 34^\circ$  and it is possible to estimate the GDOP values for  $u$  and  $v$ , i.e. the ratios  $S_u/s$  and  $S_v/s$  where  $S_u$ ,  $S_v$  and  $S$  are the errors in  $u$ ,  $v$  and  $u_r$ , respectively. Assuming that  $S$  has no bias and is uniformly distributed from  $-d_u/2$  to  $+d_u/2$ , each radar measurement has an intrinsic error  $S_u = 0.04 \text{ m s}^{-1}$  and  $S_v = 0.11 \text{ m s}^{-1}$ .

This compiled time series, extending from July 5 2006 to July 31 2008, is the basis of the following analysis. The 1200 s resolution data was averaged over 3 h blocks centered on round hours. Gaps shorter than 6 h were linearly interpolated. That time series is 97% complete, and thus covers two full years. Other parts of the radar field of view yield similar results, briefly discussed below. Due to the averaging in space and time, each point in the time series is the combination of about 30 range cells and 9 time intervals, i.e. 180 independent velocity measurements when the full radar range is obtained. The error on the velocity components are thus likely less than  $2 \text{ cm s}^{-1}$ .

Because we wish to focus on the random wind-driven currents, we also performed a tidal analysis using the T-TIDE software (Pawlowicz et al. 2002) applied to each component. This analysis on the full time series (before time averaging) allows the removal of the deterministic diurnal constituents  $K_1$ ,  $O_1$ ,  $P_1$  and  $Q_1$  that have amplitudes of 1.5 to  $0.3 \text{ cm s}^{-1}$ , with estimated errors of  $0.1 \text{ cm s}^{-1}$ . Because this only corrects for 95% of the apparent variance in the  $M_2$  and  $S_2$  semi-diurnal tides, these will be further filtered using a time filter.

#### *b. Numerical wave model and estimations of Stokes drift*

As expressed by eq. (3), the estimation of  $U_{Sf}(\theta_B)$  requires the measurement or modelling of the wave spectrum  $E(f, \theta)$ . In situ buoys were moored for restricted periods at several locations for the investigation of offshore to coastal wave transformation (Ardhuin 2006) and to provide complementary data for radar validation. The radar also measures the sea state, but the coverage is often limited, and its accuracy for a 20 minute record is typically only of the order of 25% for the significant wave height  $H_s$ . Thus, in order to use the full current time series at the offshore location (point A) we have to estimate the sea state using a numerical wave model.

The model used is an implementation of the WAVEWATCH III code, in its version 3.14 (Tolman 2007, 2008), with minor modifications of the parameterizations,

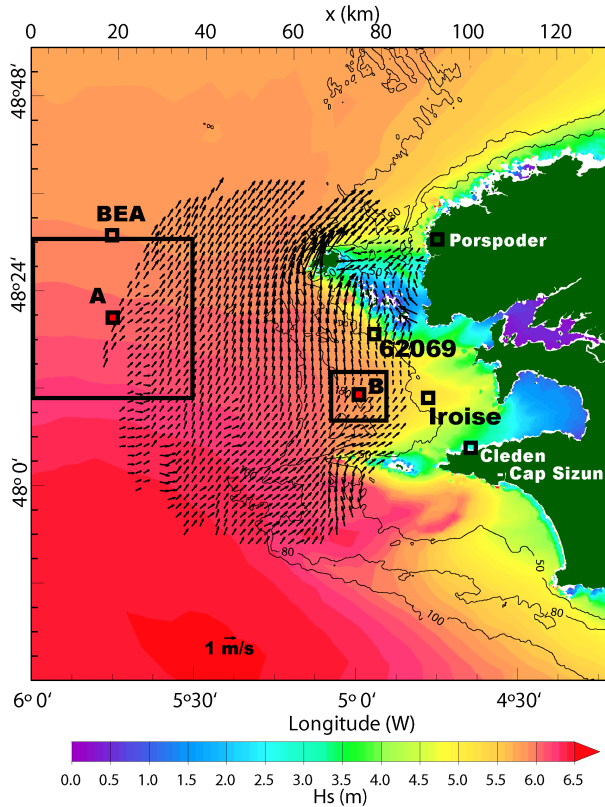


FIG. 1. Map of the area showing a map of significant wave height on January 1st 2008, at 12:00 UTC, estimated with a numerical wave model (see Appendix B), and the instantaneous surface current measured by the H.F. radars installed at Porspoder and Cléden-Cap-Sizun. In situ measurement stations include the weather buoy BEAtrice and the Pierre Noires (62069) directional Datawell waverider buoy (installed from November 2005 to March 2006 and back again since January 2008), and a previous waverider deployment (Iroise), more representative of the offshore wave conditions. The large black square around point A is the area over which the radar data has been compiled to provide the time series analyzed here, representative of offshore conditions.

see appendix B, and the addition of advection schemes on unstructured grids.

The model setting consists of a two-way nested pair of grids, covering the global ocean at 0.5 degree resolution and the Bay of Biscay and English channel at a resolution of 0.1 degree. A further zoom over the measurement area is done using an unstructured grid with 8429 wet points (figure 1). The model setting is fully described in appendix B.

In practice,  $U_{Sf}$  is dominated by the first term  $U_{ss}(f_B)$ , in eq. (3). Examining a large number of spectral data (6 buoys for 2 years spanning a range of wave climates, see appendix C), we realized that  $U_{ss}(f_B)$  is essentially a function of the wind speed  $U_{10}$  and the wave height  $H_s$ . While  $U_{10}$  explains typically only 50% of the variance of  $U_{ss}(f)$  with  $0.3 < f < 0.5$ ,  $U_{10}$  and  $H_s$  generally explain over 85% of the variance. This behaviour of  $U_{ss}(f)$  is similar to that of the fourth spectral moment,

related to the surface mean square slope (Gourrion et al. 2002; Vandemark et al. 2004). The reason for this correlation is that the wind speed is obviously related to the high frequency part of the wave spectrum, which determines most of the Stokes drift, while  $H_s$  is a surrogate variable for both the presence of swell and the stage of development of the wind sea. Here we find,

$$U_{ss}(f_c) \simeq 5.0 \times 10^{-4} \left[ 1.25 - 0.25 \left( \frac{0.5}{f_c} \right)^{1.3} \right] U_{10} \times \min \{ U_{10}, 14.5 \} + 0.025 (H_s - 0.4). \quad (4)$$

The relationship given by eq. (4) appears to be very robust, with a  $2.6 \text{ cm}^{-1} \text{ r. m. s.}$  difference compared to global hindcast values of  $U_{ss}(\infty)$ , which is a 16.9% difference. Nevertheless, when compared to buoy data, an accurate wave model generally provides a better fit to the observations (Appendix C).

We have no wave measurement at point A, and no permanent spectral measurement in the area. A detailed validation of  $U_{ss}$  was thus performed for the U.S. Northwest Pacific Coast (appendix C). The present model estimates of  $H_s$  are more accurate at buoy 62163, located 150 km west of point A, than at Pacific buoy locations. Further, the model estimate of the fourth moment  $m_4$  of the wave spectrum is better correlated in the Bay of Biscay to radar altimeter C-band cross-section, compared to other regions of the world ocean (Appendix C). We thus expect the model estimate of  $U_{ss}(f_B = 0.36 \text{ Hz})$  to have a bias smaller than 5%, with a random error less than 20% (see Appendix C). As a result, We chose to use this numerical wave model for the estimation of  $U_{ss}$  and  $U_{Sf}$ .

### 3. Analysis of wind-driven flows

#### a. Rotary spectral analysis

The rotary spectral analysis gives both the frequency distribution of the signal, and an indication of its circular polarization (Gonella 1971). The positive frequencies correspond to counter-clockwise motions, and the negative frequencies correspond to clockwise motions, the usual polarization of inertial motions in the Northern Hemisphere.

The instantaneous measurements of the radar are dominated by tidal currents, and the variance of motions with frequencies less than 1.75 count per day (cpd) only accounts for 8% of the total variance (figure 2). These low frequency motions include the diurnal tidal constituent, most importantly  $K_1$  and  $O_1$ , but these only account for 0.1% of the variance. Measured currents also contain a mean circulation not related to the wind forcing. Based on ADCP data acquired in the area over 3 months in the Fall 2007, this mean current is expected to be of the order of 10 cm/s with a subsurface maximum when the surface is stratified. The low frequency motions are generally

dominated by near-inertial motions, which are polarized clockwise with frequencies close to the inertial frequency  $f_I = 1.3$  counts per day (c.p.d.).

### b. Co-spectral analysis

Here we investigate the relationship between measured currents, processed as described above, and winds, taken from 6-hourly wind analyses from ECMWF. These analyses were verified to give excellent correlation ( $r \simeq 0.92$ ) with the BEA buoy (WMO code 62052), which unfortunately malfunctioned during large periods of time. The wind and current data are thus completely independent. The wave model was forced by these same winds, and thus the high level of coherence between the predicted Stokes drift and the wind (figure 3) is not surprising.

In order to isolate the wind-correlated dynamics from the shorter (tide) and longer (general circulation) time scales, we first perform a co-spectral analysis of the measured currents with the wind, following the method of Gonella (1971). In order to keep as much data as possible between data gaps, the Fourier transforms are taken over 264 hours, which corresponds to 21  $M_2$  tidal cycles. The measured currents are significantly coherent with the wind vector over the range  $-1.75$  to  $1.75$  cpd (figure 3). This coherence is generally reduced when the Stokes component  $U_{Sf}$  is subtracted from the radar measurements.

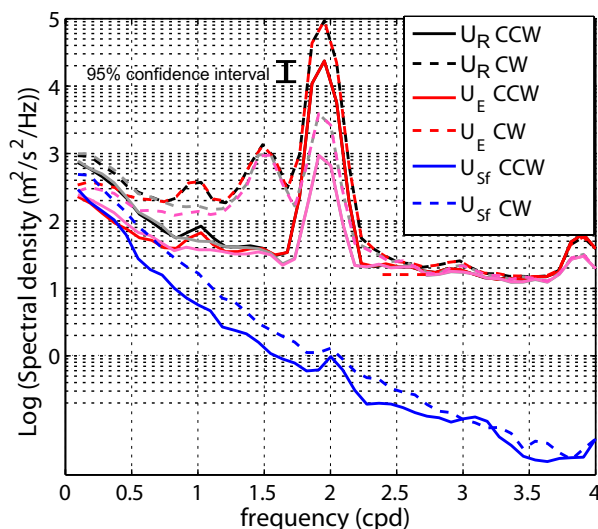


FIG. 2. Rotary power spectra of the current measured by the radar, and the contribution  $U_{Sf}$  to the surface Stokes drift estimated via eq. (A1). Clockwise (CW) motions are shown with solid lines and counter-clockwise motions are shown with dashed lines. The spectra were estimated using half-overlapping segments 264 h long over the parts of the time series with no gaps. The number of degrees of freedom is taken to be the number of non-overlapping segments, i.e. 59, at the spectral resolution of 0.09 cpd, giving a relative error of 35% at the 95% confidence level. The lighter pink and grey curves correspond to  $U_R$  and  $U_E$  from which the tidal components with frequencies larger than  $1 \times 10^{-6}$  Hz have been removed. This tide-filtered time series is used in all of the following.

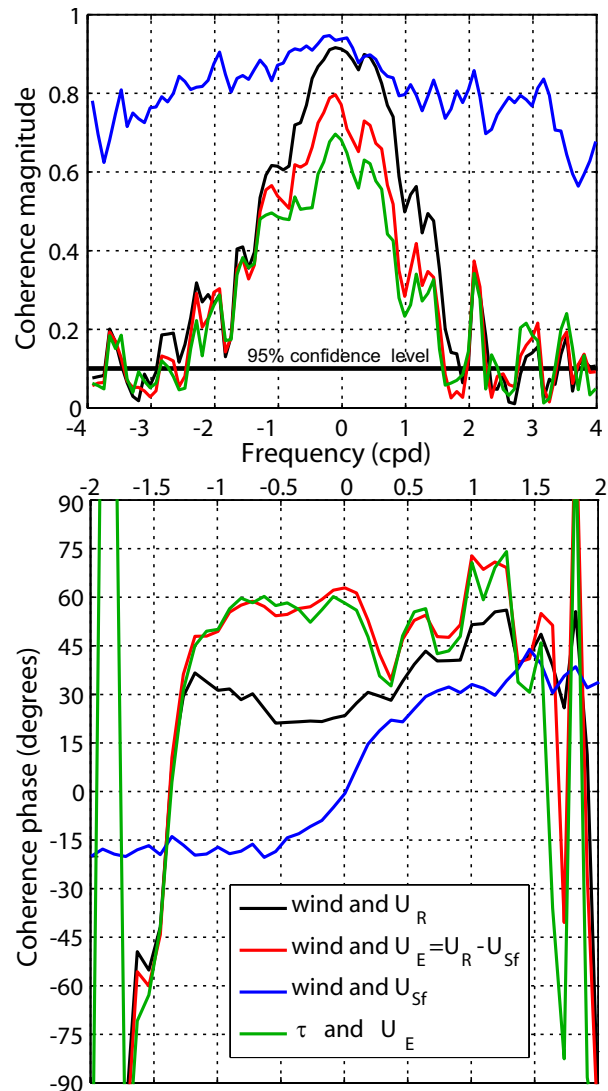


FIG. 3. Rotary co-spectra of the wind and wind stress with the radar-derived current, Stokes drift and Eulerian current. (a) magnitude and (b) phase. The number of degrees of freedom is 108 at the spectral resolution of 0.09 cpd. Coherence is significant at the 95% confidence level for a value of 0.1. Negative and positive frequencies are clockwise and counter-clockwise polarized motions, respectively.

The radar-measured current vectors  $U_R$  have stable directions relative to the wind, 20 to 40° to the right for  $f > -f_I$ , given by their coherence phase (figure 3). The coherence phase of the Stokes drift increases with frequency. This pattern is typical of a time lag, that can be estimated to about 1.5 hours, due to the relatively slow response of the wave field compared to the current. This is rather short compared to the time scale of wave development, but one should bear in mind that the Stokes drift is mostly due to short waves that respond faster to the wind forcing than the dominant waves. Because the wind preferentially turns clockwise, the Stokes drift is slightly to

the left of the wind. The asymmetry in the phase of  $U_{Sf}$  for clockwise and counter-clockwise motions is likely related to varying fetch when the wind turns.

As expected from the theory by Gonella (1972), the phase of the quasi-Eulerian current  $U_E$  jumps by about  $180^\circ$  at the inertial frequency  $-f_I$ . In the frequency range from  $-1.2$  to  $0.2$  cpd, that contains 40% of the non-tidal signal,  $U_E$  is at an angle between  $45$  and  $60^\circ$  to the right of the wind. This conclusion is not much altered when one correlates the Eulerian current against the wind stress, which, for simplicity is estimated here with a constant drag coefficient,  $\tau = 1.3 \times 10^{-3} U_{10} U_{10}$ . One may argue that the theoretical filtering of the Stokes drift is not well validated. A lower bound on the estimate of  $U_{Sf}$  can be given by removing the contribution from waves shorter than the Bragg waves. This has very little impact on the estimation of  $U_E$ .

The observed coherence phases of  $U_E$  and  $U_{10}$  are similar to the values given by Gonella (1972, figure 5), based on the constant eddy-viscosity model of Ekman (1905) ... but for a depth as large as  $1/4$  of the Ekman depth. Since the radar measurements are representative of the upper 2 meters, and the mixed layer depth is most of the time of the order of the water depth (120 m), Ekman theory does not hold. Namely, the eddy viscosity is not constant, and the velocity profiles must be closer to the ones obtained with a high surface mixing such as induced by breaking waves (e.g. Mellor and Blumberg 2004; Raschle et al. 2006).

### c. Effects of stratification

Following the theory of Gonella (1972) and the previous observations by Price and Sundermeyer (1999), it is expected that the stratification has a significant effect on the surface currents. Here we used sea surface temperature time series to diagnose the presence of a stratification. Because of the strong vertical mixing year-round at the site of buoy 62069, the horizontal temperature difference between points A and point 62069 is a good indicator of the vertical stratification at point A. This temperature difference reaches up to  $2^\circ\text{C}$ , and was present in 2006, 2007 and 2008 from early July to late October, as revealed by satellite SST data. We thus separated the data records used for the spectral analysis into "stratified" and "homogeneous" records based on the date of the midpoint in these time series.

These two series show a significant difference (at the 95% confidence level) when the spectra are smoothed over  $0.3$  c.p.d. bands, with a twice larger response in the cases expected to be stratified (dashed lines, figure 4) for frequencies in the range  $-1.7$  to  $1.5$  c.p.d. Interestingly the transfer functions decrease from a peak at the inertial frequency as  $1/(f + \omega)$  where  $\omega$  is the radian frequency. This decrease is typical of slab-like behaviors that are expected in mixed layers with a much larger surface mixing (e.g. Raschle et al. 2006) than predicted by Ekman theory, or a mixed layer depth much shallower than the Ekman

depth (Gonella 1972). Ekman theory in unstratified conditions, that should apply to our winter and spring measurements, would give a much slower decrease, proportional to  $1/\sqrt{(f + \omega)}$  (Gonella 1972). Together with this

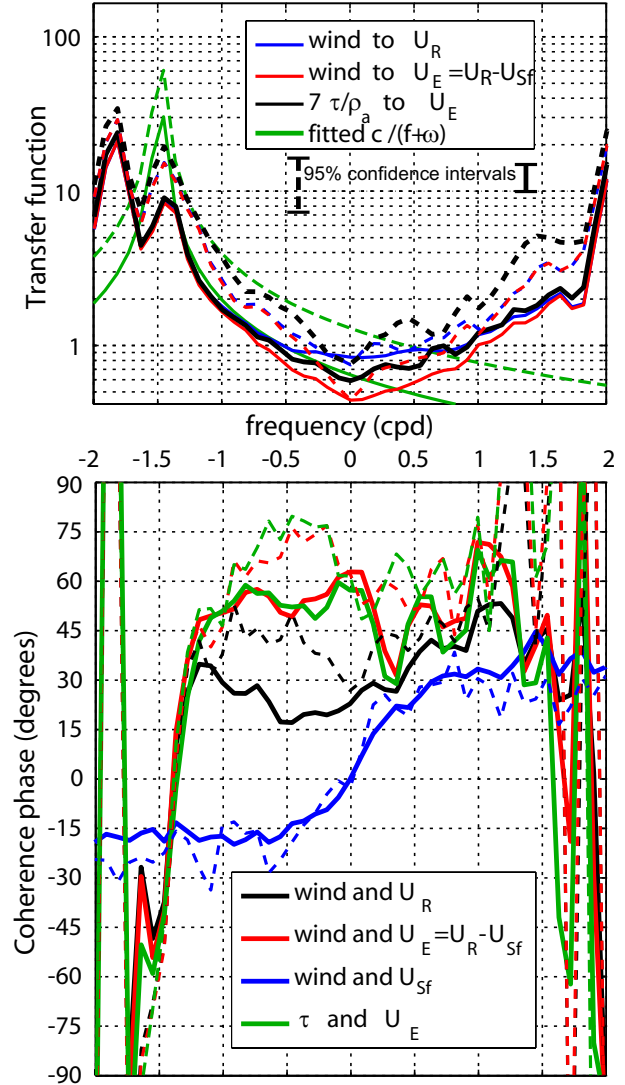


FIG. 4. Amplitude transfer functions (top) and coherence phases (bottom) between the wind forcing and the current response. The dashed lines correspond to records where a stratification is expected to be important (18 out of 108), and the solid lines correspond to the other records. Confidence intervals for the two group of records are shown for the native spectral resolution of  $0.09$  c.p.d. In order to be at a comparable level the wind stress was multiplied by 50 before estimating the transfer function. The two peaks of the transfer functions at  $\pm 2$  cpd are due to the tidal currents but do not correspond to a causal relationship between the wind forcing and the current response.

stronger amplitude of the current response in stratified conditions, we find a larger deflection angle in the  $-0.8$  to  $-0.2$  c.p.d. frequency range. This pattern of larger cur-

rents and larger deflection angles in stratified conditions is consistent with the observations of Price and Sundermeyer (1999), and the numerical model results by Raschle (2007).

#### d. Relationship between tide-filtered currents and winds

A proper model for the wind-induced current may be given by the relationship between the wind speed and wave height, giving the Stokes drift, and the complex transfer function (transfer function and phase) from the wind stress spectrum to the Eulerian current spectrum, following Gonella (1971) or Millot and Crépon (1981).

However, the transfer function is very peaked at the inertial frequency, with a large jump in phase. At any given time it is thus impossible to tell the current speed and direction just by looking at the complex transfer function, without integrating the full response to the forcing. Thus, for practical reasons, there is a long tradition, for search and rescue operations and ocean engineering applications, of directly comparing current and wind magnitudes and directions. Because of the inertial oscillations, there is usually a large scatter in the correlation of the current and wind speed vectors. In order to compare with previous analyses (e.g. Mao and Heron 2008), we thus perform such a comparison, after filtering out the dominant tidal current, by taking the inverse Fourier transform of the current, wind, and Stokes drift spectra in which the amplitudes of components with frequencies higher than 1.75 cpd, and the zero frequency, are set to zero. Again, the Fourier transforms are taken over 264 hours.

We find that the surface Eulerian  $U_E$  current lies 40 to 60° to the right of the wind, suggesting that the near-inertial motions only add scatter to the longer period motions ( $|f| < 1.3$  c.p.d.) that were found to have similar deflection angles. Interestingly, the typical magnitude of  $U_E$  decreases from about 0.8% of  $U_{10}$  at low wind to nearly 0.4% for high winds. This reduction in the relative magnitude of  $U_E$  is accompanied by a reduction of the deflection angle from 65° on average for  $U_{10} = 3 \text{ m s}^{-1}$  to 40° for  $U_{10} = 15 \text{ m s}^{-1}$ . On the contrary, the Stokes drift typically increases quadratically with the wind speed. These results contradict the usual result by Kirwan et al. (1979), Mao and Heron (2008), that the Stokes drift should be linear and the Eulerian current should be quadratic in terms of wind speed. The fact that the Stokes drift is quadratic as a function of the wind speed is well shown by observations in figure 5, and the error in Mao and Heron (2008) is likely due to their assumption that the Stokes drift is dominated by waves at the peak of the spectrum. In the analysis of Kirwan et al. (1979) and Raschle et al. (2006), the error essentially arises from the assumed shape of the wave spectrum.

The less-than-linear dependence of  $U_E$  on  $U_{10}$  contradicts the usual simple Ekman model for the quasi-Eulerian current, which would predict a current proportional to the wind stress, and thus varying as the square or cube of the wind speed. This difference is likely due

to the enhanced mixing caused by breaking waves, which tends to mix the momentum over a scale of the order of the windsea wave height, i.e. increasing with the wind speed (Mellor and Blumberg 2004; Raschle et al. 2006). Numerical models without stratification but with a realistic mixing tend to give a quasi-Eulerian current that increases with wind speed and with the inverse wave age. Here the stronger winds do not correspond to very different wave ages, and it is likely that a correlation of deeper mixed layers with stronger winds is the cause of the reduction of  $U_E$  with increasing wind speed (Raschle 2007).

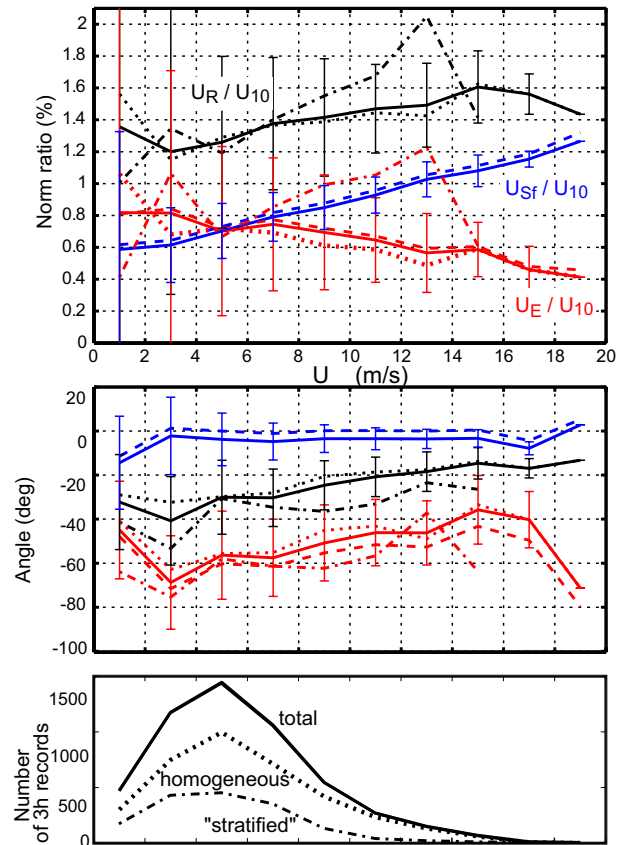


FIG. 5. Observed tide-filtered quasi-Eulerian velocities binned according to wind speed. Dash-dotted lines correspond to stratified conditions only and dotted lines correspond to homogeneous conditions. The number of data records in each of these cases is indicated in the bottom panel. The dashed line shows results when  $U_{Sf}$  is replaced by  $U_{ss}(f_B)$ . Error bars show only 1/2 of the standard deviation for all conditions combined, in order to make the plots readable. All time series (wind, current,  $U_{Sf}$  and  $U_{ss}$ ) were filtered in the same manner for consistency (except for the initial de-tiding applied only to the current data).

#### e. Effects of fetch or wave development

The same analysis was also repeated for other points in the radar field of view. For example at point B (figure 1), where the radar data quality is generally better, but



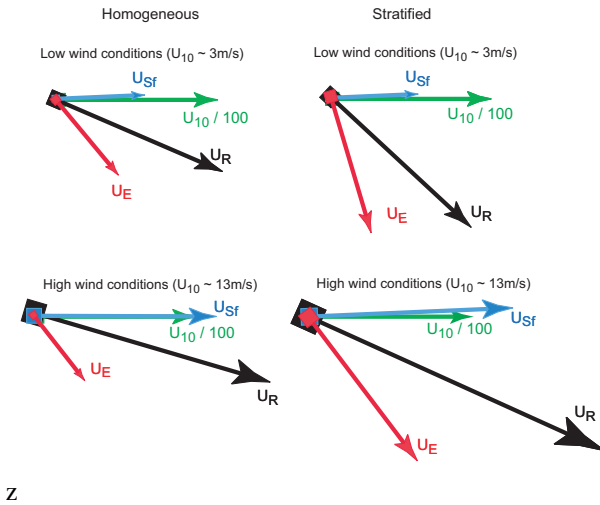


FIG. 6. Typical mean wind-correlated current vectors in low and high wind conditions, with and without stratification, measured off the West coast of France with the 12.4 MHz HF radar.  $\mathbf{U}_R$  is the radar-measured vector, that can be interpreted as a sum of a quasi-Eulerian current  $\mathbf{U}_E$ , representative of the upper two meters, and a filtered surface Stokes drift  $\mathbf{U}_{Sf}$ . The full surface Stokes drift is typically 40% larger than this filtered value.

where the wave model may have a bias of about 10% on  $U_{ss}$ , and the ECMWF wind field may be less accurate. Point B is relatively sheltered from Southerly, and North-westerly waves, and the fetch from the East is 40 km at most. If we assume that the winds are accurate at that site too, we find that the radar-derived current is weaker relative to the wind, with  $\mathbf{U}_R/U_{10}$  typically smaller by 0.2% point (i.e. a  $\sim 15\%$  reduction) compared to point A. This appears to be due to a reduction in  $\mathbf{U}_{Sf}$ , which is only partially compensated for by a small increase in  $\mathbf{U}_E$ . This difference between A and B nearly vanishes when only Westerly wind situations are considered (defined by a winds from the West with  $60^\circ$ ).

#### 4. Conclusions

Using a 2 year time series of HF radar data, and a novel numerical wave model that is shown to reproduce the observed variability of the surface Stokes drift with wind speed and wave height, we have analyzed the wind-driven surface current. When tidal currents are filtered out, we find that the measured velocities are a superposition of a filtered Stokes drift  $\mathbf{U}_{Sf}$  and a quasi-Eulerian current  $\mathbf{U}_E$ . With our 12 MHz radar,  $\mathbf{U}_{Sf}$  is of the order of 0.5 to 1.3% of the wind at 100 km from the coast, the ratio increasing linearly with wind speed. These values are a function of the radar wavelengths and would be larger, by up to 20%, with higher frequency radars that give currents representative of a shallower surface layer. The other component  $\mathbf{U}_E$  is found to be of the order of 0.6% of the wind speed, and lies, in our Northern Hemisphere, at an average 40 to 70 degrees to the right of the wind, with a large scatter

due to inertial oscillations that can be well modelled using a Laplace transform of the wind stress (Broche et al. 1983). This large deflection angle is robustly given by the coherence phase for clockwise motions in the frequency range from 0 to the inertial frequency. The magnitude of  $\mathbf{U}_E$  appears to decrease with wind speed but increases when a stronger stratification is expected (figure 5). These surface observations correspond to currents in the depth range 0 to 1.6 m, and confirm previous analysis of deeper subsurface mooring data. It thus appears that the classical picture of the Ekman spiral does not hold in the ocean, and that the surface layer is much more slab-like than assumed in many analyses, probably due to the large wave-induced mixing at the surface (Agrawal et al. 1992). This finding is very important for a proper estimation of the energy flux to the mixed layer. Instead of the wind stress work on the Ekman current, this energy flux should be dominated by the dissipation of wave energy induced by breaking (e.g. Rasclé et al. 2008), and the work of the Stokes-Coriolis force that is smaller than the Coriolis force by about a factor of 3 on average (Rasclé et al. 2008), but that may give a comparable work due to the smaller angle between that force and the quasi-Eulerian current. The accurate estimation of the surface Stokes drift using a numerical wave model also opens the way for a more accurate interpretation of space-borne measurements of surface currents using Doppler methods, that are contaminated by a Stokes-like component amplified 10 times or more (Chapron et al. 2005).

*Acknowledgments.* The efforts of Vincent Mariette and Nicolas Thomas are essential to maintain the radars in proper operating conditions. Funding for the radar purchase and maintenance was provided by DGA under the MOUTON project, and funding for the wave model development was provided under the ECORS project. Florent Birriant performed the integration of Aaron Roland's routines into the WAVEWATCH III framework. Wind and wave data were kindly provided by ECMWF, Météo-France, and the French Centre d'Etudes Techniques Maritimes Et Fluviales (CETMEF), and the sea surface temperature data used to diagnose the presence of a stratified layer was taken from the ODYSSEA Level 4 global analysis product, produced as part of the MERSEA Integrated Project. The SHOM buoy deployments were managed by David Corman with precious help from Guy Amis.

#### APPENDIX A Nonlinear correction for the wave dispersion relation in a random sea state

Based on the lowest order approximate theory of Weber and Barrick (1977) for deep water waves with  $f \simeq 2\pi\sqrt{gk}$ , the nonlinear correction to the phase speed of components with wavenumber  $k_B$  and direction  $\theta_B$ , can be expressed as an integral over the wave spectrum. Defining  $x = k/k_B$  and  $\alpha = \theta - \theta_B$ , (Broche et al.

1983, their eq. A2) give the following expression,

$$U_{Sf}(k_B, \theta_B) = \frac{\sqrt{g}}{2} k_B^{3/2} \int_0^\infty \int_0^{2\pi} F(x, \alpha) E(f, \theta) d\theta df,$$

where, correcting for typographic errors, and using  $y = x^{1/2} = f/f_B$  and  $a = \cos \alpha$ ,

$$\begin{aligned} F(x, \alpha) &= y \{2a - y + 3xa\} \\ &+ y \sum_{\varepsilon=\pm 1} \frac{\varepsilon - a}{a_\varepsilon - (1 + \varepsilon y)^2} \\ &\times \left\{ (ya - x) \left( a_\varepsilon + (1 + \varepsilon y)^2 \right) / 2 \right. \\ &\left. + (1 + \varepsilon y) (1 + \varepsilon xa + \varepsilon y (x + \varepsilon a) - a_\varepsilon) \right\}, \end{aligned} \quad (\text{A1})$$

with

$$a_\varepsilon = (1 + x^2 + 2\varepsilon xa)^{1/2}. \quad (\text{A2})$$

These expressions give the correct figures in Broche et al. (1983). For  $x < 1$  one finds that  $F(x, 0) = 4x^{3/2}$ , and for  $x > 1$ ,  $F(x, 0) = 4x^{1/2}$ , as previously given by Longuet-Higgins and Phillips (1962), Huang and Tung (1976) and Barrick and Weber (1977). As commented, by Broche et al. (1983),  $F(x, \alpha) \simeq F(x, 0) \cos \alpha$ , with the largest errors occurring for  $x = 1$  where  $F(x, \alpha) > F(x, 0) \cos \alpha$  for  $|\alpha| < \pi/3$ , which, in our case makes  $U_{Sf}$  larger by 2 to 5% than the approximation given by eq. (3).

## APPENDIX B Parameterization and numerical settings for the wave models

### a. Parameterizations

The implementation of the WAVEWATCH III model used here was ran with source functions  $S_{in}$ ,  $S_{nl}$  and  $S_{ds}$  parameterizing the wind input, nonlinear 4-wave interactions and whitecapping dissipation. An extra additional dissipation term  $S_{db}$  is also included to enhance the dissipation due to wave breaking in shallow water based on Battjes and Janssen (1978).

The parameterization for  $S_{nl}$  is taken from Hasselmann et al. (1985) and the parameterizations for  $S_{in}$  and  $S_{ds}$  are very similar the ones used by Ardhuin et al. (2008b), with modifications to further improve the high frequency part of the spectrum (Filipot et al. 2008). Namely, the whitecapping dissipation is based on recent observations of wave breaking statistics (Banner et al. 2000), and swell dissipation (Ardhuin et al. 2008a). These model settings give the best estimates so far of wave heights, peak and mean periods, but also of parameters related to the high frequency tail of the spectrum (appendix C). The present model results are thus a significant improvement over the results of Bidlot et al. (2005) and Rasclé et al. (2008). The physical and practical motivations for the parameterizations will be fully described

elsewhere, and we only give here a description of their implementation.

The parameterization of  $S_{in}$  is taken from Janssen (1991) as modified by Bidlot et al. (2005), with some further modifications for the high frequencies, and the addition of a wind output term  $S_{out}$  (or "negative wind input") based on the observations by Ardhuin et al. (2008a). The source term is thus

$$\begin{aligned} S_{in}(f, \theta) &= \frac{\rho_a \beta_{\max}}{\rho_w \kappa^2} e^Z Z^4 \left( \frac{u'_*}{C} + z_\alpha \right)^2 \\ &\times \cos^2(\theta - \theta_u) \sigma F(f, \theta) + S_{out}(f, \theta), \end{aligned} \quad (\text{B1})$$

where  $\beta_{\max}$  is a (constant) non-dimensional growth parameter,  $\kappa$  is von Kármán's constant,  $u_*$  in the friction velocity in the air,  $C$  is the phase speed of the waves,  $\sigma$  is the intrinsic frequency, equal to  $2\pi f$  in the absence of currents, and  $F(f, \theta)$  is the frequency-directional spectrum of the surface elevation variance. In the present implementation the air/water density ratio is constant. We define  $Z = \log(\mu)$  where  $\mu$  is given by Janssen (1991, eq. 16), corrected for intermediate water depths, so that

$$Z = \log(kz_1) + \kappa / [\cos(\theta - \theta_u) (u'_* + z_\alpha)], \quad (\text{B2})$$

where  $z_1$  is a roughness length modified by the wave-supported stress  $\tau_w$ , and  $z_\alpha$  is a wave age tuning parameter. The effective roughness  $z_1$  is implicitly defined by

$$U_{10} = \frac{u_*}{\kappa} \log \left( \frac{10 \text{ m}}{z_1} \right) \quad (\text{B3})$$

$$z_0 = \max \left\{ \alpha_0 \frac{u_*^2}{g}, 0.0020 \right\} \quad (\text{B4})$$

$$z_1 = \frac{z_0}{\sqrt{1 - \tau_w/\tau}}, \quad (\text{B5})$$

where  $\tau$  is the wind stress magnitude,  $\tau_w$  is the wave-supported fraction of the wind stress,  $U_{10}$  is the wind at 10 m height and  $g$  is the acceleration of gravity.

The maximum value of  $z_0$  was added to reduce the unrealistic stresses at high winds that are otherwise given by the standard parameterization. This is equivalent to setting a maximum wind drag coefficient of  $2.8 \times 10^{-3}$ . This, together with the use of an effective friction velocity  $u'_*(f)$  instead of  $u_*$  in (B2) are the only changes to the general form of Janssen's (1991) wind input. That friction velocity is defined by

$$\begin{aligned} (u'_*(f))^2 &= |u_*^2 e_\theta \\ &- |s_u| \int_0^f \int_0^{2\pi} \frac{S_{in}(f', \theta')}{C} e_{\theta'} df' d\theta'|. \end{aligned} \quad (\text{B6})$$

Here the empirical factor  $s_u = 1.0$  adjusts the sheltering effect of short waves by long waves adapted from

Chen and Belcher (2000), and helps to reduce the input at high frequency, without which a balance of source terms would not be possible (except with a very high dissipation as in Bidlot et al. 2005). This sheltering is also applied in the precomputed tables that gives the wind stress as a function of  $U_{10}$  and  $\tau_w/\tau$  (Bidlot et al. 2005).

The wind output term, is identical to the one used by Ardhuin et al. (2008b). Namely, defining the Reynolds number  $Re = 4u_{orb}a_{orb}/\nu_a$ , where  $u_{orb}$  and  $a_{orb}$  are the significant surface orbital velocity and displacement amplitudes, and  $\nu_a$  is the air viscosity, we take, for  $Re < 10^5$

$$S_{out}(f, \theta) = -1.2 \frac{\rho_a}{\rho_w} \left\{ 2k\sqrt{2\nu\sigma} \right\} F(f, \theta). \quad (B7)$$

and otherwise

$$S_{out}(f, \theta) = -\frac{\rho_a}{\rho_w} \left\{ 16f_e\sigma^2 u_{orb}/g \right\} F(f, \theta), \quad (B8)$$

where

$$f_e = 0.7f_{e,GM} + [0.015 - 0.018 \cos(\theta - \theta_u)] u_*/u_{orb}, \quad (B9)$$

where  $f_{e,GM}$  is the friction factor given by Grant and Madsen's (1979) theory for rough oscillatory boundary layers without a mean flow, using a roughness length adjusted to 0.04 times the roughness for the wind. This gives a stronger dissipation for swells opposed to winds.

The dissipation term is the sum of the saturation-based term of Ardhuin et al. (2008b) and a cumulative breaking term  $S_{ds,c}$  of Filipot et al. (2008). It thus takes the form

$$S_{ds}(f, \theta) = \sigma C_{ds} \left\{ 0.25 \left[ \max \left\{ \frac{B(f)}{B_r} - 1, 0 \right\} \right]^2 + 0.75 \left[ \max \left\{ \frac{B'(f, \theta)}{B_r} - 1, 0 \right\} \right]^2 \right\} \times F(f, \theta) + S_{ds,c}(f, \theta). \quad (B10)$$

where

$$B'(f, \theta) = 2\pi \int_{\theta-80^\circ}^{\theta+80^\circ} k^3 \cos^2(\theta - \theta') F(f, \theta') / C_g d\theta', \quad (B11)$$

$$B(f, \cdot) = \max \{ B'(f, \theta), \theta \in [0, 2\pi] \}, \quad (B12)$$

and  $B_r = 0.0009$  is a threshold for the onset of breaking consistent with the observations of Banner et al. (2000) and Banner et al. (2002), as discussed by Babanin and van der Westhuysen (2008), when including the normalization by the width of the directional spectrum (here replaced by the  $\cos^2$  factor in eq. B11).

The dissipation constant  $C_{ds}$  was adjusted to  $2.2 \times 10^{-4}$  in order to reproduce the directional fetch-limited data described by Ardhuin et al. (2007).

The cumulative breaking term represents the smoothing of the surface by big breakers with celerity  $C'$  that

wipe out smaller waves of phase speed  $C$  (Babanin and Young 2005). Due to uncertainties in the estimation of this effect from observations, we use the theoretical model of Filipot et al. (2008). Briefly, the relative velocity of the crests is the norm of the vector difference,  $\Delta_C = |C - C'|$ , and the dissipation rate of short wave is simply the rate of passage of the large breaker over short waves, i.e. the integral of  $\Delta_C \Lambda(C) dC$ , where  $\Lambda(C) dC$  is the length of breaking crests per unit surface that have velocity components between  $C_x$  and  $C_x + dC_x$ , and between  $C_y$  and  $C_y + dC_y$  (Phillips 1985). Because there is no consensus on the form of  $\Lambda$  (Gemrich et al. 2008), we prefer to link  $\Lambda$  to breaking probabilities. Based on Banner et al. (2000, figure 6), and taking their saturation parameter  $\varepsilon$  to be of the order of  $1.6\sqrt{B}$ , the breaking probability of dominant waves waves is approximately  $P = 56.8 \left( \max\{\sqrt{B} - \sqrt{B_r}, 0\} \right)^2$ . Extrapolating this result to higher frequencies, and assuming that the spectral density of crest length per unit surface  $l(\mathbf{k})$ , in the wavenumber spectral space, is  $l(\mathbf{k}) = 1/(2\pi^2 k)$ , we define a spectral density of breaking crest length,  $\Lambda(\mathbf{k}) = l(\mathbf{k})P(\mathbf{k})$ , giving the source term,

$$S_{ds,c}(f, \theta) = -c_3 F(f, \theta) \int_0^{0.7f} \int_0^{2\pi} \frac{56.3}{\pi} \times \max \left\{ \sqrt{B(f', \theta')} - \sqrt{B_r}, 0 \right\} \frac{\Delta_C}{C_g} d\theta' df' \quad (B13)$$

The tuning coefficient  $c_3$  which was expected to be of order 1, was here adjusted to 0.2. The resulting model results appear to be very accurate for sea states with significant wave heights up to 8 m. Larger wave heights are underestimated. Other parameter adjustments can correct for this defect, e.g. reducing  $s_u$  and increasing  $c_3$ , but then the Stokes drift may not be so well reproduced, especially for the average conditions discussed here. These different possible adjustments and their effects will be discussed elsewhere.

### b. Numerical schemes and model settings

Spatial advection in the finer model grid is performed using the explicit CRD-N scheme (Contour integration based Residual Distribution - Narrow stencil scheme Csík et al. 2002) that was applied to the Wave Action Equation by Roland (2008) and provided as a module for the WWIII model. The scheme is first order in time and space, it is conservative and monotone.

All model grids are forced by 6-hourly wind analysis at 0.5 degree resolution, provided by ECMWF. The model spectral grid has 24 regularly spaced directions, and extends from 0.037 to  $f_{max} = 0.72$  Hz with 32 frequencies exponentially spaced. The model thus covers the full range of frequencies that contribute most to the filtered Stokes drift  $U_{Sf}$ . The usual high frequency tail proportional to  $f^{-5}$  is only imposed for frequencies larger than the diagnostic fre-

quency  $f_d = F f_{m,0,-1}$ , with the mean frequency defined by  $f_{m,0,-1} = [\int E(f)/f df / \int E(f) df]^{-1}$ . Here we take a factor  $F = 10$ , instead of the usual value of 2.5 (Bidlot et al. 2005), so that  $f_d$  is almost always larger than the model maximum frequency of 0.72 Hz. Besides, the time step for integration of the source function is adaptively refined from 150 s for the local model down to 10 s if needed, so that virtually no limiter constrains the wave field evolution (Tolman 2002).

### APPENDIX C Model accuracy for relevant parameters

In order to define the errors on the estimations of  $U_{Sf}$  used to determine the quasi-Eulerian velocity  $U_E$  from the radar measurement, it is necessary to examine the quality of the wind forcing and model results in the area of interest. The only two parameters that are measured continuously offshore of the area of interest are the wave height  $H_s$  and mean period  $f_{02}$ , recorded at buoy 62163, 150 km to the west of point A.  $H_s$  and  $f_{02}$  can be combined to give the second moment of the wave spectrum  $m_2 = (0.25 H_s f_{02})^2$ .

Because there is no reliable wave measurement with spectral information in deep water off the French North-East Atlantic coast, we also use buoy data and model result in a relatively similar wave environment, at the location of buoy 46005, 650 km off Aberdeen (WA), on the U.S. Pacific coast. Since this buoy is not directional we first examine the third moment of the wave spectrum

$$m_3(f_c) = \int_0^{f_c} f^3 E(f) df. \quad (C1)$$

If waves were all in the same direction,  $m_3$  would be proportional to the Stokes drift  $U_{ss}(f_c)$  of waves with frequency up to  $f_c$ , as given by eq. (2). We thus define a non-directional Stokes drift

$$U_{ssnd}(f_c) = (2\pi)^3 m_3(f_c) / g. \quad (C2)$$

Looking at buoy data we found that

$$U_{ssnd}(f_c) \simeq 5.9 \times 10^{-4} \left[ 1.25 - 0.25 \left( \frac{0.5}{f_c} \right)^{1.3} \right] U_{10} \\ \times \min \{ U_{10}, 14.5 \} + 0.027 (H_s - 0.4), \quad (C3)$$

where  $f_c$  is in Hertz,  $U_{10}$  is in meters per second, and  $H_s$  is in meters.

Taking directionality into account eq. (2) yields  $U_{ss}(f_c) \simeq 0.85 U_{ssnd}(f_c)$ , for typical wave spectra, and the relationship (C3) becomes eq. (4). For buoy 46005, which is a 6 m NOMAD buoy, and  $f_c$  in the range 0.3 to 0.5 Hz, this relationship gives a root mean square (r. m. s.) error less than  $1.0 \text{ cm s}^{-1}$ , corresponding to less than

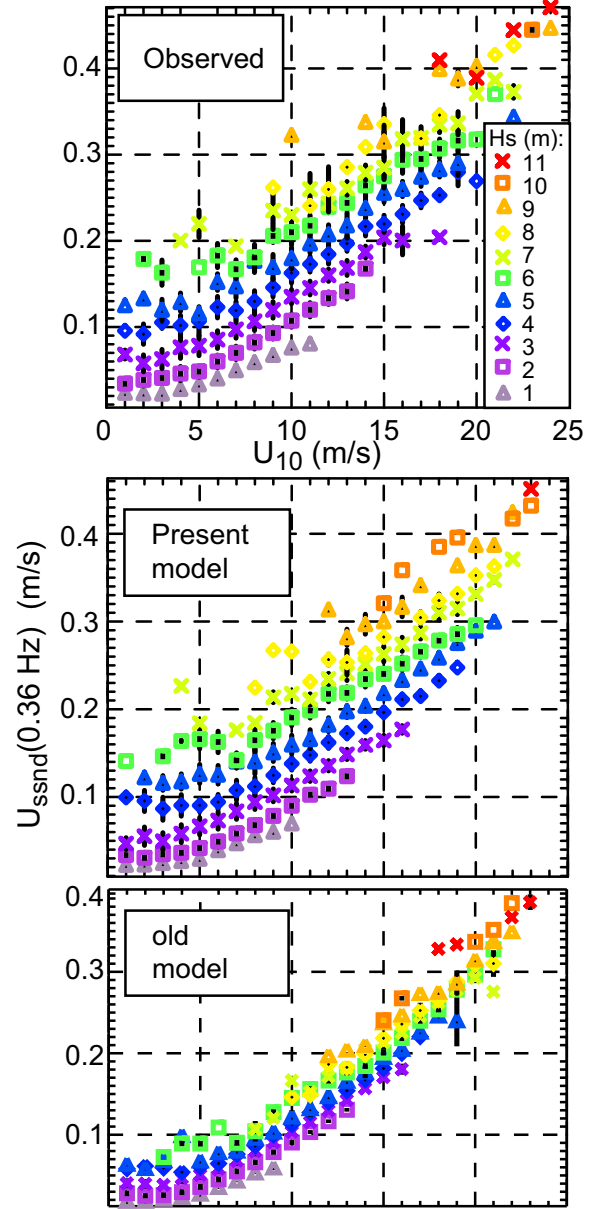


FIG. C1. Variation of the wave spectrum third moment,  $m_3$  converted to a velocity  $U_{ssnd} = (2\pi)^3 m_3(f_c) / g$ , that would equal the surface Stokes drift in deep water if all waves propagated in the same direction. For each data source a cut-off frequency of  $f_c = f_B = 0.36 \text{ Hz}$  is taken and the data is binned wind speed, at  $1 \text{ m s}^{-1}$  intervals, and significant wave height  $H_s$  (in colors) at  $1 \text{ m}$  intervals from 1 to 11 m. The top panel shows buoy data offshore of Oregon (NDBC buoy 46005), the middle panel shows present model results, and the bottom panel shows results from the same model but using the parameterization of Bidlot et al. (2005), including a factor  $F = 2.5$ . The vertical error bars indicate plus and minus half the standard deviation of the data values in each  $(U_{10}, H_s)$  class.

15% of the r. m. s. value estimated using eq. (C2). This is smaller than the error of estimates using previous wave

models (24% with the parameterization by Bidlot et al. 2005), but comparable to the 14.2% error obtained with the present model.

Table C1. Model accuracy for measured wave parameters. Buoy validation span the entire year 2007, except for buoy 62069 for which data covers the time frame 25 January to 20 August 2008, buoy Iroise covers 13 April to 20 May 2004, and JASON 1 data corresponds to January to July 2007 for the global validation (JAS-Glo: 393382 data points) and the full year for a box centered on 48.5 N and 8 W or 45 N and 128 W. (JAS-Gas or JAS-Was: 380 data points). Unless otherwise specified by the number in parenthesis, the cut-off frequency is take to be 0.5 Hz,  $C$  stands for C-band and  $f_B = 0.36$  Hz corresponds to out 12 MHz HF radar. The normalized bias (NB) is defined as the bias divided by the r.m.s. observed value, while the scatter index (SI) is defined as the r.m.s. difference between modeled and observed values, after correction for the bias, normalized by the r.m.s. observed value, and  $r$  is Pearson's correlation coefficient.

	dataset	NB(%)	SI(%)	$r$
<b>2004</b>				
$H_s$	62163	6.8	11.1	0.977
$f_{02}$	62163	10.4	8.8	0.907
$H_s$	Iroise	12.8	17.4	0.975
$f_{02}$	Iroise	-10.0	11.7	0.913
$U_{ssnd}(f_B)$	Iroise	27.2	26.9	0.968
$U_{ss}(f_B)$	Iroise	20.5	18.5	0.971
<b>2007/2008</b>				
$H_s$	JAS-Glo	-0.6	11.4	0.966
$m_4(C)$	JAS-Glo	0.6	9.1	0.939
$H_s$	62163	-1.4	8.8	0.985
$f_{02}$	62163	6.3	7.3	0.938
$H_s$	62069	10.1	14.1	0.974
$f_{02}$	62069	-7.7	11.8	0.886
$m_4(f_B)$	62069	15.8	24.1	0.955
$U_{ssnd}(f_B)$	62069	13.9	23.0	0.965
$U_{ss}(f_B)$	62069	11.1	21.0	0.963
$H_s$	JAS-Gas	-2.6	8.8	0.983
$m_4(C)$	JAS-Gas	1.0	6.7	0.962
$H_s$	46005	4.9	10.2	0.975
$f_{02}$	46005	-2.8	6.6	0.931
$m_4(f_B)$	46005	-5.4	13.5	0.965
$U_{ssnd}(f_B)$	46005	-4.9	12.6	0.973
$U_{ssnd}(0.5)$	46005	6.2	12.7	0.971
$H_s$	JAS-Was	2.4	7.9	0.985
$m_4(C)$	JAS-Was	1.8	7.3	0.953

Another source of continuous wave measurements is provided by altimeter-derived  $H_s$ , which we correct for bias following Queffeuilou (2004), and fourth spectral moment  $m_4$ . The latter is approximately given by (Vandemark et al. 2004)

$$m_4 = \frac{0.64g^2}{(2\pi)^4\sigma_0}, \quad (C4)$$

where  $\sigma_0$  is the normalized radar cross-section, corrected for a 1.2 dB bias on the C-band altimeter of JASON in order to fit airborne observations (Hauser et al. 2008). The model estimation of  $m_4(0.72$  Hz) is extrapolated to C-band by the addition of a constant  $0.011g^2/(2\pi)^4$ , con-

sistent with the saturation of the short wave slopes observed by Vandemark et al. (2004). For this parameter, the model is found to be very accurate, especially around the region of interest, relatively more than on the U.S. Pacific coast.

These indirect validations suggest that the third spectral moment including waves up to the Bragg frequency  $f_B = 0.36$  Hz, which is proportional to  $U_{ssnd}$ , is probably estimated with bias between -5 and 10%, and an r.m.s. error less than 20%. The bias on the significant wave height appears to increase from offshore (altimeter and buoy 62163 data), to the coast (buoys Iroise and 62069), and we attribute this effect to the tidal currents, not included in the present wave model, and coastal modifications of the winds that are not well reproduced at this 10-20 km scale by the ECMWF model. Because the chosen area of interest lies offshore of the area where currents are strongest (figure 1), we shall assume that, at this site, the model bias on  $U_{ss}(f_B)$  is zero, which appears most likely, or possibly weakly negative. Extreme biases of  $\pm 10\%$  only result in deflections of 5 degrees on the diagnosed quasi-Eulerian current  $U_E$ .

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**Fabrice's Draft:** October 20, 2008

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